FOURIER SERIES

Periodic Functions →

A function $f(x)$ which satisfy the relation $f(x+T) = f(x)$ for all real x and some fixed T is called a periodic function.

The smallest positive number T, for which this relation holds, is called the period of $f(x)$.

If T is the period of $f(x)$ then

$$f(x) = f(x+T) = f(x+2T) = \cdots = f(x+nT) = \cdots$$

Also $f(x) = f(x-T) = f(x-2T) = \cdots = f(x-nT) = \cdots$

$\therefore f(x) = f(x \pm nT)$ where n is a positive integer

Thus, $f(x)$ repeats itself after periods of T

For example, $\sin x$, $\cos x$, $\sec x$ and $\operatorname{cosec} x$ are periodic functions with period 2π

Since $$\tan(\theta+\pi) = \frac{\sin(\pi+\theta)}{\cos(\pi+\theta)} = \frac{-\sin\theta}{-\cos\theta} = \tan\theta.$$

and $$\cot(\theta+\pi) = \frac{\cos(\pi+\theta)}{\sin(\pi+\theta)} = \frac{-\cos\theta}{-\sin\theta} = \cot\theta$$

Therefore $\tan\theta$, and $\cot\theta$ are periodic function with period π.

The function $\sin nx$ and $\cos nx$ are periodic with period $2\pi/n$

The sum of a number of functions is also periodic.

If T_1 and T_2 are the periods of $f(x)$ and $g(x)$ then the period of $af(x) + bg(x)$ is the least common multiple of T_1 and T_2.

For Example, $\cos x$, $\cos 2x$, $\cos 3x$ are periodic functions with period 2π, $\frac{2\pi}{2} = \pi$, and $\frac{2\pi}{3}$ respectively.

$\therefore f(x) = \cos x + \frac{1}{2}\cos 2x + \frac{1}{3}\cos 3x$ is also periodic with period 2π, the L.C.M of 2π, π and $\frac{2\pi}{3}$.

FOURIER

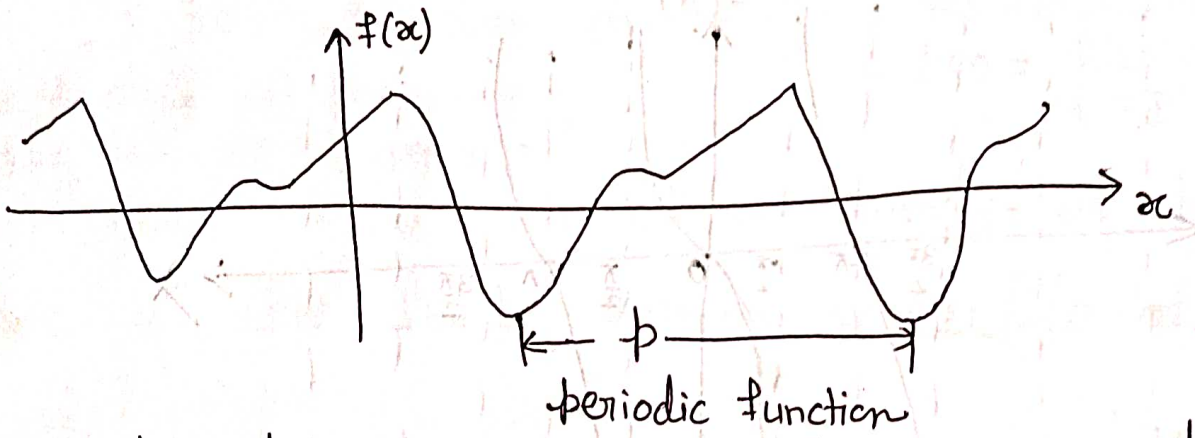
Periodic Functions → A function $f(x)$ is said to be periodic if

$$f(x+T) = f(x)$$

for all real x and some positive number T.

T is called the period of $f(x)$.

$$\Rightarrow f(x) = f(x+T) = f(x+2T) = \text{- - - - - } = f(x+nT)$$



periodic function

Familiar periodic functions are the sine and cosine functions.

Example of function that are not periodic are $x, x^2, x^3, e^x, \cosh x$ and $\ln x$

Example → $\sin x = \sin(x+2\pi) = \sin(x+4\pi) = \text{- - - } = \sin(x+n\pi)$

is a periodic function with period 2π