...D OF SOUND IN A GAS →

...pose that a small disturbance is created in non viscous gas such that

1. The motion of gas is irrotational. That is velocity potential φ exists and is given by $\vec{q} = -\nabla\varphi$ where $\vec{q}$ is the velocity of fluid or gas.

2. The square and product of all disturbances from equilibrium state are negligible (the equilibrium state is recognised as ρ_0, p_0, T_0 etc.) also $\vec{q}$ is so small that q^2 is negligible.

3. The isentropic law $p = k\rho^\gamma$ holds

Now let $\rho = \rho_0(1+s)$ where s is called condensation

Now equation of continuity is

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{q}) = 0$$

$$\frac{\partial}{\partial t}\left(\rho_0(1+s)\right) + \nabla\cdot\left(\rho_0(1+s)\vec{q}\right) = 0$$

ρ_0 is constant

$$\rho_0 \frac{\partial s}{\partial t} + \nabla\cdot(\rho_0\vec{q}) + \nabla\cdot(\rho_0 s\vec{q}) = 0$$

Product term are neglected by... so $s\vec{q}$ is negligible

$$\rho_0 \frac{\partial s}{\partial t} + \nabla\cdot(\rho_0\vec{q}) = 0$$

$$\frac{\partial s}{\partial t} + \nabla\cdot\vec{q} = 0$$

$$\frac{\partial s}{\partial t} - \frac{\partial^2\varphi}{\partial x^2} = 0$$

$\because q = -\nabla\varphi$

$\nabla\cdot q = \nabla\cdot(-\nabla\varphi)$

$= -\nabla^2\varphi$

in one dimension

$= -\frac{\partial^2\varphi}{\partial x^2}$

$$\text{so } \frac{\partial s}{\partial t} = \frac{\partial^2\varphi}{\partial x^2} \rightarrow (*)$$

[illegible]

= [illegible]

= [illegible] log [illegible] + constant [illegible]

Put this in equation of motion

[illegible] = [illegible] + constant

[illegible] log [illegible] + constant = constant

[illegible] log [illegible] + constant

[illegible] log [illegible] + constant

[illegible] log [illegible] + [illegible] log [illegible] + constant

= [illegible] log [illegible] + constant

= [illegible] + constant

[illegible] + constant [illegible]

on differentiation

[illegible]

[illegible]

$$\frac{\partial^2 \varphi}{\partial t^2} = a_0^2 \frac{\partial^2 \varphi}{\partial x^2}$$

This Equation is a wave type of Equation or This represents a wave propagation with speed a_0 and three dimensional Wave Eqn is

a_0 is the speed of sound or speed of small disturba

$$\frac{\partial^2 \varphi}{\partial t^2} = a_0^2 \nabla^2 \varphi$$

Thus a small disturbance called sound is propagated in a gas with speed $a_0 = \sqrt{\frac{\gamma p_0}{\rho_0}}$

Thus $a_0^2 = \frac{dp}{d\rho}$

$$a_0 = \sqrt{\frac{\gamma p_0}{\rho_0}}$$