Revised simplex Method :

Step II :

$B = I_m$

$Z = Cx$

where $\hat{A}$ and $\hat{b}$

$\hat{A} = \begin{bmatrix} A \\ -C^T \end{bmatrix}$ and $\hat{b} = \begin{bmatrix} b \\ 0 \end{bmatrix}$

$\hat{B}$

$\hat{B}^{-1}$

$\hat{y}_{ik} \leq 0$, $y_{ik} > 0$

$Z_j - C_j =$

$\begin{bmatrix} C_B B^{-1} & 1 \end{bmatrix} \hat{A}$

$\hat{y}_K = \hat{B}_{curr}^{-1} \hat{a}_K$.

x_B

Theorem:- If B and B' are Banach spaces and if T is a continuous linear transformation of B onto B' then T is an open mapping. (This is called then Open Mapping Theorem).

Proof: We must show that if G is an open set in B then $T(G)$ is also open set in B'. If y is a point in $T(G)$, it suffices to produce an open sphere centred on y and contained in $T(G)$.

Let x be a point in G such that $T(x) = y$. Since G is open, x is the centre of an open sphere which can be written in the form $x + S_r$ contained in G. Our lemma now implies the $T(S_r)$ contains some S'_{r_1}. It is clear that $y + S'_{r_1}$ is an open sphere centred on y and the fact that it is contained in $T(G)$ follows at once from

$$\begin{aligned} y + S'_{r_1} &\subseteq y + T(S_r) \\ &= T(x) + T(S_r) \\ &= T(x + S_r) \\ &\subseteq T(G). \end{aligned}$$

Theorem B —

A one to one continuous linear transformation of one Banach space into another is a homeomorphism.

In particular, if an one to one linear transformation T of a Banach space onto itself is continuous, then its inverse T^{-1} is automatically continuous.

Theorem - C —

If P is a projection on a Banach Space B, and if M and N are its range and null space, then M & N are closed linear subspaces of B such that

$$B = M \oplus N.$$

Proof : The null space of any continuous linear transformation is closed, so N is obviously closed; and the fact that M is also closed is a consequence of

$$M = \{P(x) : x \in B\} = \{x : P(x) = x\} = \{x : (I-P)x = 0\}.$$

which exhibits M as the null space of the operator $I-P$.

& By theorem (B), we can deduce that

$$\boxed{B = M \oplus N}.$$

Similar like vector space.

The closed Graph Theorem -

If B and B' are Banach spaces and if T is linear transformation of B into B', then T is continuous $\Longleftrightarrow$ its graph is closed.

Proof. -

Here we need to show that T is continuous if its graph is closed. We denote by B_1, the linear space B renormed by $\|x\|_1 = \|x\| + \|T(x)\|$.

Since

$$\|T(x)\| \leq \|x\| + \|T(x)\| = \|x\|_1.$$

T is continuous as a mapping of B_1 into B'. It therefore suffice to prove that B and B_1 have same topology. The identity mapping of B_1 onto B is clearly continuous, for $\|x\| \leq \|x\| + \|T(x)\| = \|x\|_1$. If we can show that B_1 is complete then Theorem 'B' will guarantee that this mapping is homeomorphism and this will conclude the theorem. Let $\{x_n\}$ be a Cauchy sequence in B_1. It follows that $\{x_n\}$ and $\{T(x_n)\}$ are also Cauchy sequence in B & B'; and since both of these space are complete, $\exists$ vectors x & y in B and B' such that

$$\|x_n - x\| \to 0 \text{ and}$$
$$\|T(x_n) - y\| \to 0.$$

Our assumption that the graph of T is closed in $B \times B'$ implies (x, y) lies on this graph, so $T(x) = y$.

The completeness of B_1 now follows from

$$
\begin{aligned}
\|x_n - x\|_1 &= \underline{\|x_n - x\| + \|T(x_n - x)\|} \\
&= \|x_n - x\| + \|T(x_n) - T(x)\| \\
&= \|x_n - x\| + \|T(x_n) - y\| \\
&\longrightarrow 0.
\end{aligned}
$$

Hence Theorem is proved.