Particular integral of non-homogeneous linear P.D.E. (reducible or irreducible) namely

$$F(D, D')z = f(x, y).$$

Case-I :- When $f(x,y) = e^{ax+by}$ and $F(a,b) \neq 0$

then $P.I. = \dfrac{1}{F(D,D')} e^{ax+by} = \dfrac{1}{F(a,b)} e^{ax+by}$

Thus $D \to a$ & $D' \to b$.

Case-II :- When $f(x,y) = \sin(ax+by)$ / $\cos(ax+by)$

then $P.I. = \dfrac{1}{F(D,D')} \sin(ax+by)$

or $P.I. = \dfrac{1}{F(D,D')} \cos(ax+by)$

which is evaluated by putting $D^2 = -a^2$, $D'^2 = -b^2$, $DD' = -ab$

Case-III – When $f(x,y) = x^m y^n$

then
$$P.I. = \frac{1}{F(D,D')} x^m y^n$$
$$= [F(D,D')]^{-1} x^m y^n$$

which is evaluated by expanding $(F[D,D'])^{-1}$ in ascending power of $\frac{D'}{D}$ or $\frac{D}{D'}$ or D or D' as the case may be.

Case (IV) When $f(x,y) = V e^{ax+by}$ where V is a f^n of x & y.

Then
$$P.I. = \frac{1}{F(D,D')} V e^{ax+by}$$
$$= e^{ax+by} \frac{1}{F(D+a, D'+b)} V.$$

Remark – If $F(a,b) = 0$ & $f(x,y) = e^{ax+by}$ then we have
$$P.I. = \frac{1}{F(D,D')} e^{ax+by}$$

$$P.I. = \frac{1}{F(D,D')} e^{ax+by} \cdot 1$$

$$= e^{ax+by} \cdot \frac{1}{F(D+a, D'+b)} \cdot 1.$$

Qus- Find the P.I. of

$$(D - D' - 1)(D - D' - 2)z = \sin(2x+3y)$$

Solⁿ- $C.F. = e^{x}\phi_1(y+x) + e^{2x}\phi_2(y+x)$

$$P.I. = \frac{1}{(D-D'-1)(D-D'-2)} \sin(2x+3y)$$

$$= \frac{1}{D^2 - 2DD' + D'^2 - 3D + 3D' + 2} \sin(2x+3y)$$

$$= \frac{1}{3(D'-D)+1} \sin(2x+3y)$$

$$= \frac{3(D'-D)-1}{9(D'^2 - 2DD' + D^2) - 1} \sin(2x+3y)$$

$$= \frac{(3D' - 3D - 1)}{9(-9) - 18(-3\cdot 2) + 9(-4) - 1} \sin(2x+3y)$$

$$= \frac{1}{10} \sin(2x+3y) - 3\cos(2x+3y).$$

Solve. $(D^2 - D')z = xe^{ax+a^2y}$

Solⁿ:- $P.I. = \frac{1}{(D^2 - D')} xe^{ax+a^2y}$

$$= e^{ax+a^2y} \frac{1}{(D+a)^2 - (D'+a^2)} x$$

$$= e^{ax+a^2y} \frac{1}{D^2+\cancel{a^2}+2Da-D'-\cancel{a^2}}$$

$$= e^{ax+a^2y} \frac{1}{D^2+2Da-D'} x$$

$$= e^{ax+a^2y} \frac{1}{2Da\left(\frac{D}{2a}+1-\frac{D'}{2Da}\right)} x$$

$$= e^{ax+a^2y} \frac{1}{2Da}\left(1+\frac{D}{2a}-\frac{D'}{2Da}\right)^{-1} x$$

$$= \frac{e^{ax+a^2y}}{2a} \frac{1}{D}\left(1-\left(\frac{D}{2a}-\frac{D'}{2Da}\right)-\cdots\right) x$$

$$= \frac{e^{ax+a^2y}}{2a} \frac{1}{D}\left(x-\frac{Dx}{2a}-\frac{D'x}{2a}-\cdots\right)$$

$$= e^{ax+a^2y}\left\{\frac{x^2}{4a}-\frac{x}{4a^2}\right\}$$