Method of finding C.F of irreducible linear PDE with constant coefficient

Ex- Solve $(D - D'^2) z = 0$ ——— (*)

Sol^n- Here $D - D'^2$ is not linear factor in D & D'. So we let $z = Ae^{hx+ky}$ be a trial sol^n of the given PDE

$Dz = Ah\, e^{hx+ky}$

$D'z = AK\, e^{hx+ky}$

$D'^2 z = AK^2 e^{hx+ky}$

putting values in (*), we get

$$A(h-K^2)\, e^{hx+ky} = 0$$

So that

$$h - K^2 = 0$$

$$\text{or } h = K^2$$

Method of finding C.F of irreducible linear P.D.E with constant coefficient

---

Replacing h by k^2, the most general solⁿ of the given P.D.E is

$$\boxed{z = \sum A e^{k^2 x + ky}}$$

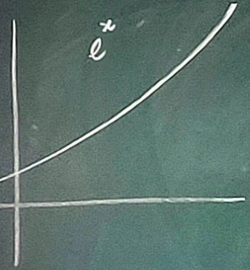
where A & k are arbitrary constants.

Ques:- Solve $(D - 2D' - 1)(D - 2D'^2 - 1)z = 0$

Solⁿ:- Since $(D - 2D' - 1)$ being linear in D & D', the part of C.F. corresponding to it is $e^{x}\phi(y + 2x)$

To find C.F corresponding non-linear in D & D' ie $D - 2D'^2 - 1$. Let ~~the~~ a trial solⁿ be

$$z = A e^{hx + ky}$$



Method of finding C.F of irreducible linear P.D.E with constant coefficient

$$Dz = Ah\,e^{hx+ky}$$

$$D'^2 z = Ak^2 e^{hx+ky}$$

Now putting Dz, $D'^2 z$ & z in the given P.D.E we get

$$A(h-2k^2-1)e^{hx+ky} = 0$$

So that $(h-2k^2-1) = 0$

Hence we have

$$h = 2k^2+1$$

Replacing h by $2k^2+1$, we get the part of CF corresponding to $(D-2D'^2-1)$ so the most general Sol^n of given P.D.E is

$$z = \sum A e^{(2k^2+1)x+ky} + e^x\phi(y+2x)$$