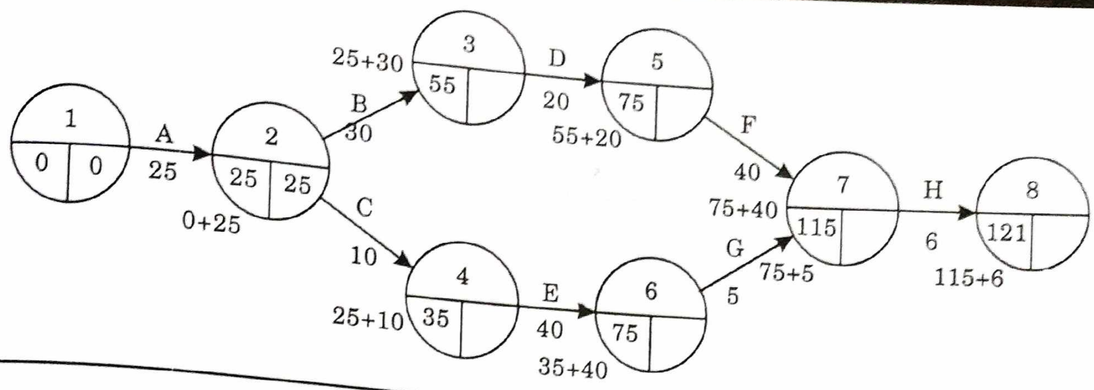
Exhibit 16.9 : Progressive Computation

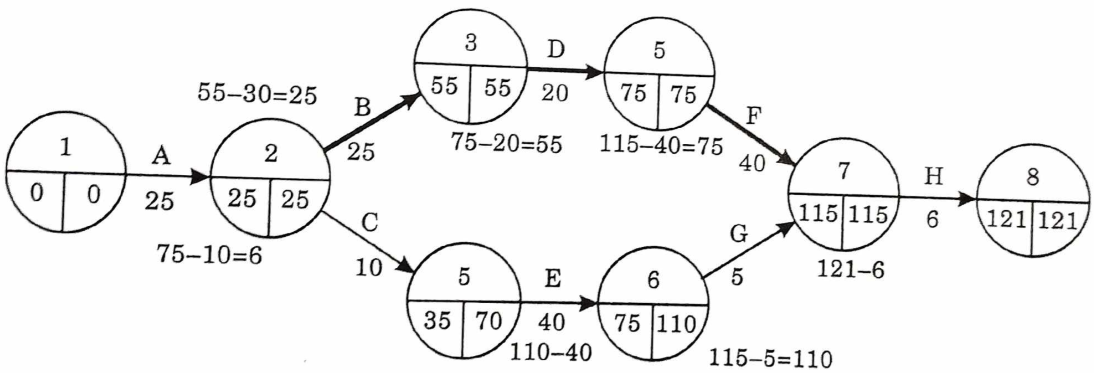


Note : Where several arrows converge, enter the biggest from EE.

(b) Retrogressive computation or Backward pass : The Latest finish and latest start dates are calculated by means of a backward pass. The LF is usually set equal to the EF of the project. Starting with the last activity, subtract the activity duration from the LF to obtain L.S.

Exhibit 16.10 illustrates the backward pass.

Exhibit 16.10 : Retrogressive Computation



Note : Where arrows branch off, insert the smallest sum as LE

(c) Calculation of Earliest Ending time of Activity (EEA) and Latest Starting time of Activity (LSA) : The earliest ending time is arrived at by adding the activity's duration D to the earliest starting time of the activity (ESA).

i.e.,

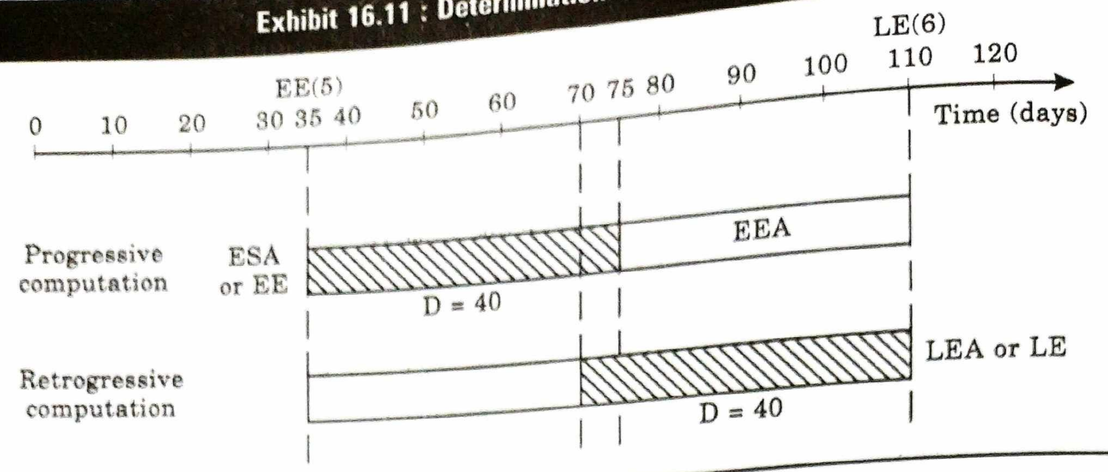
$$EEA = ESA + D \quad \text{or} \quad EEA = EE + D$$

The latest starting time (LSA) is arrived at by substracting the activity's duration D from the latest ending time of the activity (LEA).

$$LSA = LEA - D \quad \text{or} \quad LSA = LE - D$$

For example, considering the process 5 - 6 from the network diagram given at *exhibit 16.10*, we can calculate ESA, EEA, LSA & LEA as shown in *Exhibit 16.11*.

Exhibit 16.11 : Determination of EEA & LSA



ESA = Earliest starting time of the activity (5-6) *i.e.*, 35

EEA = Earliest ending time of the activity (5-6) *i.e.*, 75^{th} day

EE = Earliest time of the event (5) *i.e.*, 35^{th} day

ESA = Earliest starting time of the activity (5-6) *i.e.*, 35^{th} day

LSA = Latest starting time of activity (5-6) *i.e.*, 70^{th} day

LEA = Latest ending time of activity (5-6) *i.e.*, 110^{th} day

LE = Latest time of the event (6) *i.e.*, 110^{th} day

(iii) Assessment of the Critical Path

If, in respect of any activity, the earliest and latest times of occurrence are identical both at the start and the finish of the activity, then the activity lies on the critical path. In other words, if for two events say event 'i' and event 'j' the earliest event time and the latest event time (*i.e.*, EE & LE) are the same, the activity (i – j) connecting these two events is said to be on the critical path. In the figure 16.10, it is observed that the events 1,2,3,4,7 and 8 have identical earliest event time and latest event time *i.e.*,

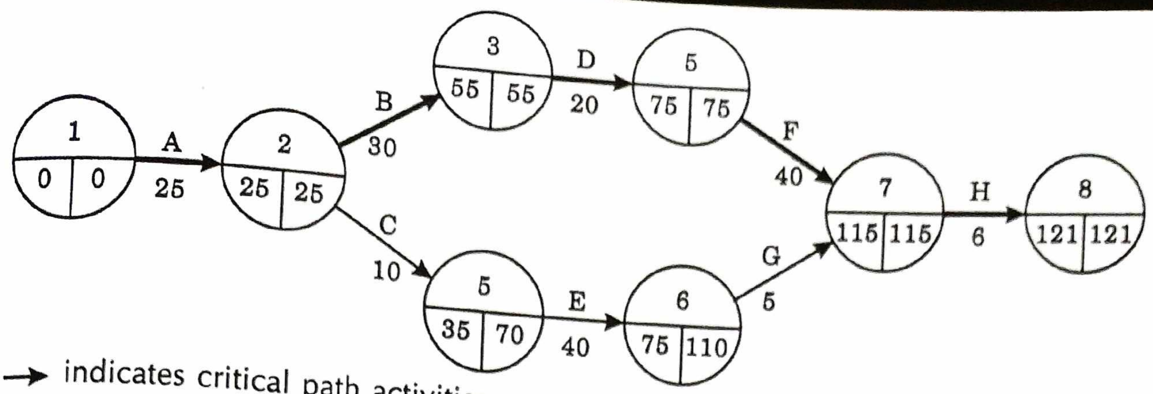
For event	1,	EE = 0	LE = 0
– do –	2	EE = 25	LE = 25
– do –	3	EE = 55	LE = 55
– do –	4	EE = 75	LE = 75
– do –	7	EE = 115	LE = 115
– do –	8	EE = 121	LE = 121

Hence the activities connecting events 1,2,3,4,7 and 8 are on critical path. In otherwards, all these activities do not posses any slack or time reserves and hence designated as critical.

Every network (where EE = LE) contains one or several such uninterrupted chains of critical activities leading from the start event to the end event. This chain is designated as critical path *exhibit 16.12* shows the critical path depicted by dark or heavier lines.

Any delay in an activity situated on the critical path thus entails directly a delay in the completion of the project. Therefore the project manager has to quite particularly focus his attention on the critical path and do his best to complete the activities on the critical path within the expected duration of their completion.

Exhibit 16.12 : Determination of Critical Path



→ indicates critical path activities.

Critical path = 1→ 2→ 3→ 4→ 7→ 8.

(iv) ***Assessment of the Floats or Slacks***

In case of the difference between earliest starting time (ESA) and latest ending time (LEA) of an activity exceeding its duration, then the respective activity is not critical *i.e.,* LEA – ESA > D. In such cases certain time reserves are ensuring termed floats or slacks. The slack analysis is done for each activity path to identify the sub-critical paths *i.e.,* activity paths which are likely to become critical if there is delay in completion of the activities by a time period more than the available slack in each activity path.

Floats or Slacks: Allowable slippage for a path. Event slack is the difference between latest event time and earliest event time.

Slack analysis can be made from the stand point of the events or activities in the network. Two type of slacks identified are

(a) Event slack *(b)* Activity slack

Event slack = Latest event time – Earliest event time

= LE – EE

All events having zero slack represent critical events *i.e.,* events on the critical path. Any schedule slippage of these events will cause delay in completion of the project.

Activity slack analysis provides with the information on the margin of allowance available for the commencement and completion of various activities. Activities with zero slack value represent activities on the critical path.

Activity slack = (LEA – ESA) – D. Where LEA = Latest ending time ESA = Earliest starting time and D = Duration of the activity.

Three types of activity slacks or floats are identified:

(i) Total float or Slack

(ii) Free float or Slack

(iii) Independent float or Slack

(i) **Total float :** Total float usually referred to as simply float or slack is the amount of time an activity can be delayed beyond its earliest possible starting time without delaying the project completion, if other activities take their estimated duration. Total float gives some indication of the criticality of an activity. An activity with little float stands a good chance of delaying the project and should be carefully monitored

Total float for activity (i – j) = LE (j) – EE (i) – D

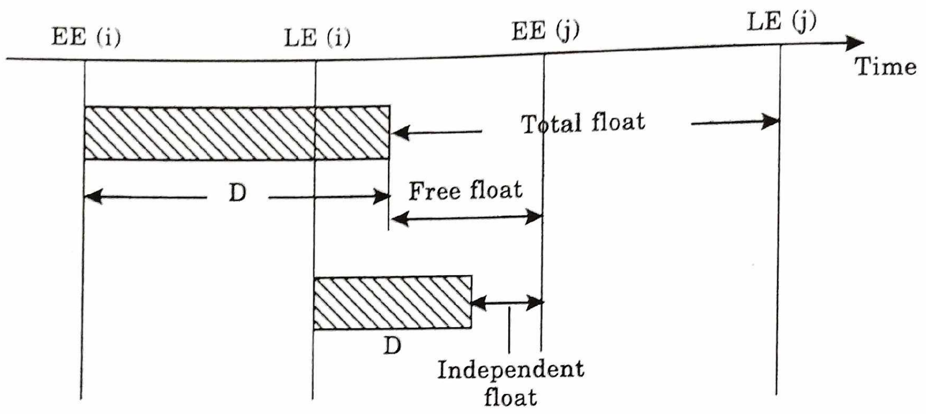
(ii) ***Free float* :** Free float is the amount of time an activity can be delayed without delaying the early start of a successor activity. To find free float, we subtract the early finish of an activity from the early start times for of its succeeding activities.

Free float for activity (i – j) = EE (j) – EE (i) – D

(iii) *Independent Float :* This indicates the time span by which the activity (i – j) can be expanded or shifted if, for the event (i) the latest and for the event (j) the earliest times of occurrence shall be maintained. A shifting of the activity in this area has no influence on the further progress of the project. Independent float can be negative also. Since a negative float has no meaning, independent float is taken as zero if negative.

Exhibit 16.13 illustrates the relationship between the three type of floats and their calculations for an activity (i – j).

Exhibit 16.13 : Relationship Between 3 Types of Floats



EE (i) = Earliest event time for event (i)

LE (i) = Latest event time for event (i)

EE (j) = Earliest event time for event (j)

LE (j) = Latest event for event (j)

D = Duration of activity i – j

Total float TF = LE (j) – EE (i) – D

Free float FF = EE (j) – EE (i) – D

Independent float IF = EE (j) – LE (i) – D

If D > [EE (j) – LE (i)], IF will be negative (assumed to be zero).

Illustration : For the net work diagram shown at *exhibit 16.12*, the activities, earliest event time (EE) and latest event time (LE) and the duration of the activity (D) are listed in the *Table 16.4*.

Table 16.4 lists the total float, free float and independent float for the activities listed in *Table 16.4*.

Programme Evaluation and Review Technique (PERT)

So far we have discussed the procedure for determining the project completion time, the earliest and latest times for the start and completion of activities and the occurrence of events. In C.P.M. Analysis, activity durations are assumed to be known whereas in PERT, the activity duration are given by probability distributions. PERT calculates the expected duration of an activity as a weighted average of the three time estimates as shown in *Exhibit 16.14*.

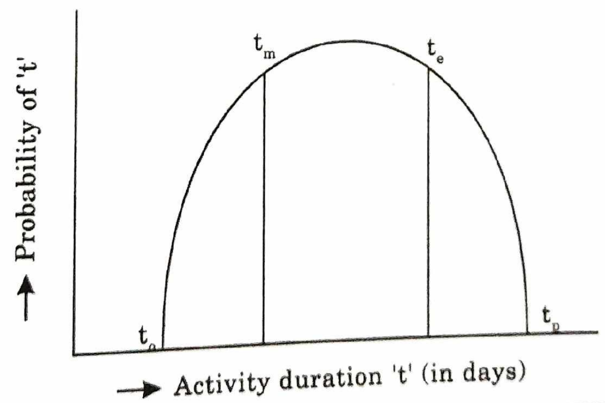
Table 16.4 : Activity Completion Time and Duration

Activity	EE (i)	LE (i)	EE (j)	LF (j)	D
1-2	0	0	25	25	30
2-3	25	25	55	55	10
2-5	25	25	35	70	20
3-4	55	55	75	75	40
5-6	35	70	75	110	40
4-7	75	75	115	115	5
6-7	75	110	115	115	6
7-8	115	115	121	121	

Table 16.5 : Calculation of Three Types of Floats

Activity	Total float (TF)	Free float (FF)	Independent float (IF)
1 - 2	(25 – 0) – 25 = 0	(25 – 0) – 25 = 0	(25 – 0) – 25 = 0
2 - 3	(55 – 25) – 30 = 0	(55 – 25) – 30 = 0	(55 – 25) – 30 = 0
2 - 5	(70 – 25) – 10 = 35	(35 – 25) – 10 = 0	(35 – 25) – 10 = 0
3 - 4	(75 – 55) – 20 = 0	(75 – 55) – 20 = 0	(75 – 55) – 20 = 0
5 - 6	(110 – 35) – 40 = 35	(75 – 35) – 40 = 0	(75 – 70) – 40 = 35 or nil
4 - 7	(115 – 75) – 40 = 0	(115 – 75) – 40 = 0	(115 – 75) – 40 = 0
6 - 7	(115 – 75) – 5 = 35	(115 – 75) – 5 = 35	(115 – 110) – 5 = 0
7 - 8	(121 – 115) – 6 = 0	(121 – 115) – 6 = 0	(121 – 115) – 6 = 0

Exhibit 16.14



The Average or expected time is, $t_e = \dfrac{t_o + 4t_m + t_p}{6}$

Where t_e = expected activity time, t_o = Optimistic time

t_p = Pessimistic time, t_m = Most likely time.

These are defined in the following lines.

(a) The optimistic time (t_o) is the particular time estimate that has a very small probability of being reached *i.e.*, a probability of 1 in 100. This particular time estimate represents the time in which an activity could be completed if every thing went along property with no problem. This would be most unusual, but it could happen with a probability of 1 in 100.

(b) The Pessimistic time (t_p) is another particular time estimate that has a very small probability of being realized, once again with a probability of 1 in 100. This time estimate represents the time in which an activity could be completed even if everything went wrong. This also would be most unusual, but could happen with a probability of 1 in 100.

(c) The most likely time (t_m) is a particular time that in the mind of the estimator, the time the activity would most often require if the work were done again and again under identical conditions. This has a probability of 50 in 100.

The PERT network provides a measure of the probability of completing the project by the *scheduled* date The probability concept is only associated with PERT and not CPM because the activity time estimates in CPM are deterministic (*i.e.*, known) and not probabilistic.

In PERT, the assessment of uncertainty for the entire network *i.e.*, the probability of occurrences of the end event of the project is related to the degree of uncertainty - associated with the three time estimates t_o, t_m and t_p PERT is almost identical to CPM in regard to its function, network diagram, calculations etc., except that the method of estimating activity times are different *i.e.*, in CPM, an activity duration is based on a single time estimate whereas there are three time estimates made for each activity in PERT which is converted into one time estimate (*i.e.*, expected time t_e) using the formula $t_e = \frac{t_o + 4t_m + t_p}{6}$.

Activity standard deviation: The measure of error associated with the expected time of an activity.

Activity standard deviation : The measure of error associated with the expected time of an activity is known as activity standard deviation.

For example, if the expected time of an activity is 8 days and its standard deviation is 2 days, it means that the probability of completing the activity between 8 – 2 = 6 days and 8 + 2 = 10 days in calculated as $\frac{10-6}{6} = \frac{4}{6} = 0.66$.

i.e., if an activity's optimistic time is t_o and its pessimistic time is t_p, then the standard deviation $\sigma = \frac{t_p - t_o}{6}$.

Event standard deviation: The measure of error associated with the cumulative expected time of an event.

Event standard deviation : The measure of error associated with the cumulative expected time of an event is called the event standard deviation. The cumulative expected time (T_E) of an event represents the sum of the expected times of all the activities leading to the event along the longest path.

If the activities leading to an event are x, y and z, then

$$\text{Event standard deviation } \sigma_{TE} = \sqrt{\sum_{i-j}^{n} \sigma_{i-j}^2}$$

$$\text{or} \quad \sigma_{TE} = \sqrt{\left(\frac{t_{px} - t_{ox}}{6}\right)^2 + \left(\frac{t_{py} - t_{oy}}{6}\right)^2 + \left(\frac{t_{pz} - t_{oz}}{6}\right)^2}$$

$$\text{or} \quad \sigma_{TE} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}$$

Where σ_x, σ_y, σ_z are the standard deviation for activities x, y and z and

$$\sigma_z = \frac{t_{px} - t_{ox}}{6}, \quad \sigma_y = \frac{t_{py} - t_{oy}}{6} \quad \text{and} \quad \sigma_z = \frac{t_{pz} - t_{oz}}{6}$$

t_{px}, t_{py} and t_{pz} are the pessimistic times of activities x, y and z respectively.

t_{ox}, t_{oy} and t_{oz} are the optimistic times of activities x, y and z respectively.

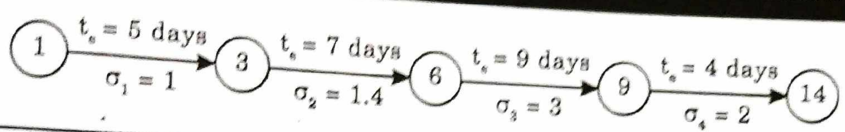
Determining Probability of Meeting Scheduled Date in PERT Analysis

Cumulative expected time of an event T_E represents a 50-50 chance. If a scheduled date T_S is given for an event and if it indicates a value higher than T_E value, then the probability of completing the event by scheduled date is greater than 50%. If T_S is lower than T_E than the probability of completing the event by scheduled date is less than 50%.

Determination of the probability of meeting the scheduled date can be best illustrated with the help of an example given below:

Exhibit 16.15 represents the critical path in a PERT network

Exhibit 16.15 : Activities Along Critical Path with Expected Times and Standard Deviations



The cumulative expected time T_E of event 14 is say 25 days.

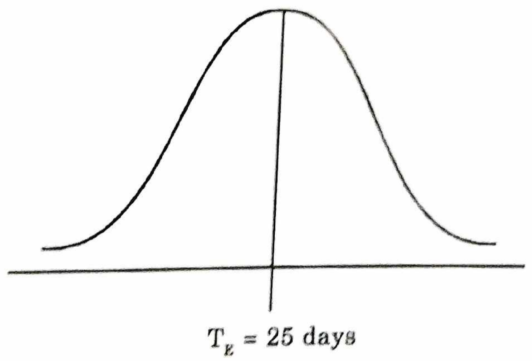
(a) Determine the probability of completing the project in 28 days and 21 days.

(b) What is the expected time of completion which assures the management a 95% confidence level of completing the project.

Solution :

The distribution of completion time of the project is assumed to follow normal distribution as shown in *exhibit 16.16.*

Exhibit 16.16 : Distribution of Completion Time of the Projects



Standard deviation for the end event (event number 14)

$$\text{is } \sigma_{TE} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2} = \sqrt{1 + 1.96 + 9 + 4} = \sqrt{15.96} \approx 4$$

To determine the probability of completing the project by 28 days : Since this completion period of 28 days is more than the expected completion time of 25 days, the probability is