Introduction to Fuzzy Logic

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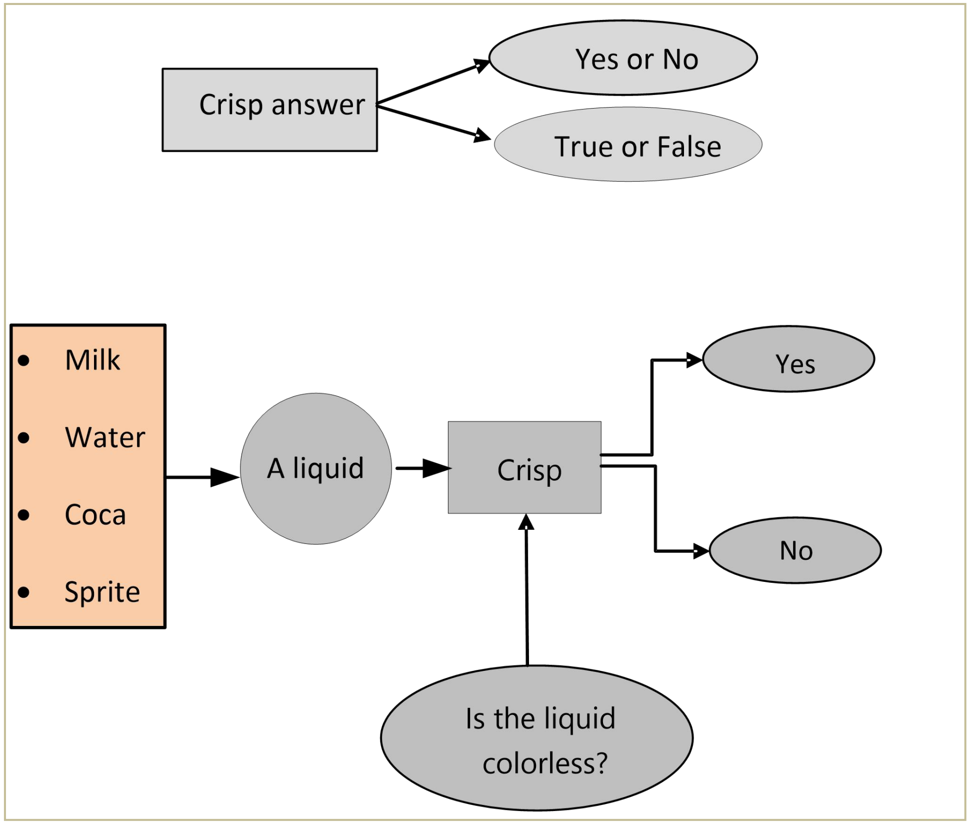
What is Fuzzy logic?

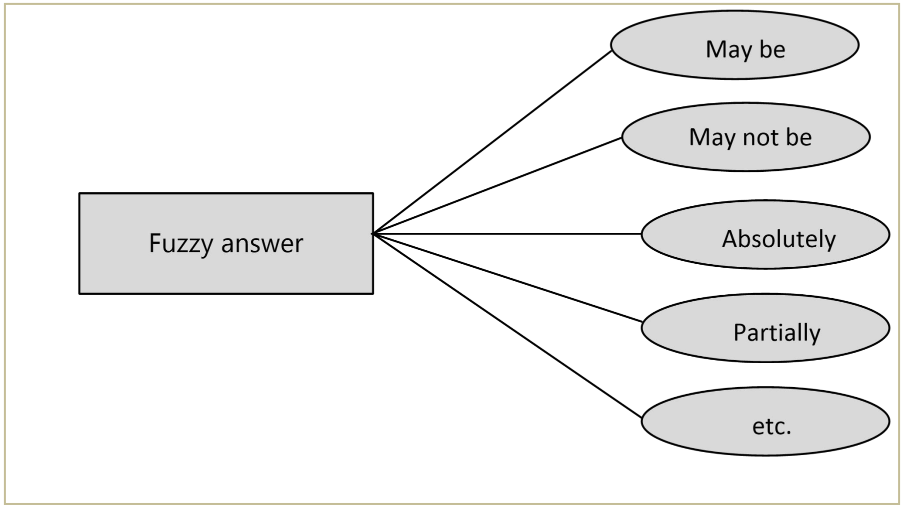
- Fuzzy logic is a mathematical language to express something.
  - This means it has grammar, syntax, semantic like a language for communication.
- There are some other mathematical languages also known
  - **Relational algebra** (operations on sets)
  - **Boolean algebra** (operations on Boolean variables)
  - **Predicate algebra** (operations on well formed formulae (wff), also called predicate propositions)
- **Fuzzy logic deals with Fuzzy set or Fuzzy algebra.**

- Like the extension of crisp set theory to fuzzy set theory, the extension of crisp logic is made by replacing the bivalent membership functions of the crisp logic with the fuzzy membership functions.
- In crisp logic, the truth value acquired by the proposition are 2-valued, namely true as 1 and false as 0.
- In fuzzy logic, the truth values are multi-valued, as absolute true, partially true, absolute false etc represented numerically as real value between 0 to 1.

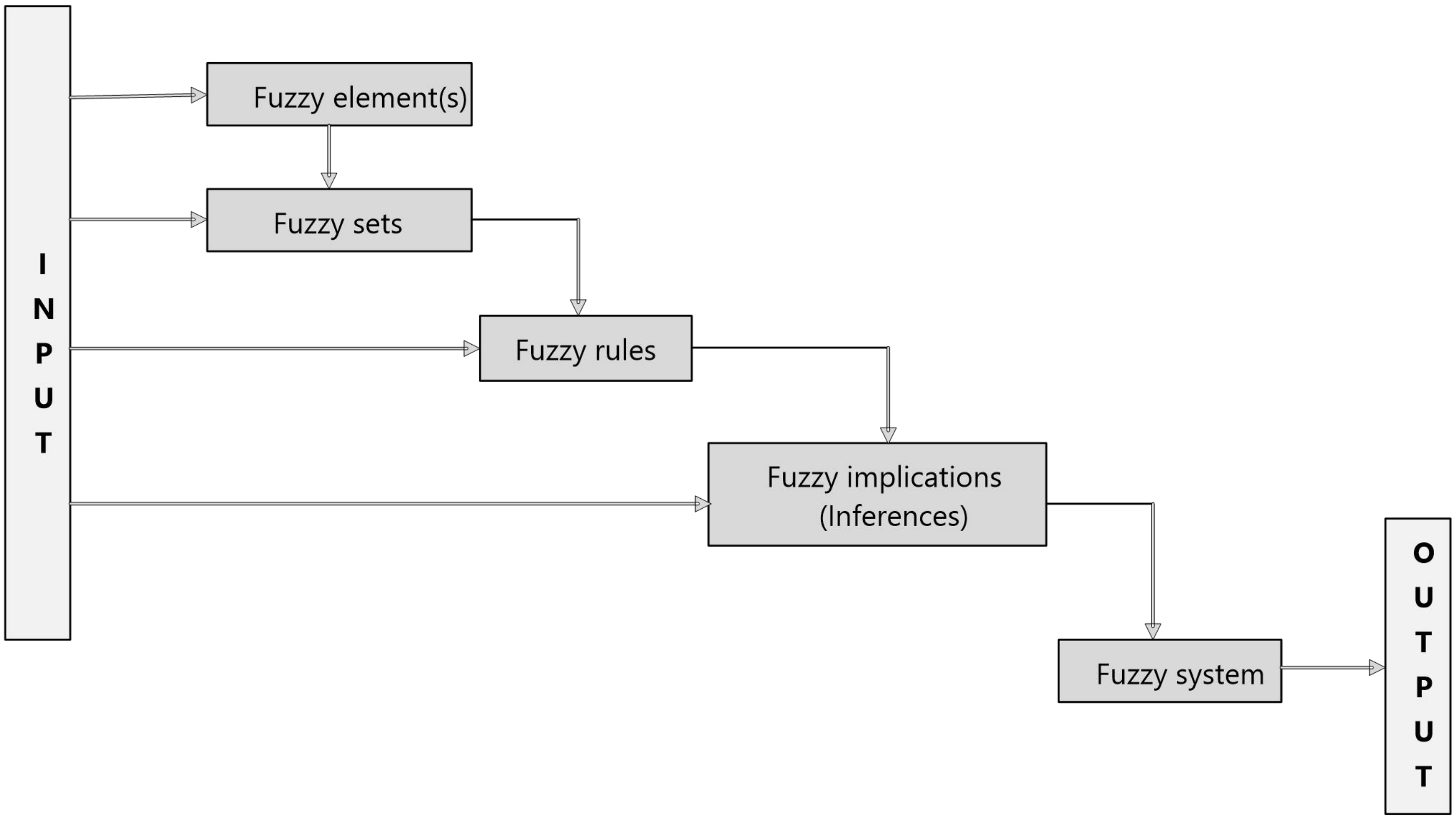
Example : Fuzzy logic vs. Crisp logic



Example : Fuzzy logic vs. Crisp logic



Concept of fuzzy system



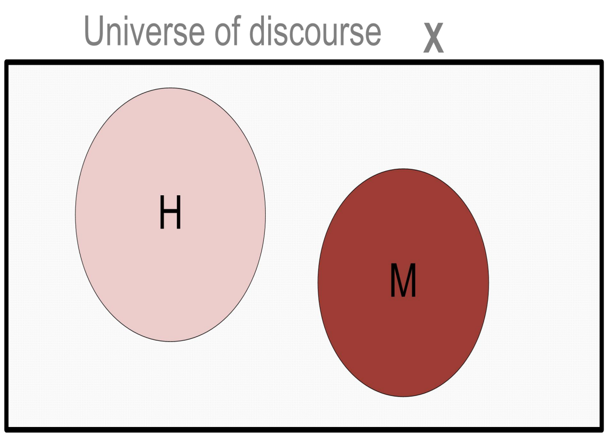
Concept of fuzzy set

To understand the concept of **fuzzy set** it is better, if we first clear our idea of **crisp set**.

X = The entire population of India.

H = All Hindu population = $\{h_1, h_2, h_3, \ldots\ldots, h_L,\}$

M= All Muslim population =$\{m_1, m_2, m_3, \ldots\ldots, m_N,\}$



Here, All are the sets of finite numbers of individuals.
Such a set is called crisp set.

Example of fuzzy set

Let us discuss about fuzzy set.

X = All students in NPTEL.

S = All **Good students**.

$S = \{(s, g(s)) \mid s \in X\}$ and *g(s)* is a measurement of goodness of the student *s*.

Example:

$S = \{(Rajat, 0.8), (Kabita, 0.7), (Salman, 0.1), (Ankit, 0.9)\}$, etc.

Fuzzy set vs. Crisp set

Note: A crisp set is a fuzzy set, but, a fuzzy set is not necessarily a crisp set.

Example:

$$\mathrm{H} = \{(h_1, 1), (h_2, 1) \ldots\ldots\ldots, (h_L, 1)\}$$

$$\mathrm{Person} = \{(p_1, 0), (p_2, 0) \ldots\ldots\ldots, (p_N, 0)\}$$

In case of a crisp set, the elements are with extreme values of degree of membership namely either 1 or 0.

Degree of membership

How to decide the degree of memberships of elements in a fuzzy set?

City	Bangalore	Bombay	Hyderabad	Kharagpur	Madras	Delhi
μ	0.95	0.90	0.80	0.01	0.65	0.75

How the cities of comfort can be judged?

Definition 1: Membership function (and Fuzzy set)

defined as a set of ordered pairs, that is
$A = \{(x,\ \mu_A(x)) | x \in X\}$ where $\mu_A(x)$ is called the membership function for the fuzzy set A.

Note: *$\mu_A(x)$ map each element of X onto a membership grade (or membership value) between 0 and 1 (both inclusive).*

Question: *How (and who) decides $\mu_A(x)$ for a fuzzy set A in X?*

Example:

X = All cities in India

A = City of comfort

A={(New Delhi, 0.7), (Bangalore, 0.9), (Chennai, 0.8), (Hyderabad, 0.6), (Kolkata, 0.3), (Kharagpur, 0)}

Fuzzy vs. Probability

Fuzzy : When we say about certainty of a thing

Example: A patient come to the doctor and he has to diagnose so that medicine can be prescribed.

Doctor prescribed a medicine with certainty 60% that the patient is suffering from flue. So, the disease will be cured with certainty of 60% and uncertainty 40%. Here, in stead of flue, other diseases with some other certainties may be.

Probability: When we say about the chance of an event to occur

Example: India will win the T20 tournament with a chance 60% means that out of 100 matches, India own 60 matches.

Prediction vs. Forecasting

The Fuzzy vs. Probability is analogical to Prediction vs. Forecasting

Prediction : When you start guessing about things.

Forecasting : When you take the information from the past job and apply it to new job.

The main difference:

Prediction is based on the best guess from experiences.

Forecasting is based on data you have actually recorded and packed from previous job.

Fuzzy membership functions

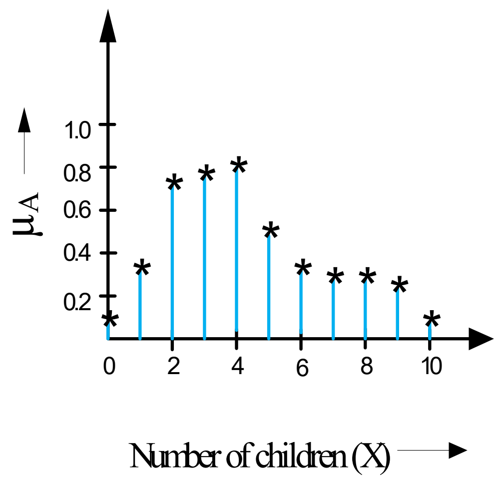
A fuzzy set is completely characterized by its membership function (sometimes abbreviated as MF and denoted as μ). So, it would be important to learn how a membership function can be expressed (mathematically or otherwise).

Example:

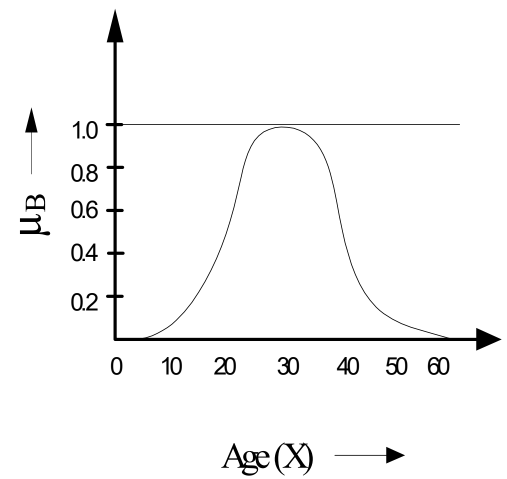
Note: A membership function can be on

a) a discrete universe of discourse and

b) a continuous universe of discourse.



A= Fuzzy set of "Happy family"

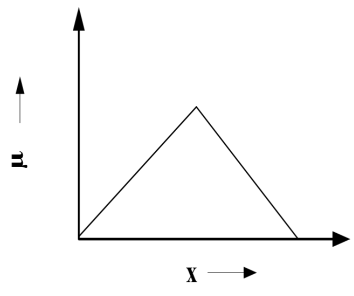


B= "Young age"

Fuzzy membership functions

So, membership function on a discrete universe of course is trivial. However, a membership function on a continuous universe of discourse needs a special attention.

Following figures shows typical examples of membership functions.



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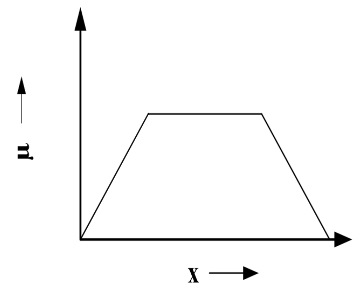
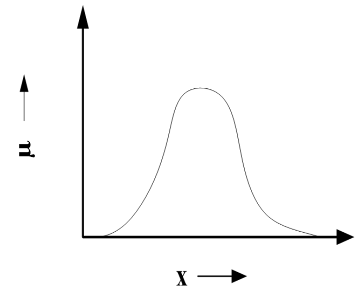


< trapezoidal >



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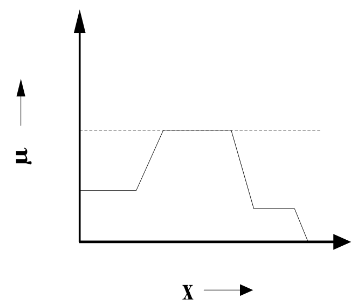

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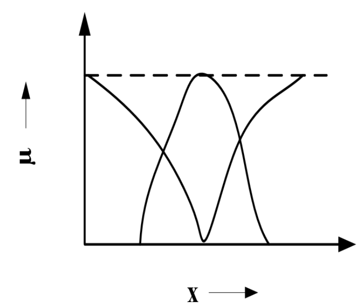

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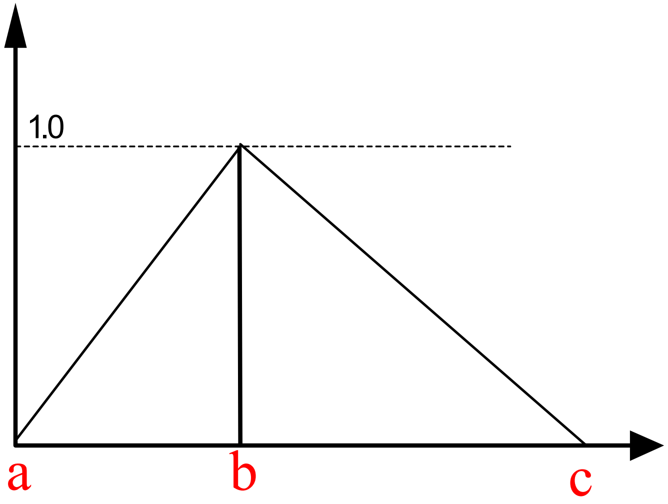
Fuzzy MFs : Formulation and parameterization

In the following, we try to parameterize the different MFs on a continuous universe of discourse.

Triangular MFs : A triangular MF is specified by three parameters $\{a,\ b,\ c\}$ and can be formulated as follows.

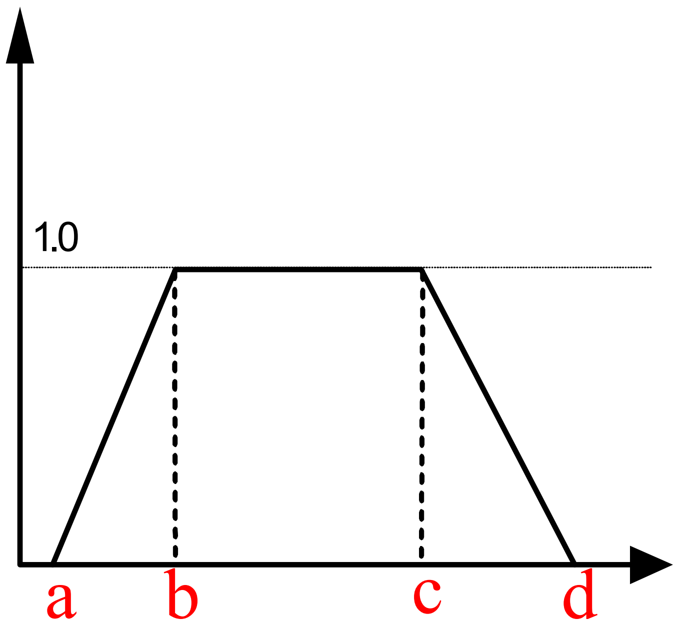


$$\text{triangle}\,(x; a, b, c) = \begin{cases} 0 & if\ x \le a \\ \dfrac{x-a}{a-b} & if\ a \le x \le b \\ \dfrac{c-x}{c-b} & if\ b \le x \le c \\ 0 & if\ c \le x \end{cases}$$

Fuzzy MFs: Trapezoidal

A **trapezoidal MF** is specified by four parameters $\{a,\ b,\ c,\ d\}$ and can be defined as follows:

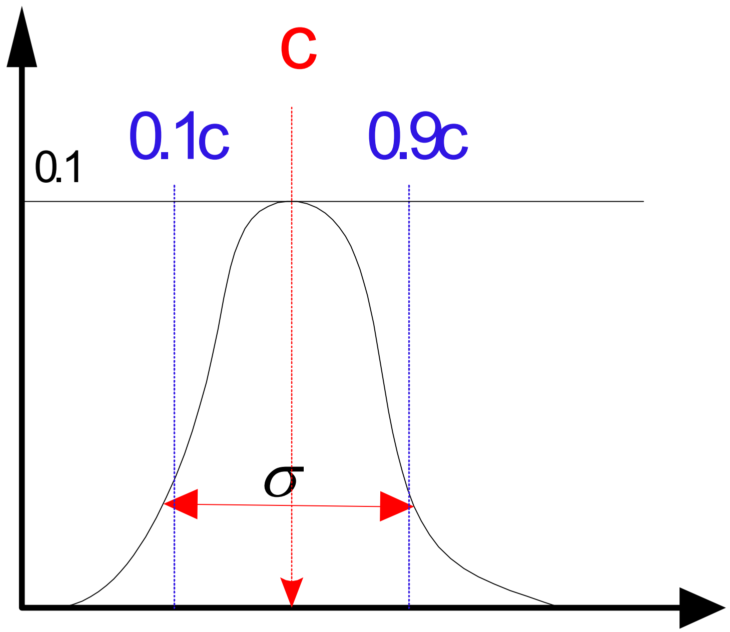
$$\text{trapozoid}\ (x; a, b, c,\ d) = \begin{Bmatrix} 0 & if\ x \leq a \\ \dfrac{x-a}{b-a} & if\ a \leq x \leq b \\ 1 & if\ b \leq x \leq c \\ \dfrac{d-x}{d-c} & if\ c \leq x \leq d \\ 0 & if\ d \leq x \end{Bmatrix}$$



Fuzzy MFs: Gaussian

A **Gaussian MF** is specified by two parameters $\{c, \sigma\}$ and can be defined as below:

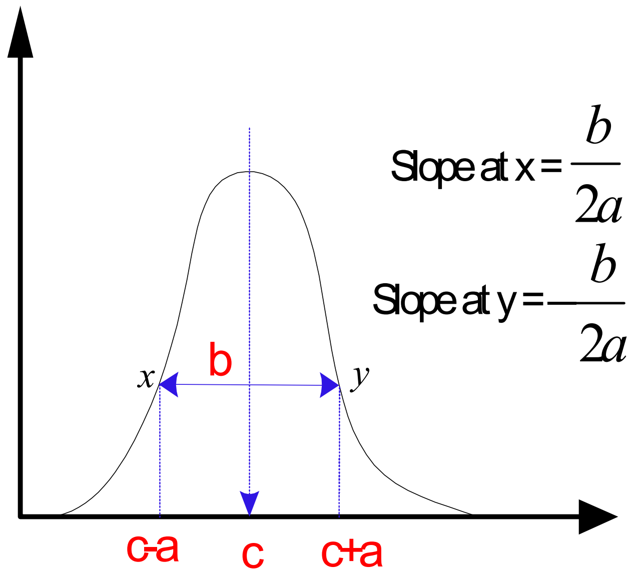
$$gaussian\ (x; c, \sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}$$



Fuzzy MFs: Generalized bell

It is also called Cauchy MF. A generalized bell MF is specified by three parameters $\{a,\ b,\ c\}$ and is defined as:

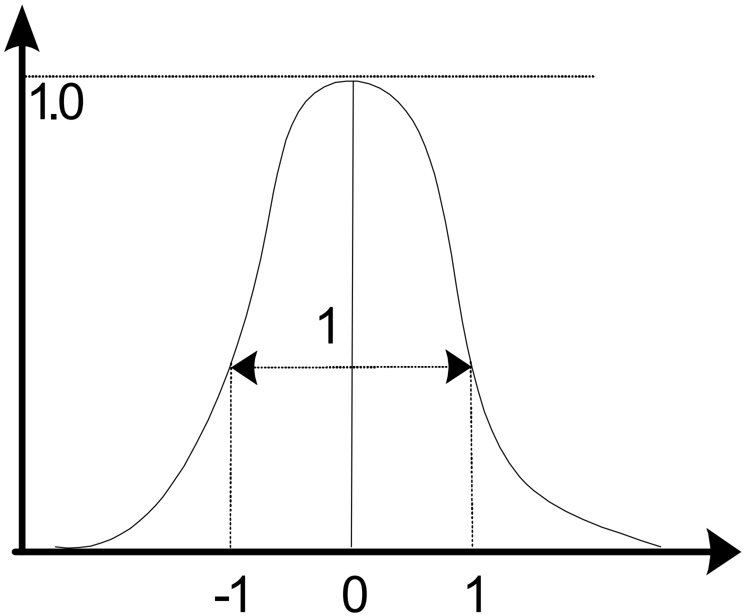
$$bell(x;\ a,\ b,\ c) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}}$$



Example: Generalized bell MFs

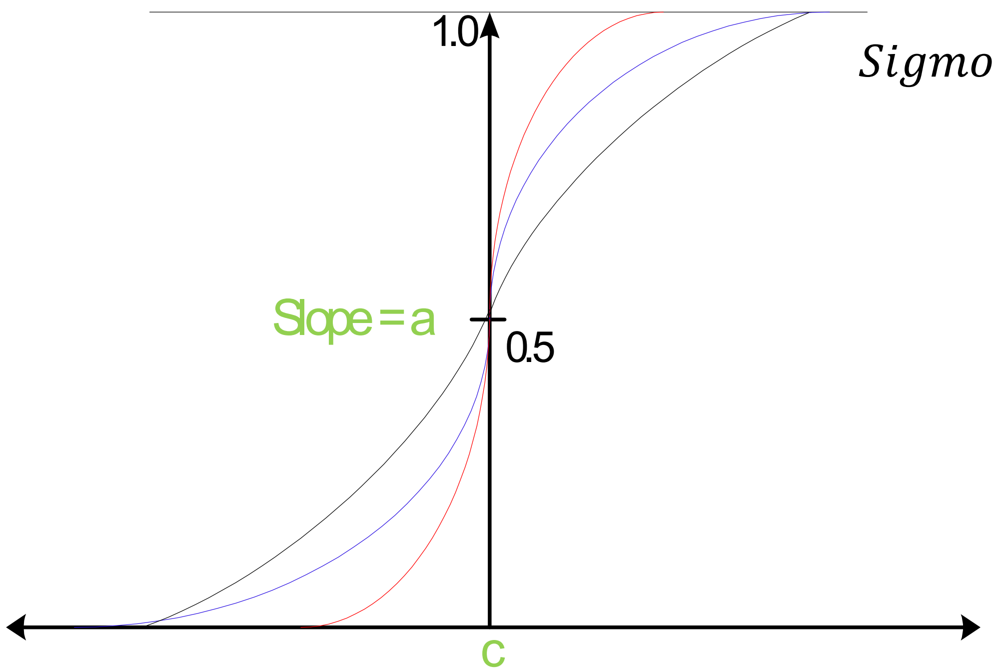
Example: $\mu(x) = \frac{1}{1+|x|^2}$;

a = b = 1 and c = 0;



Fuzzy MFs: Sigmoidal MFs

Parameters: $\{a, c\}$; where c= crossover point and a= slope at c;



$$Sigmoid(x;\ a,\ c) = \frac{1}{1 + e^{-\left[\frac{x-c}{a}\right]}}$$

Fuzzy MFs : Example

Example : Consider the following grading system for a course.

Excellent = Marks $\leq$ 90

Very good = 75$\leq$ Marks$\leq$ 90

Good = 60$\leq$ Marks$\leq$ 75

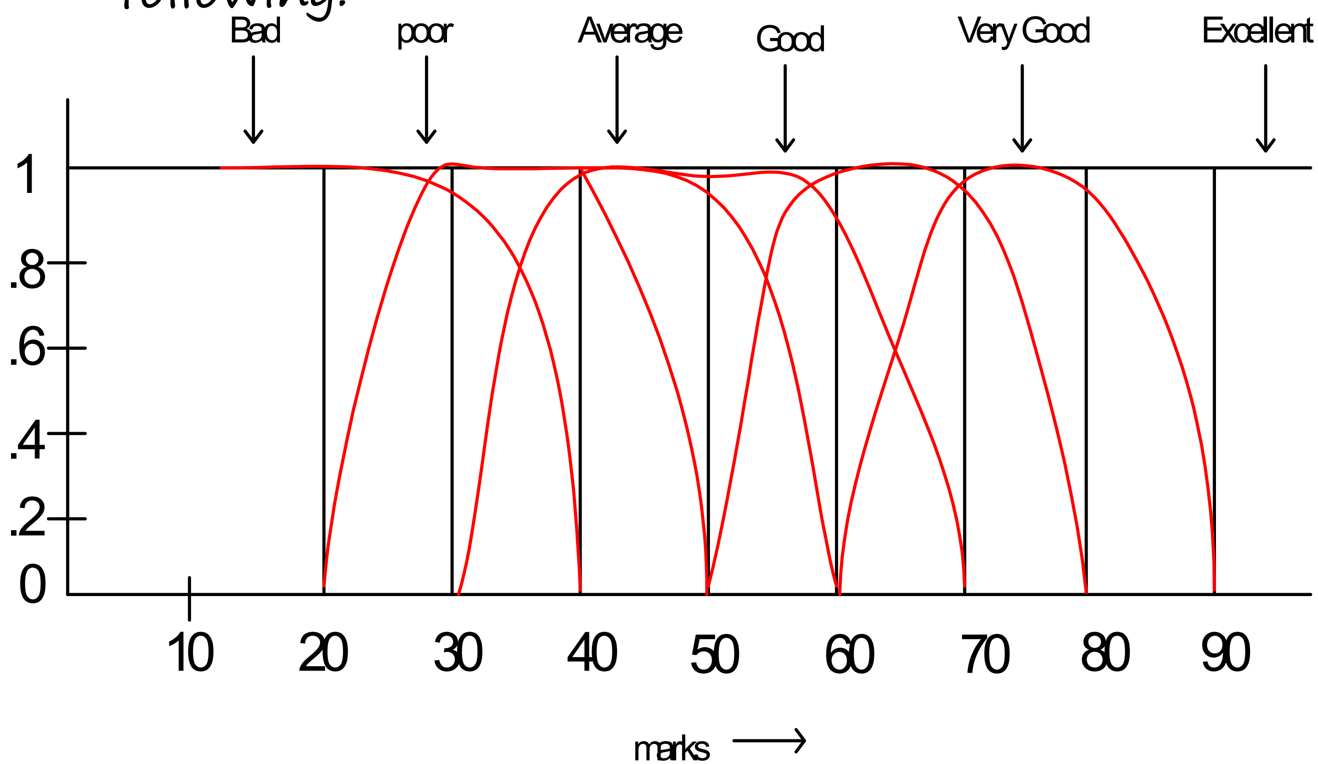
Average = 50$\leq$ Marks$\leq$ 60

Poor = 35$\leq$ Marks$\leq$ 50

Bad= Marks$\leq$ 35

Grading System

A fuzzy implementation will look like the following.



You can decide a standard fuzzy MF for each of the fuzzy grade.

References: 1. DIP by Gonzales & Woods
2. NPTEL & other online sources