

In a test an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability he makes a guess is  $\frac{1}{3}$  and the probability that he copies the answer is  $\frac{1}{6}$ . The probability that his answer is correct, given that he copied it, is  $\frac{1}{8}$ . Find the probability that he knew the answer to the question, given that he correctly answered it.

Solution  $\rightarrow$  Let  $A_1$  be the event that the examinee guesses the answer  
 $A_2$  be the event that he copies the answer  
 $A_3$  be the event that he knows the answer.  
 Also let  $B$  be the event that ~~the~~ he answers correctly. Then, as given, we have

$$P(A_1) = \frac{1}{3} \quad P(A_2) = \frac{1}{6} \quad P(A_3) = 1 - \frac{1}{3} - \frac{1}{6} = \frac{1}{2}$$

Here we have assumed that the events  $A_1$ ,  $A_2$  and  $A_3$  are mutually exclusive & exhaustive. Now

$$P(B|A_1) = \frac{1}{4} \text{ since there are 4 choices of the question}$$

$$P(B|A_2) = \frac{1}{8} \text{ as given \& } P(B|A_3) = 1$$



We need to find  $P(A_3|B)$

Using Bayes theorem, we have

$$\begin{aligned} P(A_3|B) &= \frac{P(A_3) \cdot P(B|A_3)}{P(A_1) \cdot P(B|A_1) + P(A_2) \cdot P\left(\frac{B}{A_2}\right) + P(A_3) \cdot P\left(\frac{B}{A_3}\right)} \\ &= \frac{\frac{1}{2} \times 1}{\frac{1}{3} \times \frac{1}{4} + \frac{1}{6} \times \frac{1}{8} + \frac{1}{2} \times 1} \\ &= \frac{24}{29} \end{aligned}$$