Que:- Solve $yp + p = \cos(x+y) - y\sin(x+y)$

Sol^n:-

$$y\frac{\partial p}{\partial y} + p = \cos(x+y) - y\sin(x+y)$$

$$\Rightarrow \frac{\partial p}{\partial y} + \frac{p}{y} = \frac{1}{y}\cos(x+y) - \sin(x+y) \longrightarrow$$

p & y as variable & x as constant.

$$I.F. = e^{\int \frac{1}{y}dy} = e^{\log y} = y.$$

Solⁿ will be

$$p\cdot(I.F.) = \int (I.F.)\left(\frac{1}{y}\cos(x+y) - \sin(x+y)\right) dy.$$

$$py = \int \left(\cos(x+y) - y\sin(x+y)\right) dy.$$

$$= \sin(x+y) - \left[-y\cos(x+y) - \int -\cos(x+y)\, dy\right].$$

$$= \sin(x+y) + y\cos(x+y) - \sin(x+y) + F(x).$$

$$= y\cos(x+y) + F(x).$$

$$p = \cos(x+y) + \frac{1}{y} F(x)$$

$$\frac{\partial z}{\partial x} = \cos(x+y) + \frac{1}{y} F(x)$$

Integrating w.r.to 'x' we get

$$z = \sin(x+y) + \frac{1}{y}\int F(x)\,dx + \frac{1}{y}\phi(y)$$

$$\boxed{yz = y\sin(x+y) + \psi(x) + \phi(y)}$$

Que- Solve $2yq + y^2 z = 1$.

Ans: $\boxed{z = \log y - \frac{1}{y}\phi_1(x) + \phi_2(x)}$

Type - III -

Under this type, we consider eqn of the form

$$Rr + Ss + Pp = F \quad \text{or} \quad R\left(\frac{\partial p}{\partial x}\right) + S\left(\frac{\partial p}{\partial y}\right) = F - Pp.$$

and

$$Ss + Tt + Qq = F \quad \text{or} \quad S\left(\frac{\partial q}{\partial x}\right) + T\left(\frac{\partial q}{\partial y}\right) = F - Qq$$

These are linear P.D.E of order one with p or q as dependent variable and x, y as independent variable. In such situation, we shall apply well known Lagrange's Method

Recall that

$$Pp + Qq = R$$

is solved by considering its auxiliary eqns

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}.$$

Que: Solve $t + s + q = 0$.

Sol^n:

$$\frac{\partial q}{\partial y} + \frac{\partial p}{\partial y} + \frac{\partial z}{\partial y} = 0$$

Integrating w.r.to 'y', we get

$$q + p + z = \phi(x)$$

$$\Rightarrow \quad p + q = \phi(x) - z$$

L.A.E.

$$\underset{\text{I}}{\frac{dx}{1}} = \underset{\text{II}}{\frac{dy}{1}} = \underset{\text{III}}{\frac{dz}{\phi(x) - z}}$$

I & II ——.

$$dx = dy$$

$$\boxed{x - y = c_1}$$

I & III. ——.

$$\frac{dx}{1} = \frac{dz}{\phi(x) - z}$$

$$\frac{dz}{dx} = \phi(x) - z$$

$$\frac{dz}{dx} + z = \phi(x)$$

$$I.F. = e^{\int dx} = e^{x}$$

Soln will be

$$z \cdot e^{x} = \underline{\int e^{x} \phi(x)\, dx}.$$

$$\boxed{z e^{x} = \psi(x) + C_2} \Rightarrow \boxed{z e^{x} - \psi(x) = C_2}$$

Hence the soln will be

$$\boxed{z e^{x} = \phi(x-y) + \psi(x).}$$

Que:- Solve $p+r+s=1$.

Sol^n:-

$$\frac{\partial z}{\partial x}+\frac{\partial p}{\partial x}+\frac{\partial q}{\partial x}=1.$$

Ans:- $$\boxed{ze^{y}-xe^{y}-\phi(y)=\psi(x-y).}$$