$$\left\{1+\frac{\sqrt{2}}{n}\right\} \longrightarrow 1 \notin \mathbb{Q}^c.$$

$\mathbb{Q}$. $\qquad \underline{\mathbb{R}}$ $\qquad \mathbb{Q}^c$.

$2.999\ldots$

3

$x_1 = 1.4$

$x_2 = 1.41$

$x_3 = 1.414$

$\vdots$

$$\{x_n\} \longrightarrow \sqrt{2} \notin \mathbb{Q}.$$

$\pi = 3.14\ldots$

Ex: $\mathbb{R}^n$ with $\|x\| = \left(\sum_{i=1}^{n} x_i|^2 \right)^{1/2}$

$x \in \mathbb{R}^n$

$(x_1, x_2, \cdots x_n)$

Ex: Space ℓ^p.

with

$$\|x\| = \left(\sum_{i=1}^{\infty} |x_i|^p \right)^{1/p}$$

Ex. $\mathbb{C}^n$ with

$$\|x\| = \left(\sum |x_i|^2 \right)^{1/2}$$

Ex Space ℓ^2.

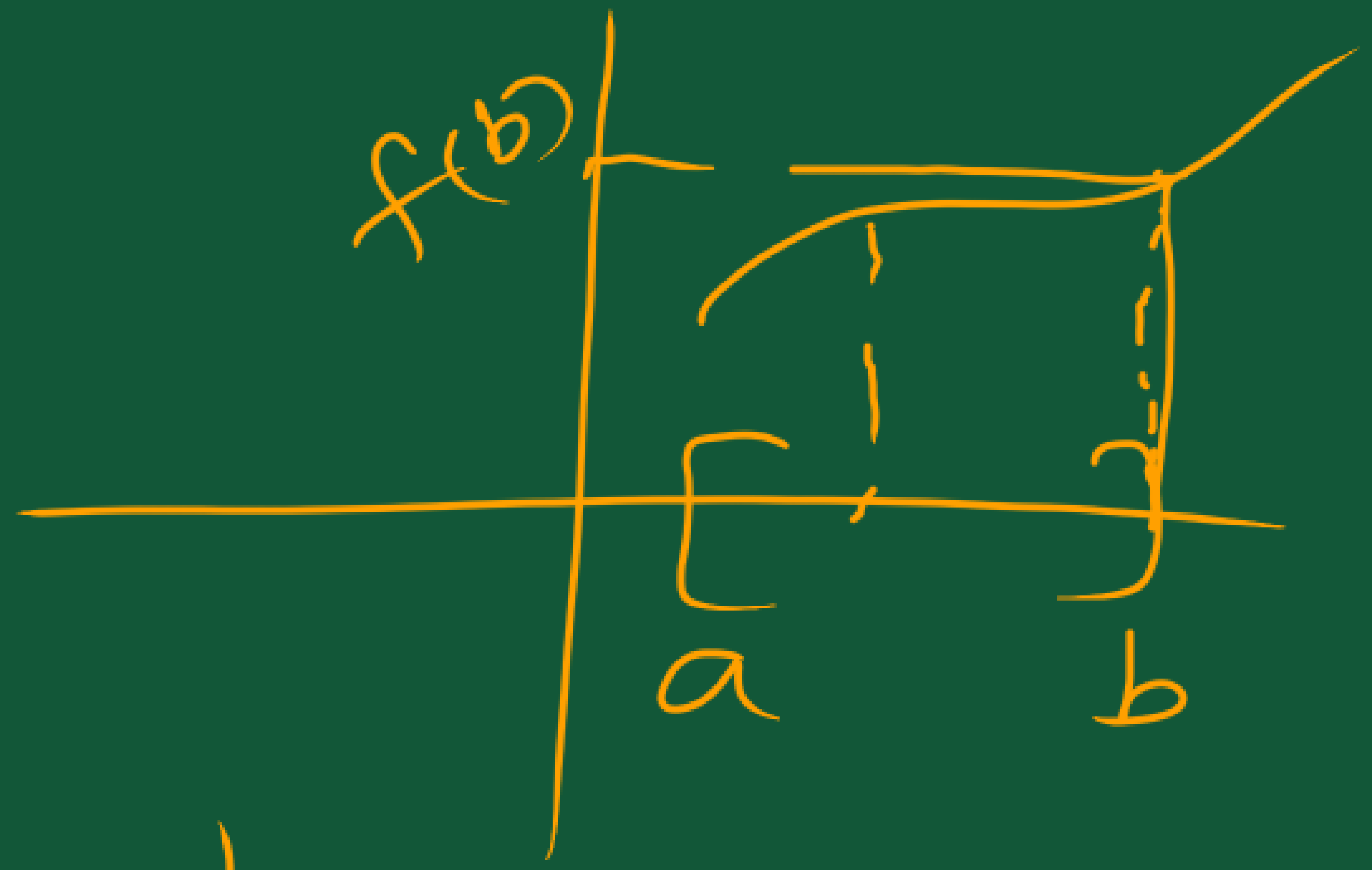
with
$$\|x\| = \left(\sum_{i=1}^{\infty} |x_i|^2\right)^{1/2}$$

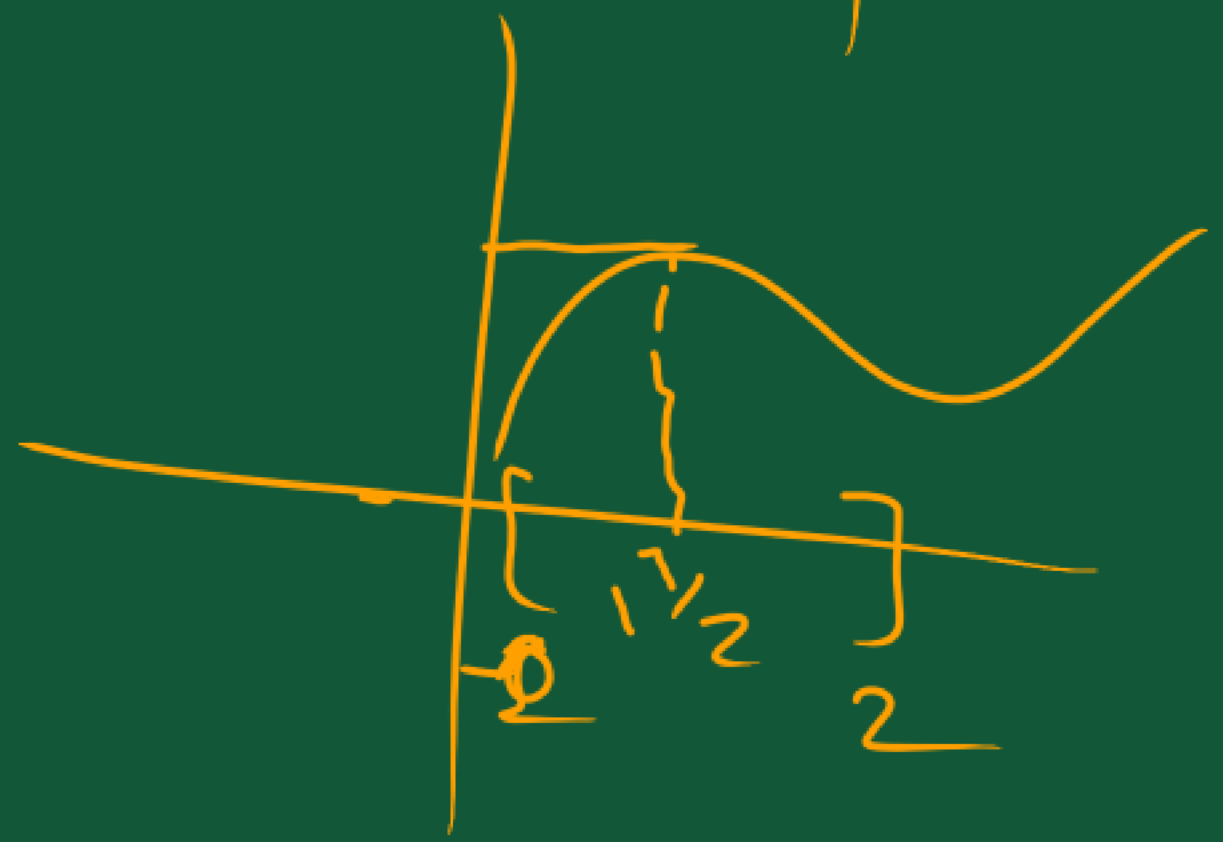
Ex - Space ℓ^∞.

$$\|x\| = \sup_i |x_i|$$

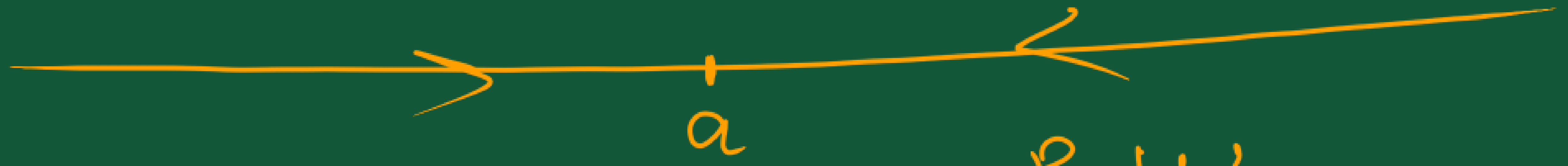
Ex: Space $C[a,b]$.

with norm $\|x\| = \max_{t \in J} |x(t)|$ where $J = [a,b]$





$f(x)$



L.H.L. R.H.L.

$f(x, y)$

$$\|x\| = \left(\int_0^1 (x(t))^2 \, dt \right)^{1/2}.$$

If we take

$$x_n(t) = 0 \qquad \text{if } t \in [0, 1/2]$$

$$x_n(t) = 1 \qquad \text{if } t \in [a_n, 1].$$

where $a_n = \frac{1}{2} + \frac{1}{n}$

Hence for every $x \in X$

$$\|x_m - x\| = \left(\int_0^1 (x_m(t) - x(t))^2 \, dt \right)^{1/2}$$

$$\leq \left(\int_0^{1/2} (x_m(t) - x(t))^2 \, dt \right)^{1/2} + \left(\int_{1/2}^{a_m} (x_m(t) - x(t))^2 \, dt \right)^{1/2} + \left(\int_{a_m}^{1} (1 - x(t))^2 \, dt \right)^{1/2}.$$

(By Holder's inequality)

$$d(x,y) + d(y,z) \geq d(x,z)$$

$x(t) = 0$ if $t \in [0, 1/2]$

& $x(t) = 1$ if $t \in (1/2, 1]$.

$$\boxed{d(x,y) = \|x-y\|.}$$

$\|\cdot\| \longrightarrow$ any norm.

(i) $d(x,y) = d(y,x)$

$\|x-y\|$

$\|-1(y-x)\|$

$= |-1|\,\|y-x\|$

$= \|(y-x)\|$

$= d(y,x)$

$$d(x,y) = \sum_{j=1}^{\infty} \frac{1}{2^j} \frac{|\xi_i - \eta_i|}{1+(\xi_i - \eta_i)}$$

where $x = (\xi_1, \xi_2, \ldots \xi_n \ldots)$

$y = (\eta_1, \eta_2, \ldots \eta_n \ldots)$

$0v.$

$$0 = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 \quad \times$$

$$S = \{v_1 \ldots v_k\}$$

$$\sum_{i=1}^{k} \alpha_i v_i = 0$$

$$\alpha_i = 0$$

$$\forall i$$

(a) $d(x+a, y+a) = d(x,y)$

(b) $d(\alpha x, \alpha y) = |\alpha|\, d(x,y)$.

$\forall\ x, y, a \in X$ & $\alpha \in \mathbb{F}$.

Proof :- We have

(i) $d(x+a, y+a) = \|x+a-(y+a)\| = \|x-y\| = d(x,y)$.

(ii) $d(\alpha x, \alpha y) = \|\alpha x - \alpha y\| = |\alpha|\,\|x-y\| = |\alpha|\, d(x,y)$.

$V = \mathbb{R}^3(\mathbb{R})$

$S = \{(1,0,0), (1,1,0), (1,1,1)\}$

$\boxed{[L(S)] = V}$

$$v = (v_1, v_2, v_3) = \underline{\alpha}(1,0,0) + \underline{\beta}(1,1,0) + \underline{\gamma}(1,1,1)$$

$\neq 2$

$$\alpha(1,0,0) = (0,0,0) \qquad \alpha = 0.$$

$$\alpha(0,0,0) = (0,0,0) \qquad \underline{\alpha = 0}$$