

§ OF SOUND IN A GAS →

Suppose that a small disturbance is created in non viscous gas such that

1. The motion of gas is irrotational. That is velocity potential ϕ exists and is given by $\vec{q} = -\nabla\phi$ where \vec{q} is the velocity of fluid or gas.
2. The square and product of all disturbances from equilibrium state are negligible (the equilibrium state is recognised as p_0, ρ_0, T_0 etc) also \vec{q} is so small that q^2 is negligible.
3. The isentropic law $p = k \rho^\gamma$ holds

Now let $p = p_0(1+s)$ where s is called condensation
 Now Equation of continuity is

$$\frac{\partial p}{\partial t} + \nabla \cdot (p\vec{q}) = 0$$

$$\frac{\partial}{\partial t} (p_0(1+s)) + \nabla \cdot (p_0(1+s)\vec{q}) = 0$$

p_0 is constant

$$p_0 \frac{\partial s}{\partial t} + \nabla \cdot (p_0\vec{q}) + \nabla \cdot (p_0 s\vec{q}) = 0$$

$$p_0 \frac{\partial s}{\partial t} + \nabla \cdot (p_0\vec{q}) = 0$$

Product term are neglected by so $s\vec{q}$ is negligible

$$\frac{\partial s}{\partial t} + \nabla \cdot \vec{q} = 0$$

$$\frac{\partial s}{\partial t} - \frac{\partial^2 \phi}{\partial x^2} = 0$$

$$\text{so } \frac{\partial s}{\partial t} = \frac{\partial^2 \phi}{\partial x^2} \rightarrow (*)$$

$$\begin{aligned} \because q &= -\nabla\phi \\ \nabla \cdot q &= \nabla \cdot (-\nabla\phi) \\ &= -\nabla^2\phi \\ &\text{in one dimension} \\ &= -\frac{\partial^2 \phi}{\partial x^2} \end{aligned}$$

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$$\frac{\partial^2 \phi}{\partial t^2} = a_0^2 \frac{\partial^2 \phi}{\partial x^2}$$

a_0 is the speed of sound. The equation of sound for small disturbance

This equation is a wave type of equation or This represents a wave and three dimensional wave eqn is

$$\frac{\partial^2 \phi}{\partial t^2} = a_0^2 \nabla^2 \phi$$

Thus a small disturbance called sound is propagated in a gas with speed $a_0 = \sqrt{\frac{\gamma p_0}{\rho_0}}$

$$\text{Thus } a_0^2 = \frac{dp}{d\rho}$$

$$a_0 = \sqrt{\frac{\gamma p_0}{\rho_0}}$$