Sampling Theorem

- Sampling is Process of converting analog signal in to a corresponding sequence of numbers(Digital form) that are uniformly spaced in time.

- For efficient Process ,the sampling rate(the distance between sample to sample) should be chosen properly and is explained by Sampling Theorem.

Sampling Theorem

Part 1: a band limited signal of finite energy ,which has no frequency components higher than fm Hz is completely described by specifying the values of the signal at instants of time separated by 1/2fm seconds

Part 2: a band limited signal of finite energy ,which has no frequency components higher than fm Hz may be completely recovered from the knowledge of its samples taken at the rate of separated by 2fm per seconds

Combining the two parts, the sampling theorem states that

A continuous time signal, Band limited to f_m Hz, may be completely represented in its samples and recovered back from the knowledge of its samples taken if the sampling frequency is $f_s >= 2f_m$.

Here f_s is the sampling frequency f_m is the maximum frequency present in the signal

Proof

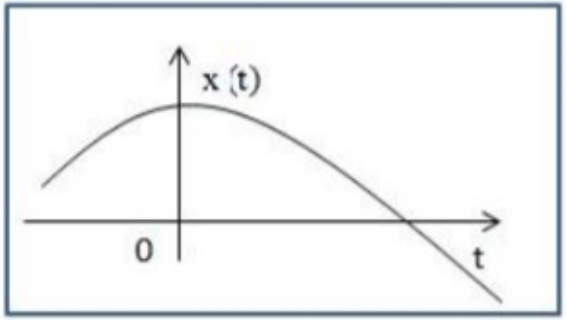


Fig (a) Modulating signal

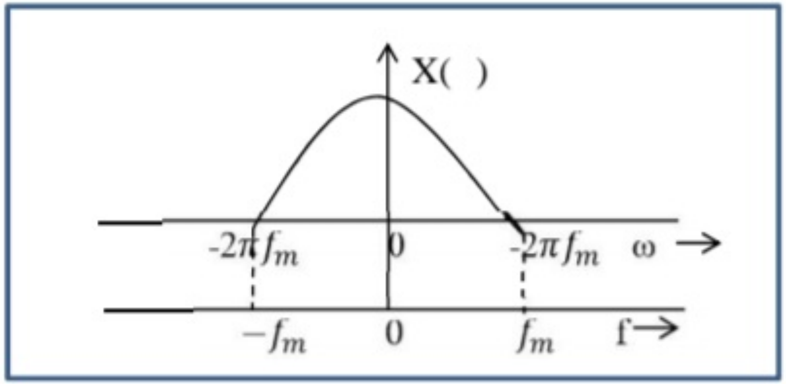


Fig (b) Spectrum

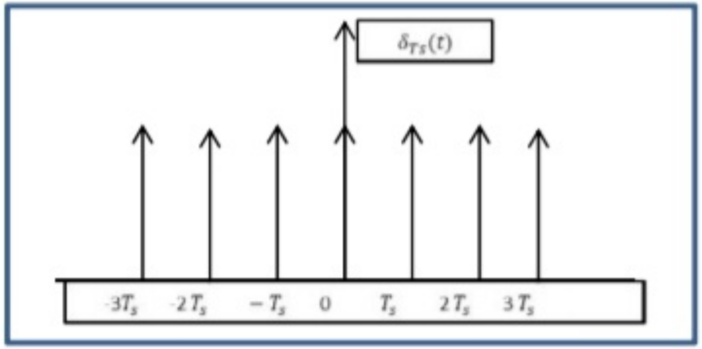


Fig (c) Impulse train

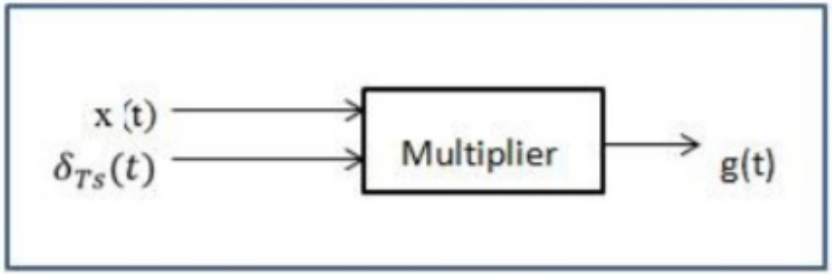


Fig (d)

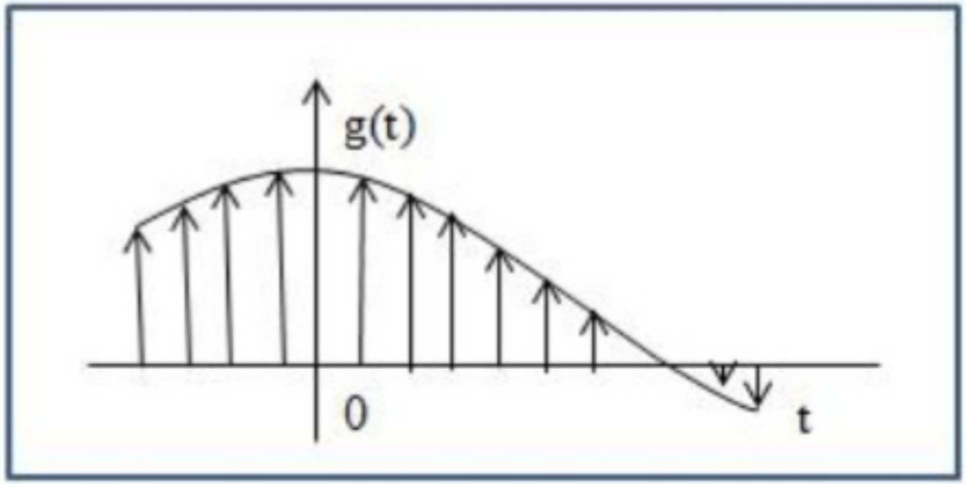
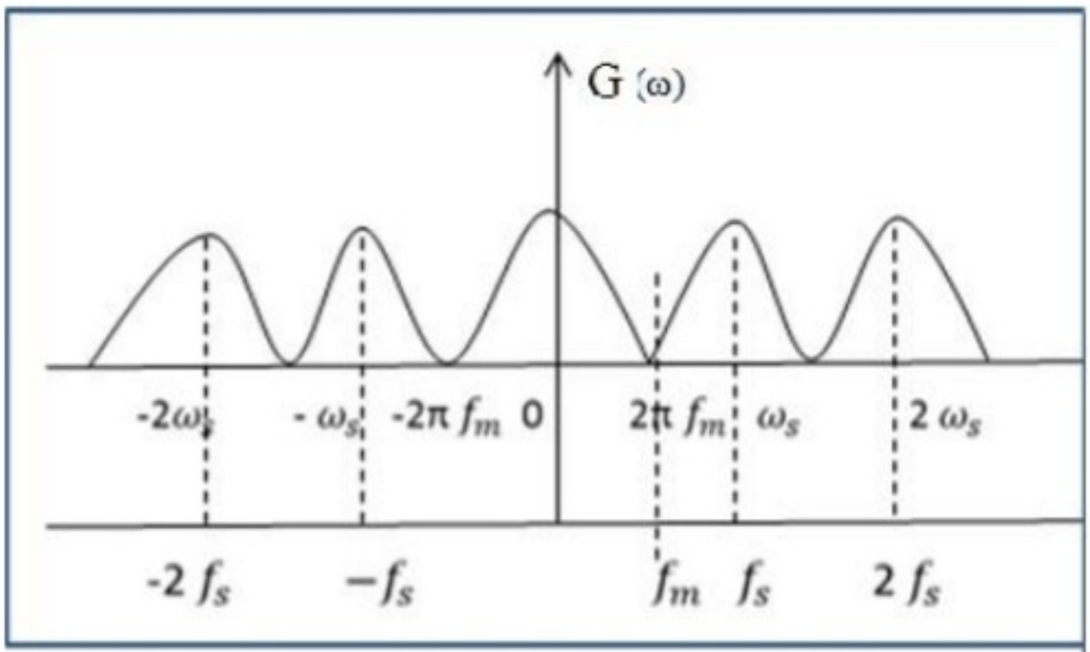


Fig (e) Sampled signal



$$g(t) = x(t).\delta_{TS}(t) \longrightarrow \quad (1)$$

$$\delta_{TS}(t) = \frac{1}{T_s}[1 + 2cos\omega_s t + 2cos2\omega_s t + 2cos3\omega_s t + \ldots\ldots] \longrightarrow \quad (2)$$

$$\omega_s = 2\pi f_s = \frac{2\pi}{T_S}$$

Substituting eq(2) in eq(1)

$$g(t) = \frac{1}{T_S}[x(t) + 2x(t)cos\omega_s t + 2x(t)cos2\omega_s t + 2x(t)cos3\omega_s t + \ldots\ldots] \longrightarrow \quad (3)$$

$$x(t) \longleftrightarrow X(\omega)$$

$$g(t) \longleftrightarrow G(\omega)$$

$$2x(t)cos\omega_s t \longleftrightarrow x(\omega - \omega_s) + x(\omega + \omega_s)$$

According to frequency shifting property

The F.T of eq (3) is

$$G(\omega)=\frac{1}{T_s}[\,x(\omega)+x(\omega-\omega_s)+x(\omega+\omega_s)+x(\omega-2\omega_s)+ x(\omega+\omega_s)+\dots\dots\dots] \longrightarrow \quad (4)$$

$$\text{Therefore } G(\omega)=\frac{1}{T_s}\sum_{n=-\infty}^{\infty}x(\omega-n\omega_s) \longrightarrow \quad (5)$$

From equation (4) & (5) it is clear that the spectrum G(ω) consists of x(ω) repeating periodically with period

$$\omega_s=\frac{2\pi}{T_s}\ radsec$$

$$or\ \ f_s=\frac{1}{T_s}\ Hz$$

Now if we want to reconstruct x(t) from g(t) we must be able to recover X(ω) from G(ω) .

This is possible if there is no overlap between successive cycles of G(ω) (as shown in fig f)

This can be happened if and only if $f_s \geq 2f_m$

But the sampling interval $T_s < \frac{1}{f_s}$

$$\text{Hence } \frac{1}{T_S} \geq 2f_m$$

$$\text{or } T_s < \frac{1}{2f_m}$$

Therfore as long as the sampling frequency f_s is greater than twice the maximum signal frequency f_m, G(ω) will consists of non overlapping repetition of X(ω).

Nyquist rate and Nyquist Interval

- The minimum sampling rate of ($2f_m$) samples per second, for an analog signal bandwidth of fm Hz, is called the Nyquist rate. $f_s=2f_m$

- The reciprocal of Nyquist rate,the maximum sampling interval $1/(2f_m)$, is called the Nyquist interval, that is, $Ts = 1/(2f_m)$.

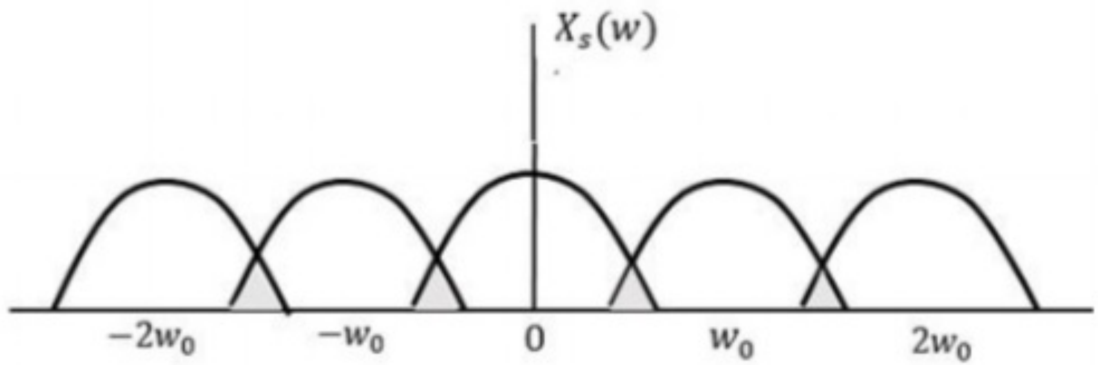
- The phenomenon of the presence of high spectrum of the original analog signal is called aliasing or simply fold over.

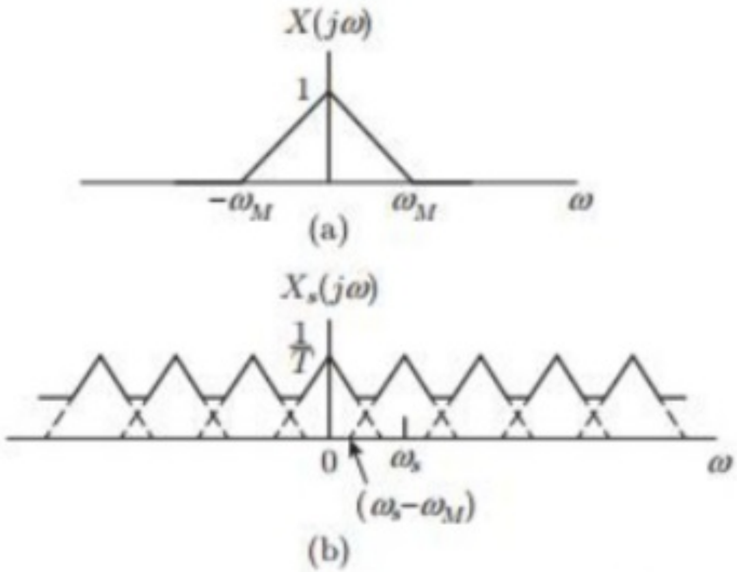
- When the continuous time band limited signal is sampled at Nyquist rate , the sampled spectrum contains non overlapping $G(\omega)$ repeating periodically.
- The successive cycles of $G(\omega)$ touch each other .
- The original spectrum $X(\omega)$ can be recovered from the sampled spectrum by using a LPF with a cut off frequency ω_m

Aliasing or fold over

- When the sampling frequency is less than the Nyquist frequency, the original signal is not recoverable by low pass filtering. The sampled signals overlap in the frequency domain and this process is referred to as aliasing.

- When the continuous time band limited signal is sampled at a rate lower than Nyquist rate , $f_s < 2f_m$ then The successive cycles of G(ω) of the sampled signal g(t) overlap with each other .
- The signal is under sampled .This is referred to as Aliasing affect





Aliasing phenomenon: (a) Original signal; (b) Sampled signal.

- Prealiasing Filter is used to avoid aliasing
- Prealiasing filter must be used to limit band of frequencies at signal f_m
- Sampling Frequency must be selected such that fs>2fm

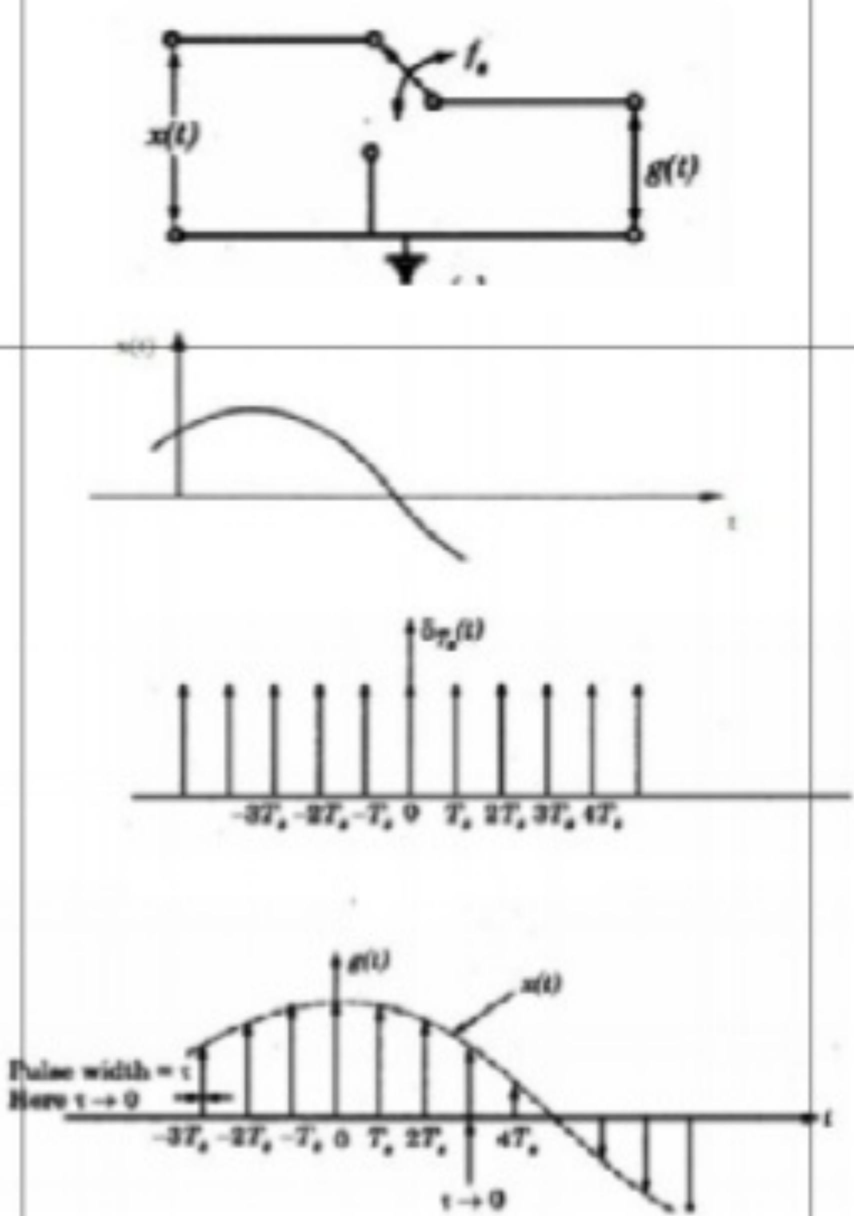
Types of Sampling

- Instantaneous Sampling or Ideal Sampling
- Natural Sampling
- Flat Top Sampling

❖Instantaneous Sampling or Ideal Sampling

- ❖principle used is known as multiplication principle
- ❖Not Practically Realizable since it is impossible to generate true impulses and the spectrum of such an ideal sampled signal occupies the entire bandwidth (-α α),i.e. it contains components of all frequencies
- ❖The out put energy is very low since the width of the impulses are approximately zero so high noise interference

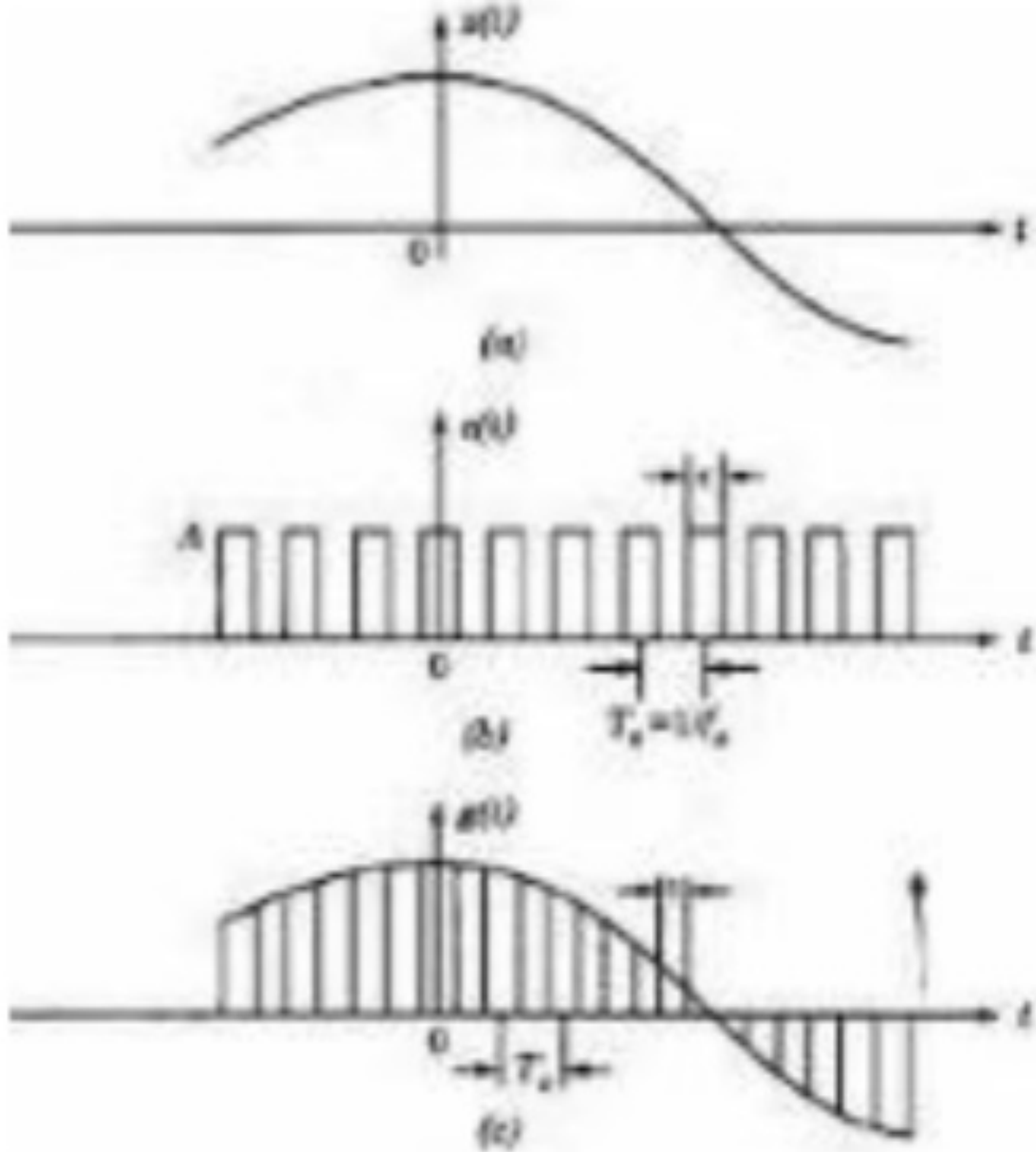
Instantaneous Sampling or Ideal Sampling



Natural Sampling

- principle used is known as Chopping principle
- The sampled signal is obtained by switching action
- In the natural sampling the tops of the sampled signal follows the natural envelope of the message signal hence called natural sampling

- Drawbacks:
  - needs complex electronic circuitry
  - The pulse shape needs to be maintained

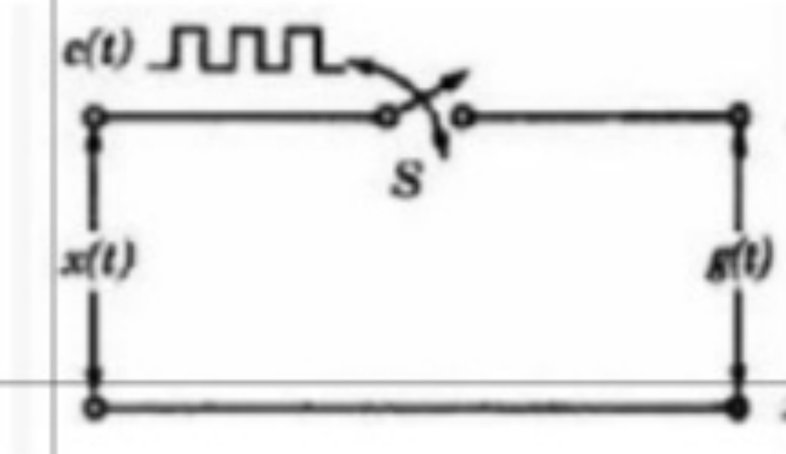


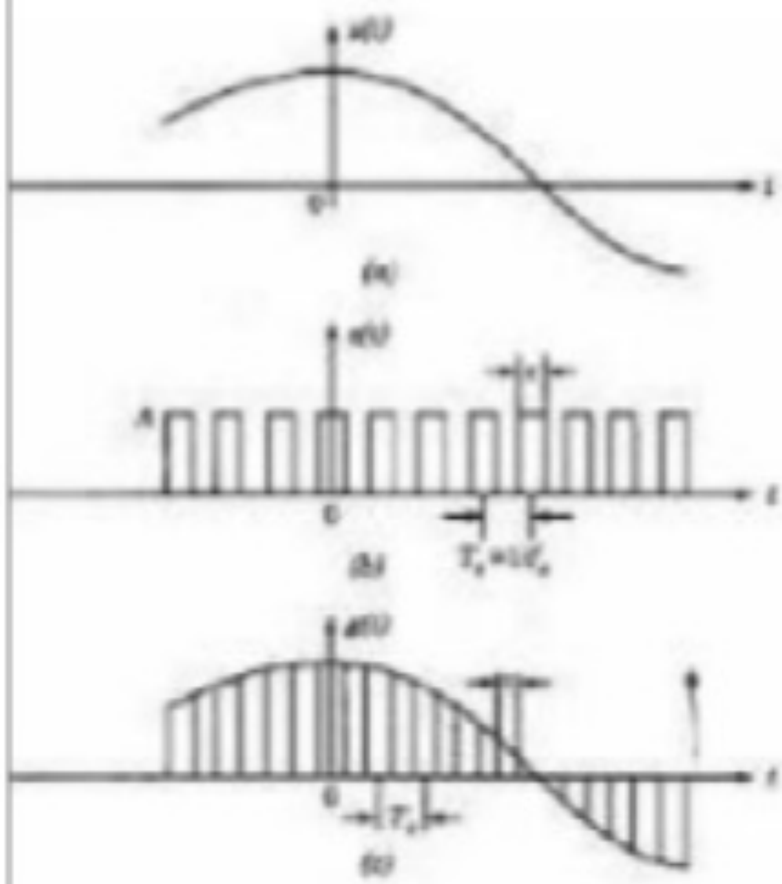
t
0
(a)
A
0
t
$T_s = 1/f_s$
(b)
0
t
T_s
(c)

❖ Flat Top Sampling

- ❖ principle used is known as Sample and hold action
- ❖ In the Flat top sampling the tops of the sampled signal are flat
- ❖ Pulses have a constant amplitude with in the pulse interval and is equal to instantaneous value of the message signal at the beginning of the sampling process.
- ❖ The pulse shape need not be maintained
- ❖ Avoids the need of complex electronic circuitry

Natural Sampling





Aperture effect

- The amplitude of the flat top signal must be constant, but sometimes it is not constant due to the high frequency roll off of the sampling signal.
- This results in the attenuation in the high frequency part of the message spectrum.
- Thus the sampled signal in the flat top sampling consists of attenuated high frequency components and this effect is known as Aperture effect.
- Aperture effect can be reduced by selecting value of
  pulse width τ to be very small and
  by using equalizer circuit.

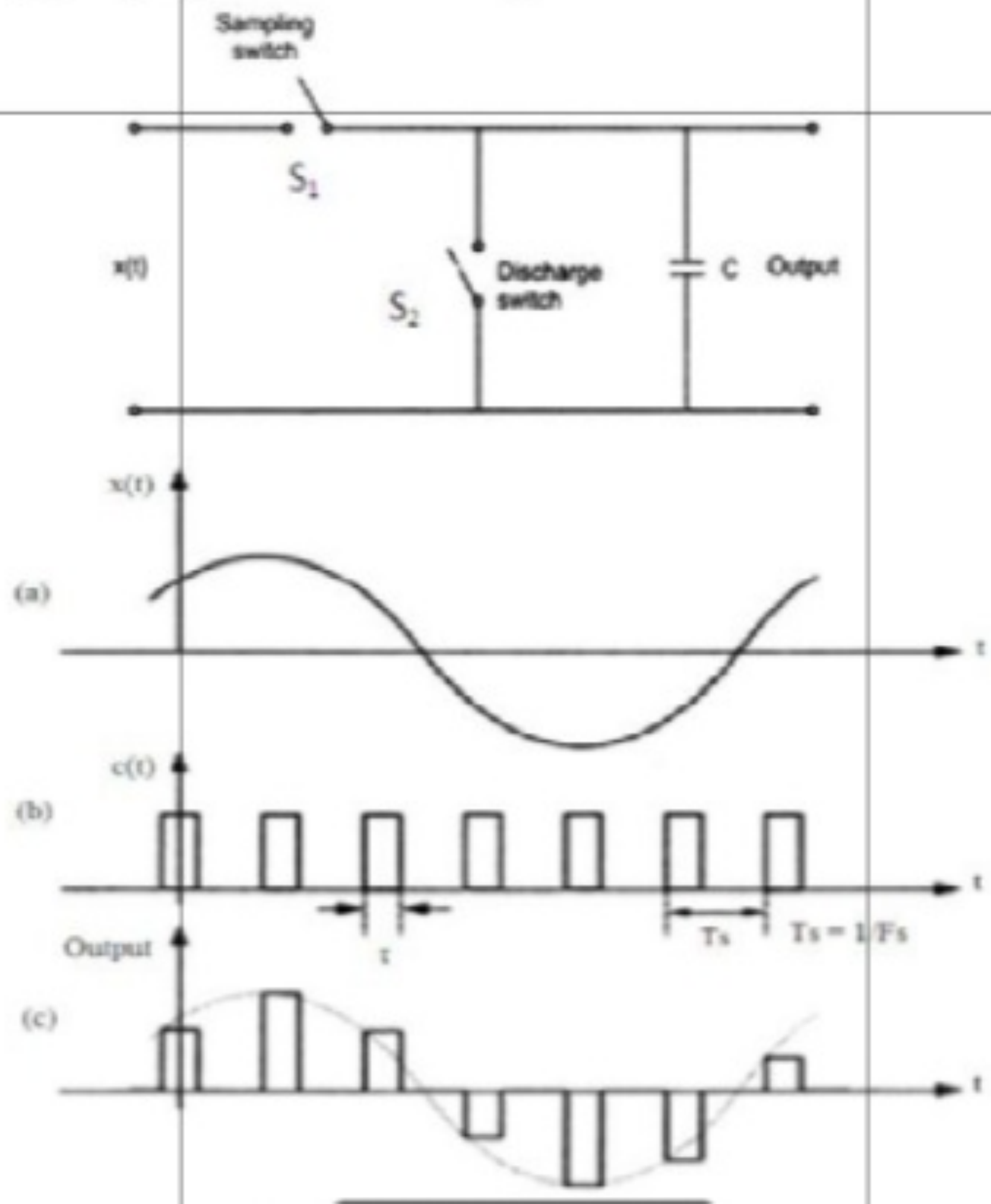
Under Sampling

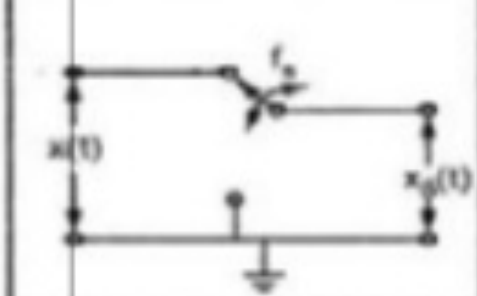
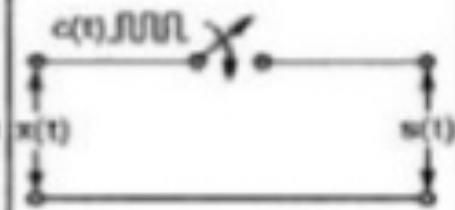
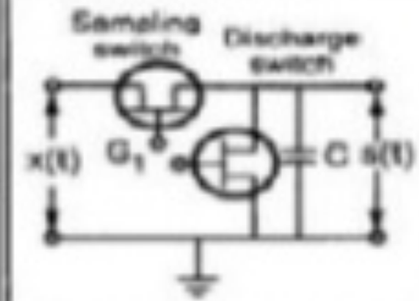
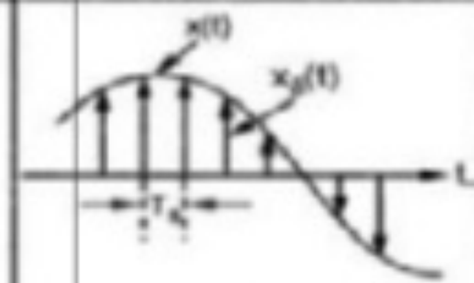
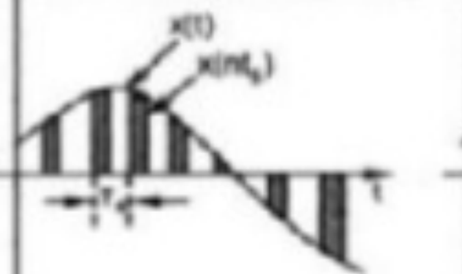
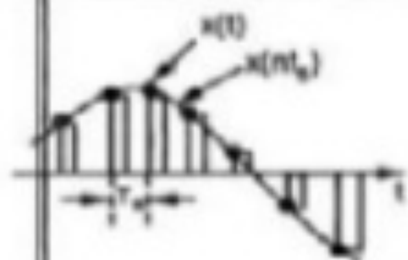
- In Undersampling a band pass signal is sampled slower than its Nyquist rate.
- When one undersamples a bandpass signal, the samples are indistinguishable from the samples of a low-frequency samples of the high-frequency signal.
- In such a way that the lowest-frequency alias satisfies the Nyquist criterion, because the bandpass signal is still uniquely represented and recoverable. Such undersampling is also known as *bandpass sampling*, *harmonic sampling*, *IF sampling*, and *direct IF to digital conversion*.

Oversampling

- In Oversampling a signal is sampled faster than its Nyquist rate.
- Oversampling is used in most modern analog-to-digital converters to reduce the distortion or noise effects introduced by practical digital-to-analog converters.

Flat Top Sampling



Sr. No.	Parameter of comparison	Ideal or instantaneous sampling	Natural sampling	Flat top sampling
1	Principle of sampling	It uses multiplication by an impulse function	It uses chopping principle	It uses sample and hold circuit
2	Circuit of sampler			
3	Waveforms			
4	Realizability	This is not practically possible method	This method is used practically	This method is used practically
5	Sampling rate	Sampling rate tends to infinity	Sampling rate satisfies Nyquist criteria	Sampling rate satisfies Nyquist criteria
6	Noise interference	Noise interference is maximum	Noise interference is minimum	Noise interference is maximum
7	Time domain representation	$x_\delta(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(t - nT_s)$	$s(t) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} x(t)\,\text{sinc}(n f_s \tau)\, e^{j 2\pi n f_s t}$	$s(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, h(t - nT_s)$
8	Frequency domain representation	$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$	$S(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}(n f_s \tau)\, X(f - nf_s)$	$S(f) = f_s \sum_{n=-\infty}^{\infty} X(f - n f_s)\, H(f)$