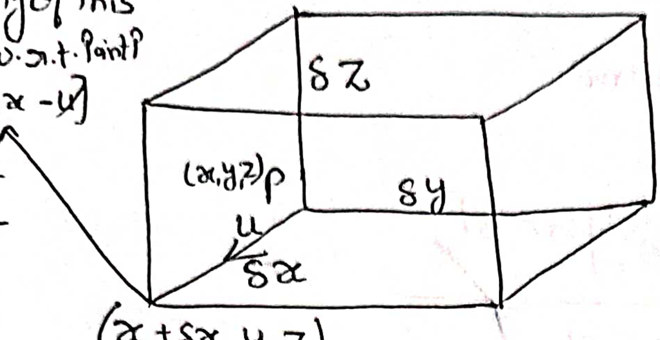
...ation Between Stress and Rate of Strain

Velocity of this corner w.r.t. Point P
$\left[u + \frac{\partial u}{\partial x}.\delta x - u\right]$
$= \frac{\partial u}{\partial x}.\delta x$
$= a\delta x$



$(x+\delta x, y, z)$

velocity $\left(u + \frac{\partial u}{\partial x}.\delta x\right)$

$$\frac{\partial u}{\partial x} = a$$

$$\frac{\partial v}{\partial y} = b$$

$$\frac{\partial w}{\partial z} = c$$

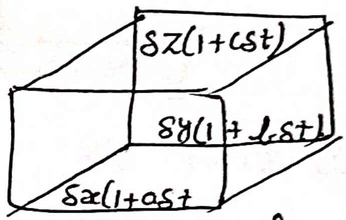
at time δt, the ~~Area~~ edge is

$\delta x + a.\delta x.\delta t$

$\delta x(1 + a\delta t)$

Thus Volumetric increment in time (or in interval δt)

δt = Volume at time $(t+\delta t)$ − volume at time t



$\delta x(1+a\delta t).\delta y(1+b\delta t).\delta z(1+c\delta t) - \delta x\delta y\delta z$

volume $= \delta x(1+a\delta t).\delta y(1+b\delta t)$
$.\delta z(1+c\delta t)$
at time $t+\delta t$

$= (\delta x + a\delta t\delta x)(\delta y + b\delta t\delta y)(\delta z + c\delta t\delta z) - \delta x\delta y\delta z$

$= (\delta x\delta y + b\delta x\delta y\delta t + a\delta x\delta y\delta t + ab(\delta t)^2\delta x\delta y)(\delta z + c\delta t\delta z) - \delta x\delta y\delta z$

$= \cancel{\delta x\delta y\delta z} + b\delta x\delta z\delta z\delta t + a\delta x\delta y\delta z\delta t + ab(\delta t)^2\delta x\delta y\delta z$
$+ c\delta x\delta y\delta z\delta t + bc\delta x\delta y\delta z(\delta t)^2 + ac\delta x\delta y\delta z(\delta t)^2 + ab(\delta t)^3 \delta x\delta z\delta z$
$- \cancel{\delta x\delta y\delta z}$

$\simeq (a+b+c)\,\delta x\,\delta y\,\delta z\,\delta t$

Volumetric strain or ~~dilatation in time~~ dilatation in time δt

$$= \frac{\text{change in volume}}{\text{original volume}}$$

$$= \frac{(a+b+c)\,\delta x\,\delta y\,\delta z\,\delta t}{\delta x\,\delta y\,\delta z}$$

$$= (a+b+c)\,\delta t$$

Rate of Volumetric strain $(\Delta) = \dfrac{(a+b+c)\cancel{\delta t}}{\cancel{\delta t}}$

$$= a+b+c$$

$$\Delta = a+b+c$$

$$\Delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

If u, v, w are components of $\vec{q}$ then for incompressible fluid

$$\Delta = \operatorname{div} \vec{q} = 0$$

for compressible fluid

$$\Delta = \operatorname{div} q \neq 0$$

This quantity is invariant $= \operatorname{div} \vec{q}$ at each point of the fluid, its value is also $A+B+C$

also $\Delta = a+b+c = A+B+C$ in terms of principal rates of strain

where A, B, C are principal rate of strain and

a, b, c are non principal rate of strain. We know that Equation of Continuity for incompressible fluid is $\Delta = 0$, but for a compressible fluid $\Delta \neq 0$