MOMENTS

The r^{th} central moment of a variable x about the mean $\bar{x}$ is usually denoted by μ_r is given by

$$\mu_r = \frac{1}{N} \sum_{i=1}^{n} f_i (x_i - \bar{x})^r \qquad \sum f_i = N$$

The r^{th} moment of a variable x about any point a is defined by

$$\mu'_r = \frac{1}{N} \sum_{i=1}^{n} f_i (x_i - a)^r$$

Moments about Mean →

Let $\bar{x}$ be the arithmetic mean, then

$$\mu_r = \frac{1}{N} \sum_{i=1}^{n} f_i (x_i - \bar{x})^r \qquad r = 0, 1, 2, 3 \text{ —}$$

where $N = \sum_{i=1}^{n} f_i$

If $r = 0$ $\quad \mu_0 = \frac{1}{N} \sum_{i=1}^{n} f_i (x_i - \bar{x})^0 = \frac{1}{N} \cdot N \cdot 1 = 1$

If $r = 1$ $\quad \mu_1 = \frac{1}{N} \sum_{i=1}^{n} f_i (x_i - \bar{x})^1$

If $r = 2$, $\quad \mu_2 = \frac{1}{N} \sum_{i=1}^{n} f_i (x_i - \bar{x})^2$ $\qquad \mu_2 =$ Variance

If $r = 3$ $\quad \mu_3 = \frac{1}{N} \sum_{i=1}^{n} f_i (x_i - \bar{x})^3$

If $r = 4$ $\quad \mu_4 = \frac{1}{N} \sum_{i=1}^{n} f_i (x_i - \bar{x})^4$

Moments about any Number (Raw Moments)

Let a be a arbitrary number then

$$\mu'_r = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - a)^r \qquad r = 0, 1, 2, 3 \text{ ---}$$

where $N = \sum_{i=1}^{n} f_i$

If $r = 0$, $\mu'_0 = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - a)^0 = 1$

$$\Rightarrow \boxed{\mu'_0 = 1}$$

If $r = 1$ $\mu'_1 = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - a)^1$

$$= \frac{1}{N}\sum_{i=1}^{n} f_i x_i - \frac{a}{N}\underbrace{\sum_{i=1}^{n} f_i}_{=N}$$

$$= \bar{x} - \frac{a}{N} \cdot N$$

$$\boxed{\mu'_1 = \bar{x} - a}$$

If $r = 2$ $\mu'_2 = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - a)^2$

If $r = 3$ $\mu'_3 = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - a)^3$

If $r = 4$ $\mu'_4 = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - a)^4$

Moments about the origin →

$$\nu_r = \frac{1}{N} \sum_{i=1}^{n} f_i x_i^r$$

If $r=0$ $\quad \nu_0 = \frac{1}{N} \sum_{i=1}^{n} f_i x_i^0 = \frac{1}{N} \sum_{i=1}^{n} f_i \cdot 1 = \frac{N}{N} = 1$

If $r=1$ $\quad \nu_1 = \frac{1}{N} \sum_{i=1}^{n} f_i x_i = \bar{x}$

If $r=2$ $\quad \nu_2 = \frac{1}{N} \sum_{i=1}^{n} f_i x_i^2$

If $r=3$ $\quad \nu_3 = \frac{1}{N} \sum_{i=1}^{n} f_i x_i^3$

If $r=4$ $\quad \nu_4 = \frac{1}{N} \sum_{i=1}^{n} f_i x_i^4$