POWER SERIES

Power Series →

A series of the form $\sum_{n=0}^{\infty} a_n z^n$ or $\sum_{n=0}^{\infty} a_n (z-a)^n$ is called a power series, where a_n, a are called complex constants and z is a complex variable.

The second form namely $\sum a_n (z-a)^n$ can be reduced to the first form by the substitution $z = \xi + a$ so that

$$\sum a_n (z-a)^n = \sum a_n \xi^n$$

The first form is simpler than the second form. Hence we will consider only the first form $\sum_{n=0}^{\infty} a_n z^n$ or simply $\sum a_n z^n$ in our discussion.

Theorem 1 Cauchy - Hadamard Theorem →

Every power series $\sum a_n z^n$, there exists a number R such that $0 \le R \le \infty$ with the following properties:

(i) the series converges for every z such that $|z| < R$.

(ii) the series diverges for every z such that $|z| > R$.

Circle of Convergence → The circle $|z| = R$ such that the power series $\sum a_n z^n$ is convergent for every z within it is called the circle of convergence of the series.

Radius of Convergence → The number R such that the power series $\sum a_n z^n$ is convergent inside the circle $|z| = R$ is called the radius of convergence of the series.

Thus the radius of the circle of convergence is the radius of convergence of the series.

There are three possibilities for R

(i) $R = 0$, In this case, the series is convergent only at $z = 0$

(ii) R is finite and positive. In this case, the series is convergent at every point within the circle $|z| < R$ and is divergent at every point outside it.

(iii) R is infinite. In this case, the series is convergent for all value of z.