Mean is average of a given set of data. Let us consider below example

$$2,\ 4,\ 4,\ 4,\ 5,\ 5,\ 7,\ 9$$

These eight data points have the mean (average) of 5:

$$\frac{2+4+4+4+5+5+7+9}{8} = 5.$$

$$Formula : \mu = \frac{\sum_{i=1}^{N} x_i}{N}$$

Where μ is mean and x_1, x_2, x_3...., x_i are elements.Also note that mean is sometimes denoted by $\bar{x}$

Variance is the sum of squares of differences between all numbers and means.

Deviation for above example. First, calculate the deviations of each data point from the mean, and square the result of each:

$$\begin{array}{ll} (2-5)^2 = (-3)^2 = 9 & (5-5)^2 = 0^2 = 0 \\ (4-5)^2 = (-1)^2 = 1 & (5-5)^2 = 0^2 = 0 \\ (4-5)^2 = (-1)^2 = 1 & (7-5)^2 = 2^2 = 4 \\ (4-5)^2 = (-1)^2 = 1 & (9-5)^2 = 4^2 = 16. \end{array}$$

variance =

$$\frac{9+1+1+1+0+0+4+16}{8}$$

= 4.

$$Formula : \sigma^2 = \frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}$$

Where μ is Mean, N is the total number of elements or frequency of distribution.

Standard Deviation is square root of variance. It is a measure of the extent to which data varies from the mean.

Standard Deviation (for above data) = $\sqrt{4} = 2$

1. Value of standard deviation is 0 if all entries in input are same.
2. If we add (or subtract) a number say 7 to all values in the input set, the mean is increased (or decreased) by 7, but the standard deviation doesn't change.
3. If we multiply all values in the input set by a number 7, both mean and the standard deviation is multiplied by 7. But if we multiply all input values with a negative number say -7, the mean is multiplied by -7, but the standard deviation is multiplied by 7.
4. Standard deviation and variance is a measure that tells how spread out the numbers is. While variance gives you a rough idea of spread, the standard deviation is more concrete, giving you exact distances from the mean.
5. Mean, median and mode are the measure of central tendency of data (either grouped or ungrouped).

Continuous Random Variable Definition

A continuous random variable can be defined as a random variable that can take on an infinite number of possible values. Due to this, the probability that a continuous random variable will take on an exact value is 0. The cumulative distribution function and the probability density function are used to describe the characteristics of a continuous random variable.

Continuous Random Variable Example

Suppose the probability density function of a continuous random variable, X, is given by $4x^3$, where $x \in [0, 1]$. The probability that X takes on a value between 1/2 and 1 needs to be determined. This can be done by integrating $4x^3$ between 1/2 and 1. Thus, the required probability is 15/16.

PDF of Continuous Random Variable

The probability density function of a continuous random variable can be defined as a function that gives the probability that the value of the random variable will fall between a range of values. Let X be the continuous random variable, then the formula for the pdf, f(x), is given as follows:

$$f(x) = \frac{dF(x)}{dx} = F'(x)$$

where, F(x) is the cumulative distribution function.

For the pdf of a continuous random variable to be valid, it must satisfy the following conditions:

- $\int_{-\infty}^{\infty} f(x)dx = 1$. This means that the total area under the graph of the pdf must be equal to 1.
- $f(x) \geq 0$. This implies that the probability density function of a continuous random variable cannot be negative.

CDF of Continuous Random Variable

The cumulative distribution function of a continuous random variable can be determined by integrating the probability density function. It can be defined as the probability that the random variable, X, will take on a value that is lesser than or equal to a particular value, x. The formula for the cdf of a continuous random variable, evaluated between two points a and b, is given below:

$$P(a < X \leq b) = F(b) - F(a) = \int_a^b f(x)dx$$

Mean of Continuous Random Variable

The mean of a continuous random variable can be defined as the weighted average value of the random variable, X. It is also known as the expectation of the continuous random variable. The formula is given as follows:

$$E[X] = \mu = \int_{-\infty}^{\infty} xf(x)dx$$

Variance of Continuous Random Variable

The variance of a continuous random variable can be defined as the expectation of the squared differences from the mean. It helps to determine the dispersion in the distribution of the continuous random variable with respect to the mean. The formula is given as follows:

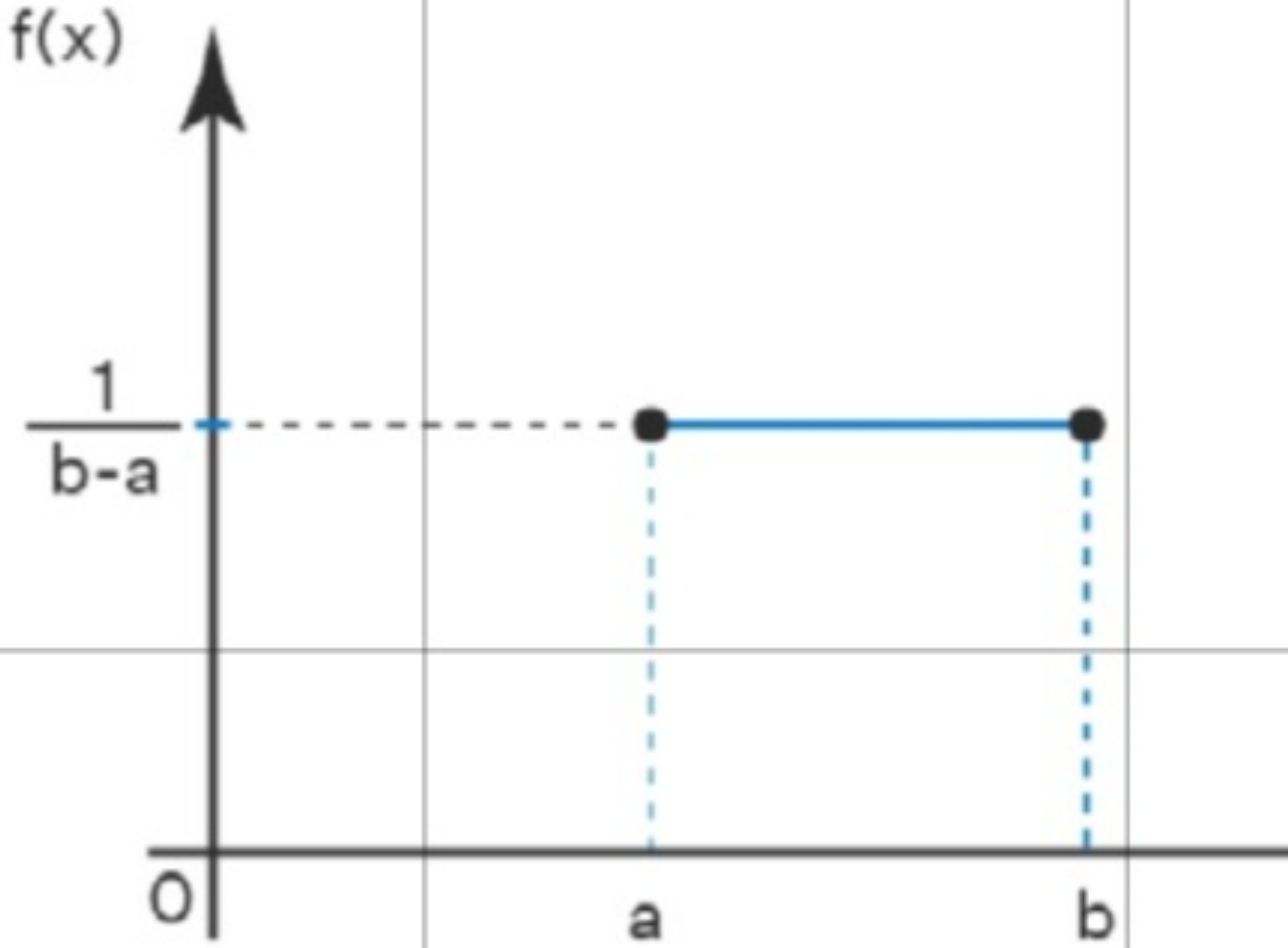
$$Var(X) = \sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$$

Uniform Random Variable

A continuous random variable that is used to describe a uniform distribution is known as a uniform random variable. Such a distribution describes events that are equally likely to occur. The pdf of a uniform random variable is as follows:

$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

Uniform Random Variable



A continuous random variable that is used to model a normal distribution is known as a normal random variable. If the parameters of a normal distribution are given as $X \sim N(\mu, \sigma^2)$ then the formula for the pdf is given as follows:

$$f(x) = \frac{1}{\sigma\sqrt{2\Pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where,

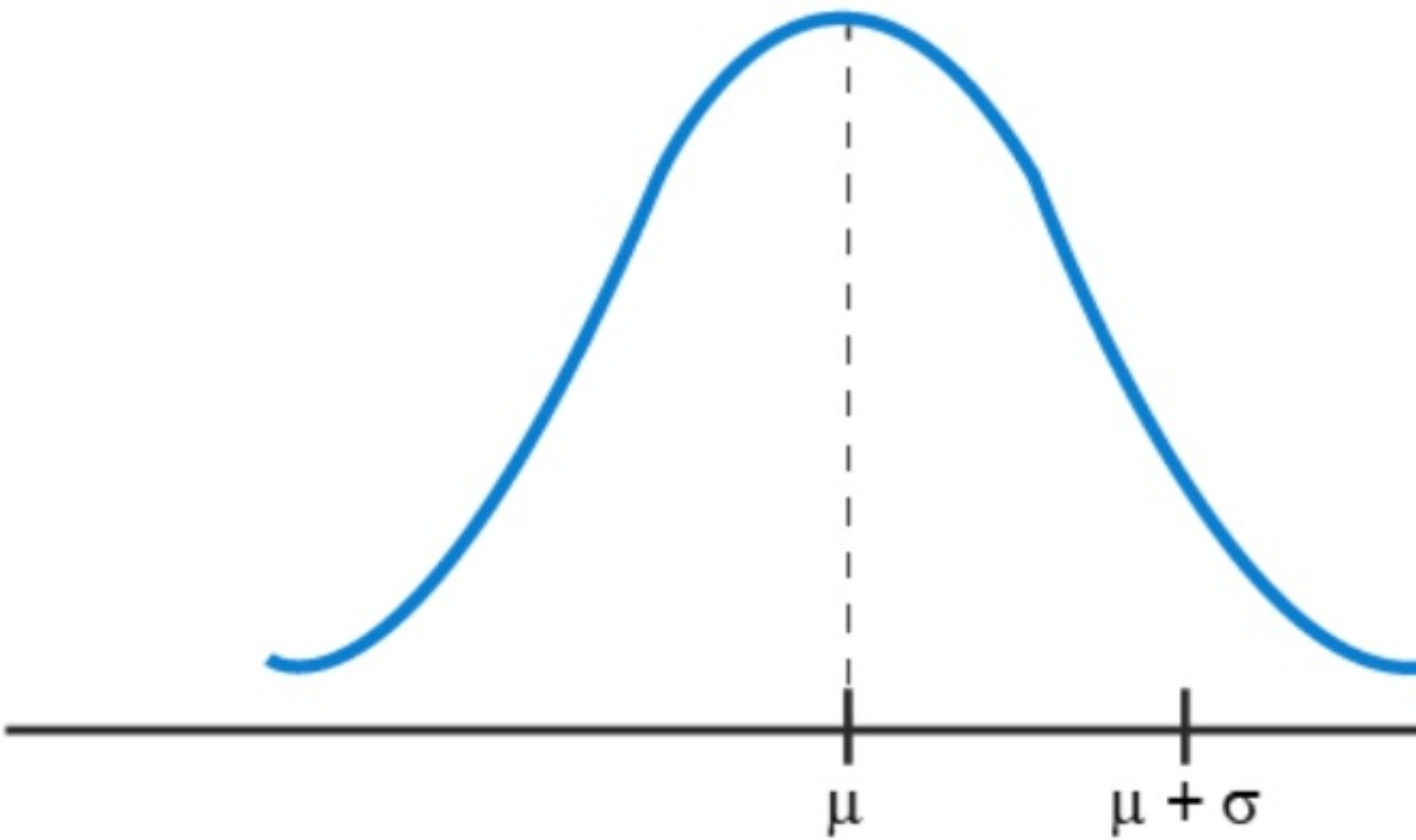
μ = mean

σ = standard deviation

σ^2 = variance.

A normal distribution where $\mu = 0$ and $\sigma^2 = 1$ is known as a standard normal distribution.

Normal Random Variable



Exponential Random Variable

Exponential distributions are continuous probability distributions that model processes where a certain number of events occur continuously at a constant average rate, $\lambda \geq 0$. Thus, a continuous random variable used to describe such a distribution is called an exponential random variable. The pdf is given as follows:

$$f(x) = \lambda e^{-\lambda x}$$

- A continuous random variable is a variable that is used to model continuous data and its value falls between an interval of values.
- The probability density function of a continuous random variable is given as $f(x) = \frac{dF(x)}{dx} = F'(x)$.
- The cumulative distribution function is given by $P(a < X \leq b) = F(b) - F(a) = \int_a^b f(x)dx$.
- The mean of a continuous random variable is $E[X] = \mu = \int_{-\infty}^{\infty} xf(x)dx$ and variance is $Var(X) = \sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)dx$.
- Uniform random variable, exponential random variable, normal random variable, and standard normal random variable are