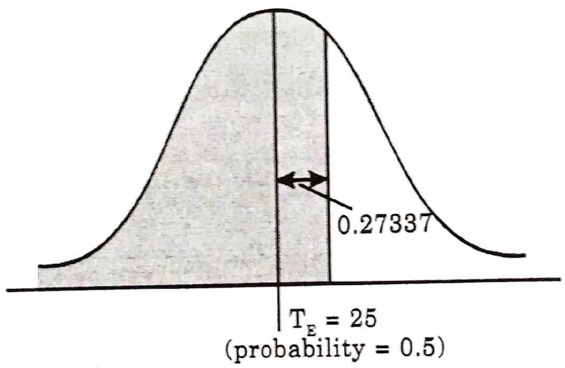
greater than 0.5. But the question is by how much. To find this, the number of standard deviations (Z) has to be calculated as

$$Z_1 = \frac{T_S - T_E}{\sigma_{TE}} = \frac{28 - 25}{4} = 0.75 \text{ standard deviation}$$

Where T_S = Scheduled time ;
T_E = Cumulative expected time
T_E = Event standard deviation

Exhibit 16.17 shows the area under the normal distribution curve which corresponds to 0.75. This can be ascertained from the table of values under a normal curve (Appendix A) that the area corresponding to 0.75σ is 0.27337. The probability of completing the project by 28 weeks is 0.5 + 0.27337 *i.e.*, 0.77337 or a probability of 77%.

Exhibit 16.17 : Area Corresponding to 0.75σ to to the Right of the Mean

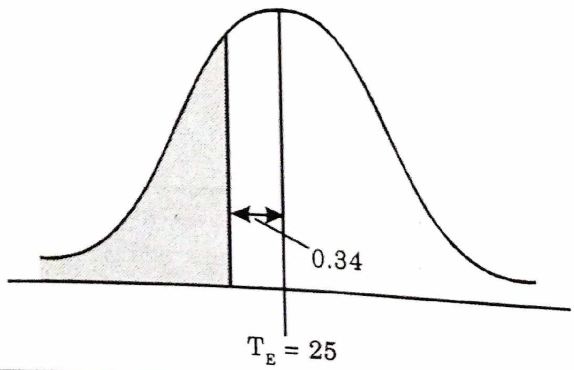


In a similar manner the probability of completing the project by 21 weeks is determined as below:

The number of standard deviation $Z_2 = \frac{T_S - T_E}{\sigma_{TE}} = \frac{21 - 25}{4} = \frac{-4}{4} = -1\sigma$

From the table of values of area under normal curve (Appendix A) it is found that 1s corresponds to an area of 0.34134 (Ref. Exhibit 16.18).

Exhibit 16.18 : Area Corresponding to 1σ to the Left of the Mean



The probability of completion of the project in 21 days is found by subtracting the area under the curve corresponding to 1 σ *i.e.*, 0.34 from 0.5 which is equal to 0.16. This means the probability of completing the project in 21 days is 16%.

To determine the scheduled time for completing the project which assures 95% probability. The area under the curve corresponding to 0.95 is taken into consideration and from the table of values for the area under the normal curve, the number of standard deviation which corresponds to 0.95 has to be found out. This is observed from the table (Appendix A) as 1.65σ to the right of the mean (*i.e.*, T_E) The scheduled time is calculated as $T_E + 1.64\sigma$ *i.e.*, 25 + 1.64 × 4 = 31.6 days.

Crashing CPM Networks

Crashing CPM Networks: To reduce the normal project duration to the desired shorter duration activities on the critical path are crashed by the use of additional resources\expenses.

In CPM network, deterministic activity time estimate are used. These time estimates are developed under conditions of normal working and known as normal time estimates. A normal time estimate does not take into consideration overtime working or use of abnormal resources for completion of an activity. The project duration time in CPM networks is determined based on normal time estimates for various activities in the network. Hence this project duration time is known as normal time to complete the project. However if the project manager desires to reduce the total duration of the project to be below the normal duration, it is necessary to analyse the critical path for a possible reduction of the original time estimates for some of the activities on the critical path. This reduce time estimates is called "crash time estimate". The project manager has same options such as hiring additional workers and machines or increasing control with supervision on critical activities to reduce their completion time.

In these instances, the goal is to identify the time versus cost trade-offs and to evaluate alternative plans for minimising the sum of the indirect and direct project costs.

The cost of resources required to complete an activity in the normal time is known as 'normal cost' and the cost of completing an activity in less than normal time by crashing or reducing the duration by using more resources is known as 'crash cost'. The crash cost is bound to be higher than the normal cost.

To compress the normal project duration to the desired duration time, we have to concentrate on the crashing of activities lying on the critical path. As we crash the activities on the critical path, we must ensure that no other path on the network becomes critical. If in the processing of crashing the activities, some other path becomes critical then the activities on that path must be considered for crashing.

Steps in Network Crashing

Step 1 : Determine the time-cost ratio for each activity in the network. This ratio represents the increase in cost for a unit decrease in time.

$$\text{Time-cost ratio} = \frac{\text{Increase in cost}}{\text{decrease in time}}$$

$$= \frac{\text{crash cost} - \text{normal cost}}{\text{normal time} - \text{crash time}}$$

Step 2 : Identify the activities on the critical path and select that activity on the critical path which has the smallest time-cost ratio and crash that activity to the extent possible.

Step 3 : Observe whether there is any change in the critical path *i.e.*, any other path becomes critical. If so, calculate the time-cost ratio for the activities in the new critical path.

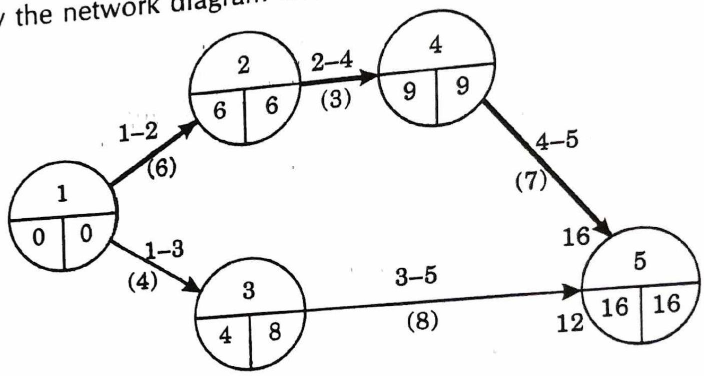
Step 4 : Repeat step 2 and 3, till the activities are crashed to reduce the project duration to the desired time period.

The network crashing is better understood through the following illustration.

Illustration :

Activity	Time in weeks		Cost (Rs.)	
	Normal	Crash	Normal	Crash
1-2	6	4	10,000	14,000
1-3	4	3	5,000	8,000
2-4	3	2	4,000	5,000
3-5	8	6	9,000	12,000
4-5	7	4	7,000	8,000
		Total cost	**Rs. 35,000**	**Rs. 47,000**

Step 1 : Draw the network diagram with normal times for activities.



Critical path is 1 → 2, 2 → 4, 4 → 5

Project completion duration = 16 weeks

The normal estimate of project completion is 16 weeks at a cost of Rs. 35,000/-.

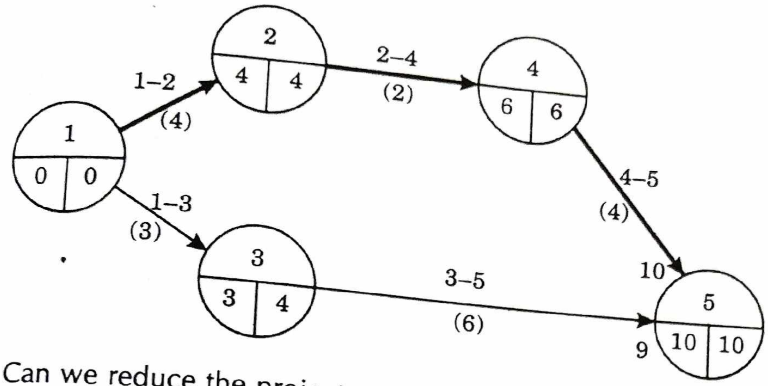
Step 2 : Calculate time cost ratio for each activity.

$$\text{Time-cost ratio} = \frac{\text{Increase in cost}}{\text{Decrease in time}} = \frac{\text{crash cost} - \text{normal cost}}{\text{normal time} - \text{crash time}}$$

Activity	Time-cost ratio
* 1 – 2	$\frac{14{,}000 - 10{,}000}{6 - 4} = \frac{4{,}000}{2}$ = Rs. 2,000/week
1 – 3	$\frac{8{,}000 - 5{,}000}{4 - 3} = \frac{3{,}000}{1}$ = Rs. 3,000/week
* 2 – 4	$\frac{5{,}000 - 4{,}000}{3 - 2} = \frac{1{,}000}{1}$ = Rs. 1,000/week
3 – 5	$\frac{12{,}000 - 9{,}000}{8 - 6} = \frac{3{,}000}{2}$ = Rs. 1,500/week
* 4 – 5	$\frac{8{,}000 - 7{,}000}{7 - 4} = \frac{1{,}000}{3}$ = Rs. 333.33/week
* – indicates activities on critical path.	

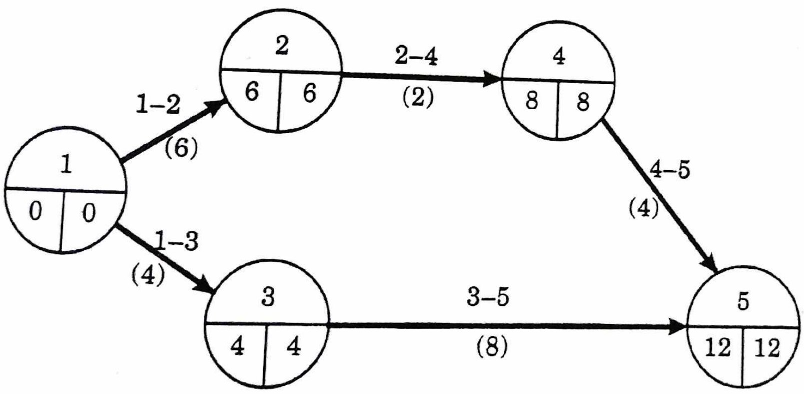
Step 3 : Draw the network diagram with all crash timings for the activity and find the project completion duration as below:

With crash times for each activity it is found that the project can be completed in 10 weeks and the corresponding cost is Rs. 47,000.



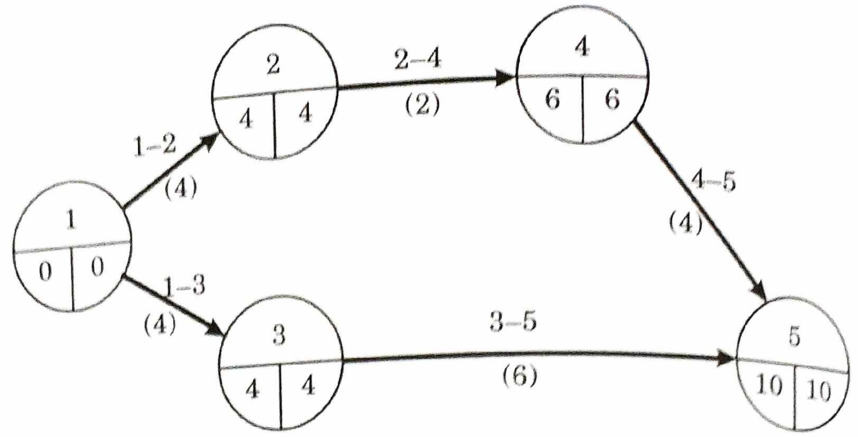
Step 4 : Can we reduce the project completion time to 10 weeks without increasing the cost by Rs. 12,000? To determine this, we have to crash the network by 16 – 10 = 6 weeks by crashing the time for each activity on the critical path to the extent possible. The activities on critical path are selected for crashing one by one based on their time-cost ratio, taking the activity having the least time-cost ratio first for the purpose of crashing (*i.e.,* to reduce the normal time to crash time).

Since activity 4-5 on critical path has the smallest T-C ratio (*i.e.,* Rs. 333.33/week), crash this activity to the extent possible *i.e.,* by 3 weeks (*i.e.,* from normal time of 7 weeks to crash time of 4 weeks) Since we have to reduce the project duration by 6 weeks *i.e.,* from 16 to 10 weeks, the next activity on critical path to be selected is activity 2-4, which has the next lowest Time-Cost ratio (*i.e.,* Rs. 1000/week). Activity 2-4 can be crashed by one week (*i.e.,* from 3 to 2 weeks). The network diagram is redrawn with crash times for activity 4-5 and 2-4 as below:



It can be observed that both paths 1→ 2, 2→ 4, 4→ 5 and 1→ 3, 3→ 5 become ritical paths. The project completion time is now reduced to 12 weeks from 16 weeks.

To reduced the project duration to 10 weeks, from 12 weeks. We have to reduce both itical paths by 2 weeks. On path 1-2, 2-4, 4-5, reduce the activity time of activity 1-2 by 2 eeks *i.e.,* from 6 weeks to 4 weeks on path 1-3, 3-5, reduce the activity time of activity 5 by 2 weeks *i.e.,* from 8 weeks to 6 weeks (Note that activity 3-5 has lesser Time-cost ratio in 1-3). The modified network diagram is show below:



The crash cost to complete the project in 10 weeks is calculated as below:

Activity	Activity Time (weeks)	Cost (Rs.)
1-2	4 (crash)	14,000 (crash)
1-3	4 (normal)	5,000 (normal)
2-4	2 (crash)	5,000 (crash)
3-5	6 (crash)	12,000 (crash)
4-5	4 (crash)	8,000 (crash)
	Total cost	Rs. 44,000/-

The increase in cost to crash the project completion time from 16 weeks to 10 weeks Rs. 9,000/- *i.e.,* (Rs. 44,000 – Rs. 35,000) instead of Rs. 12,000/- is we use crash time and crash cost for all the activities to complete the project in 10 weeks.

Comparison of PERT and CPM Methods

(i) PERT is a network method for project scheduling which was first developed in the 1950s for the Polaris submarine project. This technique was used to schedule more than 3000 contractors, suppliers and agencies and was successful in completing the Polaris submarine project ahead of schedule by about two years.

Critical path method (CPM) was developed by E.I. du Pont De Nemours & Co as a way to schedule the start up and shut down of major plants. Since these plant activities were repeated frequently the times were fairly well known.

(ii) The main difference between PERT and CPM networks is whether the time estimates are *probabilistic* or *deterministic.*

If the time estimates can be made with high degree of certainty, such time estimates are called deterministic time estimates. On the contrary, if the actual times are subject to variation and can only be estimated with probability of variation, such time estimates are called "probabilistic time estimates".

(iii) PERT requires three time estimates for each activity : an **optimistic time estimate** (t_o), **most likely time estimate** (t_m) and a **pessimistic time estimate** (t_p). This is necessary because of the uncertainty in activity time that is typical of R & D projects. The **mean time** or **expected time** (t_e) is calculated as $\frac{t_o + 4t_m + t_p}{6}$.

In CPM , the activity time estimates are **deterministic** and may be determined fairly accurately based on previous experience.

(iv) In a PERT network, the individual activity times are random and hence the project completion time will also be random. Hence it is not possible to set concrete project

completion dates. Each project completion date has a probability of being met, which is a function of the uncertainties in the individual activities and the precedence relationships. The probability associated with start and completion times of each activity or event is also calculated for a PERT network.

In CPM network, the activity time is estimated based on the resources employed to carryout the activity. However, the time of any activity can be compressed by allocating more resources to carryout that activity (*i.e.*, by spending more money). Thus CPM assures a time-cost trade-off rather than the probabilistic times used in PERT. To express the time-cost relationship, in CPM network, **normal time**, **normal cost**, **crash time** and **crash cost** are used.

Applying Project Scheduling to Service Firms

Every firm whether it is a manufacturing or a service firm, needs to plan, schedule and complete large projects at one point of time or another. For example, installing a new computer system or relocating a hospital are large service projects for which PERT and CPM networks can be used. For relocating a hospital, a large variety of planning considerations had to be taken into account such as vehicles and ambulances to be used to move patients, police escorts for the vehicles etc. To coordinate all the activities, a project network will have to be developed before the move.

Evaluation of PERT and CPM

Advantages

(*i*) Useful at several stages of project management, especially in the scheduling and control of large projects.

(*ii*) Simple in concept and not complex mathematically.

(*iii*) Networks help to perceive quickly relationships among project activities.

(*iv*) Critical path and slack time analysis help pinpoint activities that need to be closely watched.

(*v*) Useful in monitoring both time schedules and costs.

(*vi*) Applicable to a wide variety of projects and industries.

Limitations

(*i*) Project activities must be clearly defined, independent and stable in their relationship.

(*ii*) Precedence relationships must be specified and networked together.

(*iii*) Time estimates tend to be subjective and are subject to judging by managers.

(*iv*) Too much emphasis is placed on the longest or critical path and near critical path is not monitored closely.

Use of Computers in Project Management

Project network may contain thousands of activities and may represent work that spans several years. Drawing such network would require enormous amount of time and it is difficult to keep it up to date as the project progresses and it would continually have to be redrawn. Computers are highly useful to support project management especially for large projects. A computer can store, update and selectively retrieve data about a project and can provide valuable outputs in terms of plans and status reports. Actual start dates, completion dates, cost etc., can be recorded as the project progresses.

The computer can provide a rolling wave of detailed information as the project progresses. It may be valuable to sort activities in the order of increasing slack to identify the most critical activities in the project. The activities may be sorted to retrieve all activities with early start dates within a specified planning horizon to find where available resources may be utilised. The activities may be sorted to show all late start dates within that time to keep the project on schedule.

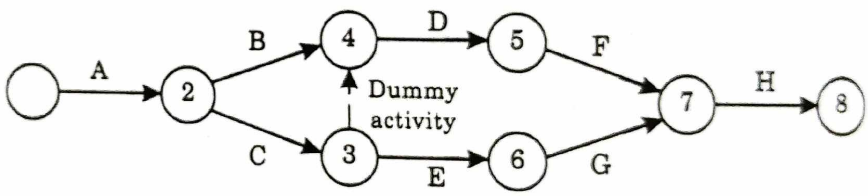
Many software packages are available for support of project management. The programs can provide graphics for networks, Gantt Charts and a variety of reports.

SOLVED PROBLEMS

1. *Draw the network diagram for the data given below:*

Activity	Immediate predecessor Activity
A	-
B	A
C	A
D	B, C
E	C
F	D
G	E
H	F, G

Solution :



The dummy activity 3-4 has been introduced here because activity B and C both precede activity D and activity C precedes activity E.

2. *Draw a network diagram for the activities given below and determine the critical path.*

Activity	Description of activity	Duration of activity (month)	Immediate Predecessor Activity
A	Design Plant	12	-
B	Select Site	8	A
C	Select Vendor	4	A
D	Select personnel	3	A
E	Prepare site	12	B
F	Manufacture Generator	18	C
G	Prepare Operations Manual	5	C
H	Install Generator	4	E, F
I	Train Operators	9	D, G
J	Obtain Licence	6	H, I