

# Normal Distribution or Gaussian Distribution →

The normal or (Gaussian) Distribution is defined by the probability density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty$$

where  $\mu$  and  $\sigma > 0$  are the parameters of the distribution.

Clearly (i)  $f(x)$  is non negative

$$f(x) \geq 0$$

(ii)  $\int_{-\infty}^{\infty} f(x) dx = 1$  i.e. total area under the normal curve above the X-axis is

Proof →

$$\int_{-\infty}^{\infty} f(x) dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2} dz$$

let  $z = \frac{x-\mu}{\sigma}$

$$= \frac{2}{\sigma\sqrt{2\pi}} \int_0^{\infty} e^{-\frac{1}{2}z^2} dz$$

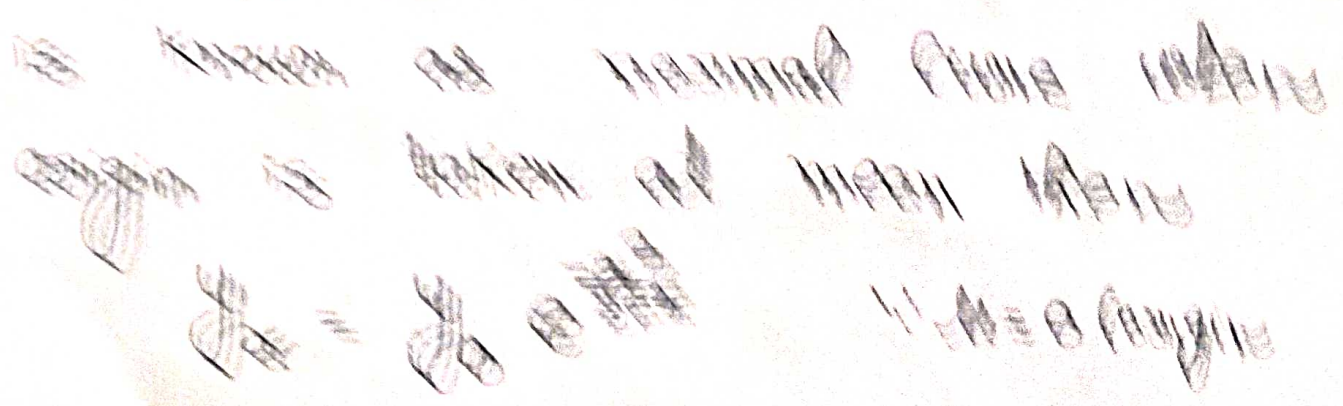
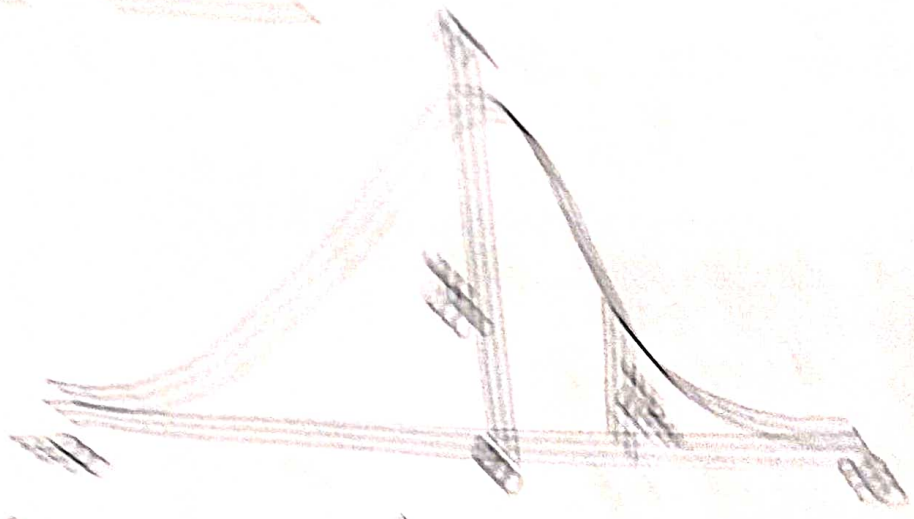
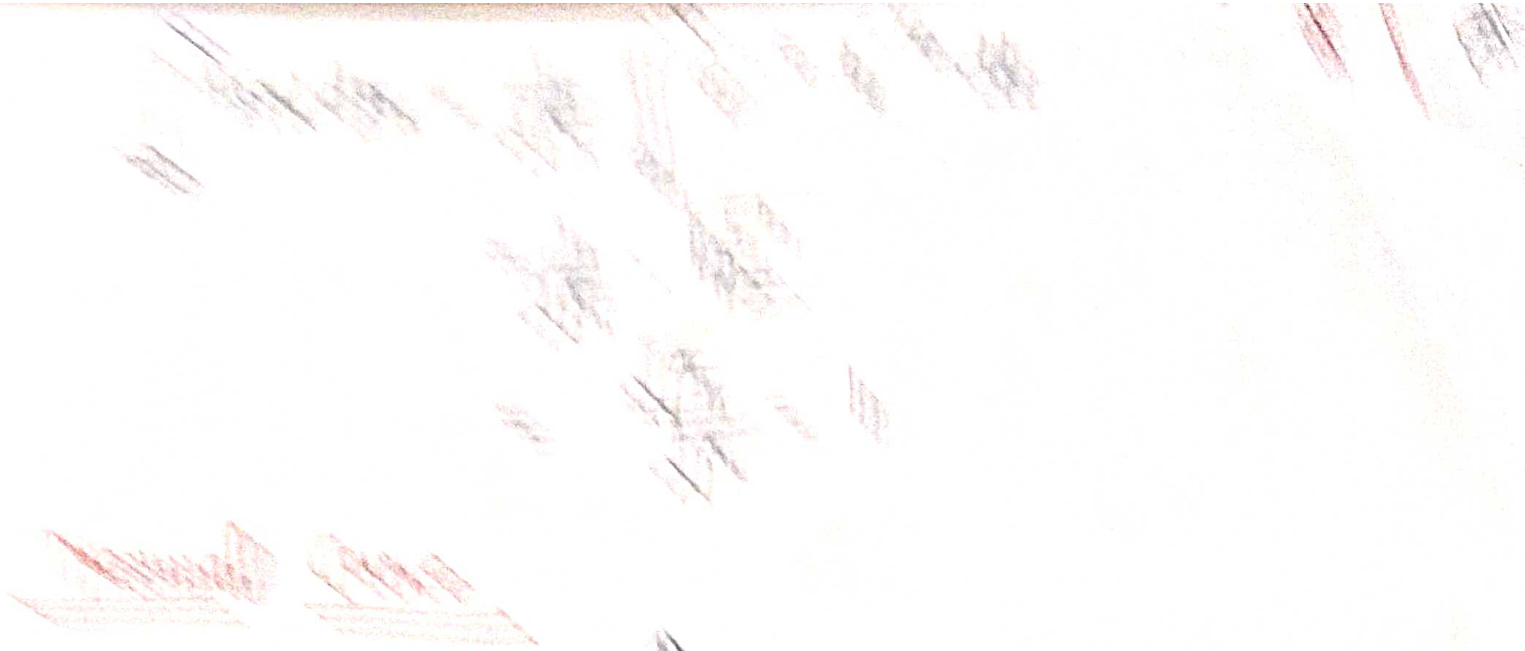
$\sigma dz = dx$   
 $\int_{-\infty}^{\infty} f(x) dx = 2 \int_0^{\infty} f(x) dx$

$$= \frac{2}{\sigma\sqrt{2\pi}} \int_0^{\infty} e^{-\frac{1}{2}z^2} dz$$

let  $\frac{1}{2}z^2 = t$

$$= \frac{2}{\sigma\sqrt{2\pi}} \int_0^{\infty} e^{-t} \cdot \frac{dt}{\sqrt{2t}}$$

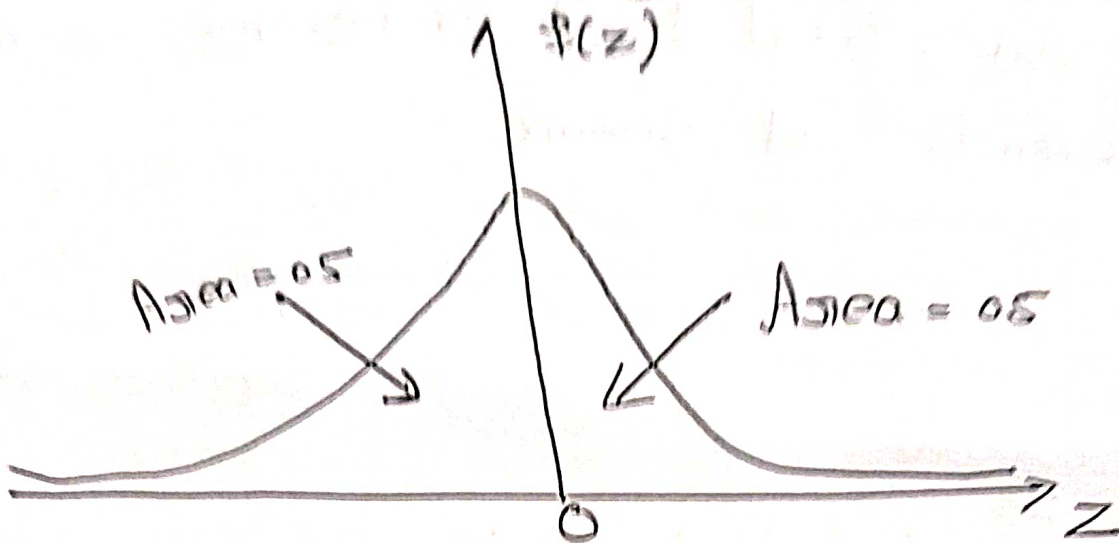
$\frac{1}{2} \cdot 2z dz = dt$   
 $z dz = \frac{dt}{\sqrt{2}}$   
 $dz = \frac{dt}{\sqrt{2t}}$



## Area Under Normal Curve

By taking  $Z = \frac{x - \mu}{\sigma}$ , standard normal curve is formed.

The total area under this curve is 1.



The area under the curve is divided into two equal parts by  $Z=0$ .

The area between the ordinate  $Z=0$  and any other ordinate can be noted from the supplied table.

It should be noted that mean of the Normal Distribution is 0