CONTROL

Er. Yastuti Rao Gautam
Asst. Prof.
Mech. Engg. Dept.
U.I.E.T. Kanpur
C.S.J.M.U. Kanpur

Course Code: MEE- S404 T **Breakup: 3 – 0 – 0 – 3**

Course Name: MEASUREMENT AND CONTROLS

Course Details:

Similarity , errors , dynamic response , pitot tube , hot wire anemometer,Laser doppler velocimeter , optical tecniques for field measurement,image processing, volume averaged measurment ,uncertanity analysis signal processing and compensation for probev characteristics. Laplace transform , Inverse Laplace transform.Block diagrams, Transfer functions ,Signal flow graphs , state variable characterization of dynamic systems. Modeling of mechanical system

elements, error sensing devices in control systems.stability,routh hurwitz criterion , Ny quist criterion.Root locus techniques , frequency domain analysis,Brief Introduction of Mecatronics.

Text Books and References:

Automatic control Engineering, Ogatta

Automatic control Engineering , Kuo Mecatronics,HMT

1. Basic Concept of Control System

Control Engineering is concerned with techniques that are used to solve the following six problems in the most efficient manner possible.

(a)The identification problem : to measure the variables and convert data for analysis.
(b)The representation problem: to describe a system by an analytical form or mathematical model
(c)The solution problem: to determine the above system model response.
(d)The stability problem: general qualitative analysis of the system
(e)The design problem: modification of an existing system or develop a new one
(f)The optimization problem: from a variety of design to choose the best.

The two basic approaches to solve these six problems are conventional and modern approach. The electrical oriented conventional approach is based on complex function theory.

The modern approach has mechanical orientation and based on the state variable theory.

Therefore, control engineering is not limited to any engineering discipline but is equally applicable to aeronautical, chemical, mechanical, environmental, civil and electrical engineering.

For example, a control system often includes electrical, mechanical and chemical components. Furthermore, as the understanding of the dynamics of business, social and political systems increases; the ability to control these systems will also increase.

Basic terminologies in control system

System: A combination or arrangement of a number of different physical components to form a whole unit such that that combining unit performs to achieve a certain goal.

Control: The action to command, direct or regulate a system.

Plant or process: The part or component of a system that is required to be controlled.

Input: It is the signal or excitation supplied to a control system.

Output: It is the actual response obtained from the control system.

Controller: The part or component of a system that controls the plant.

Disturbances: The signal that has adverse effect on the performance of a control system.

Control system: A system that can command, direct or regulate itself or another system to achieve a certain goal.

Automation: The control of a process by automatic means

Control System: An interconnection of components forming a system configuration that will provide a desired response.

Actuator: It is the device that causes the process to provide the output. It is the device that provides the motive power to the process.

Design: The process of conceiving or inventing the forms, parts, and details of system to achieve a specified purpose.

Simulation: A model of a system that is used to investigate the behavior of a system by utilizing actual input signals**.**

Optimization: The adjustment of the parameters to achieve the most favorable or advantageous design.

Feedback Signal: A measure of the output of the system used for feedback to control the system.

Negative feedback: The output signal is feedback so that it subtracts from the input signal.

Block diagrams: Unidirectional, operational blocks that represent the transfer functions of the elements of the system.

Signal Flow Graph (SFG): A diagram that consists of nodes connected by several directed branches and that is a graphical representation of a set of linear relations.

Specifications: Statements that explicitly state what the device or product is to be and to do. It is also defined as a set of prescribed performance criteria.

Open-loop control system: A system that utilizes a device to control the process without using feedback. Thus the output has no effect upon the signal to the process.

Closed-loop feedback control system: A system that uses a measurement of the output and compares it with the desired output.

Regulator: The control system where the desired values of the controlled outputs are more or less fixed and the main problem is to reject disturbance effects.

Servo system: The control system where the outputs are mechanical quantities like acceleration, velocity or position.

Stability: It is a notion that describes whether the system will be able to follow the input command. In a non-rigorous sense, a system is said to be unstable if its output is out of control or increases without bound.

Multivariable Control System: A system with more than one input variable or more than one output variable.

Trade-off: The result of making a judgment about how much compromise must be made between conflicting criteria.

System

A combination or arrangement of a number of different physical components to form a whole unit such that combining unit performs to achieve a certain goal

Example: a lamp (made up of glass, filaments)

CONTROL SYSTEM

In a system when the output quantity is controlled by varying the input quantity the system is called control system.

Example: a lamp controlled by a switch

Example of control system

Robotics

Biomedical

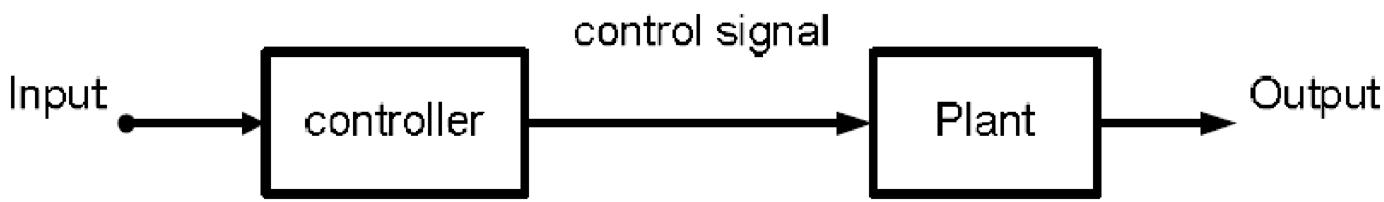
Automated surgical

Space industry

Types of Control system

- **Open-loop control system**
- **Closed-loop control system:**

Open-loop control system: It is a control system where its control action only depends on input signal and does not depend on its output response



An open-loop system

Examples: traffic signal, bread toaster, Automatic washing machine ,Coffee or Tea making machine ,Volume on the stereo system , Electric hand drier , Bread toaster
Inkjet printers , Servo motor/Servo motor ; Electric bulb; Clothes drier based on a timer ,Light switch ,TV (remote control) Water faucet , Door lock system etc.

Advantages:

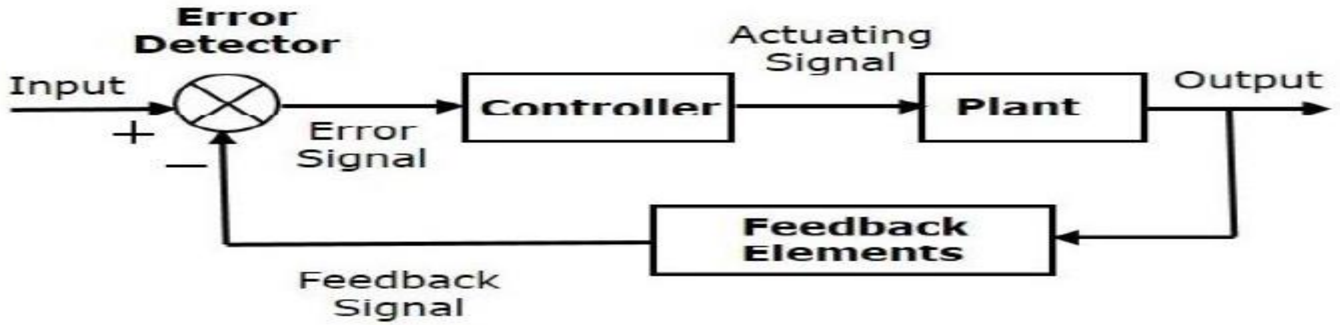
- ✓ Simple design and easy to construct
- ✓ Economical
- ✓ Easy for maintenance
- ✓ Highly stable operation

Dis-advantages:

- ➢ Not accurate and reliable when input or system parameters are variable in Nature
- ➢ Recalibration of the parameters are required time to time

Closed-loop control system:

It is a control system where its control action depends on both of its input signal and output response



Examples:

automatic electric iron, missile launcher, speed control of DC motor, etc

Advantages:

- ✓ More accurate operation than that of open-loop control system
- ✓ Can operate efficiently when input or system parameters are variable in nature
- ✓ Less nonlinearity effect of these systems on output response
- ✓ High bandwidth of operation
- ✓ There is facility of automation
- ✓ Time to time recalibration of the parameters are not required

Dis-advantages:

- ❑ Complex design and difficult to construct

Comparison between open loop system and closed loop system

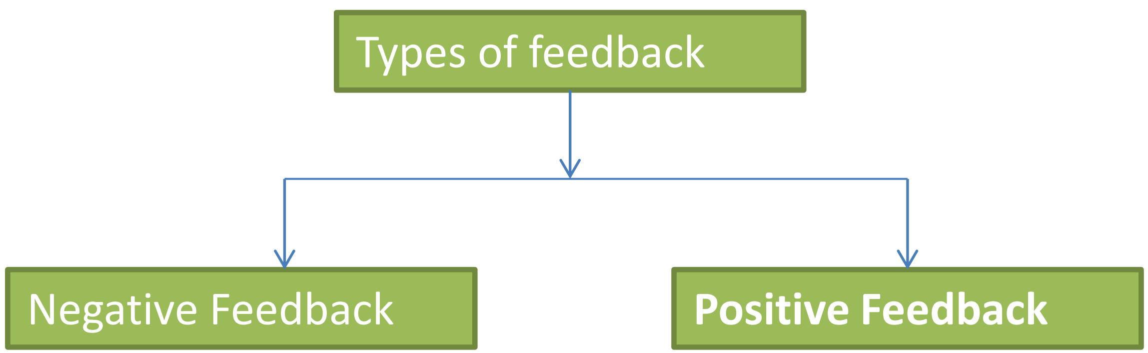
Open loop system	Closed loop system
these are not reliable	these are reliable
it is easier to build	it is difficult to built
if calibration is good they perform accurately	they are accurate because of feedback
operating systems are generally more stable	these are less stable
Optimization is not possible	Optimization is possible

Feedback

Feedback acts as the characteristic of the system that allows comparison between achieved output and reference input of system.

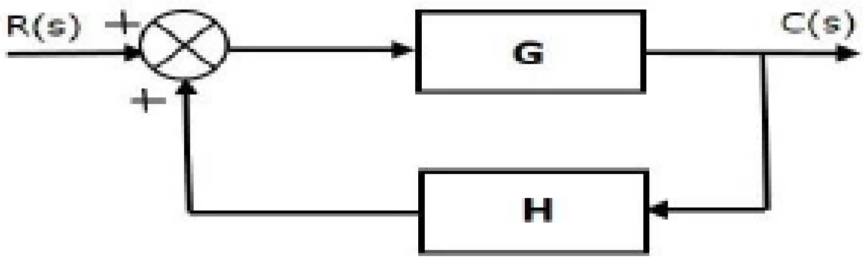
This difference between the actual and desired value is called the error. Feedback control manipulates an input to the system to minimize this error. Thus F.B. loop is considered as the key parameter of a closed loop control system

The major objective of feedback control is to ensure that the output value of the system must be similar to the required value.



Positive Feedback

The positive feedback adds the reference input, *R*(*s*) and feedback output. The following figure shows the block diagram of **positive feedback control system**.



the transfer function of positive feedback control system is,

$$T = \frac{G}{1-GH}$$

Where,

T is the transfer function or overall gain of positive feedback control system.

G is the open loop gain, which is function of frequency.

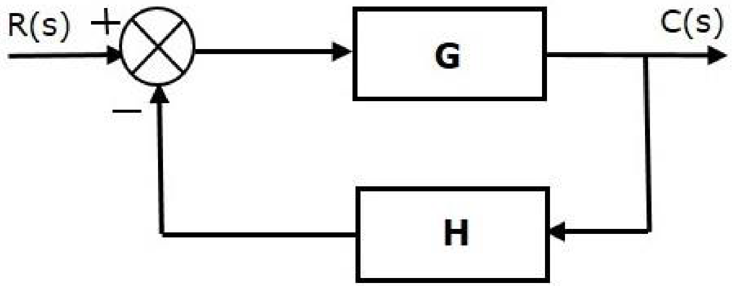
H is the gain of feedback path, which is function of frequency.

Negative Feedback

Negative feedback reduces the error between the reference input, *R*(*s*) and system output. The following figure shows the block diagram of the **negative feedback control system**.

Transfer function of negative feedback control system is,

$$T = \frac{G}{1+GH}$$



Where,
T is the transfer function or overall gain of negative feedback control system.
G is the open loop gain, which is function of frequency.
H is the gain of feedback path, which is function of frequency.

Advantages of feedback control

- Rejection of disturbances
- The system sensitivity will be very less to the plant uncertainties
- The steady-state error reduction of the system is increased
- The transient response can be easily controlled
- System stability is improved
- The overall system gain is improved
- High bandwidth
- The effect of variation of the system parameters are reduced
- There is no need to know what disturbance will affect the process
- The relation between the process and the final control element is not a problem

Disadvantages of a feedback system

- If the system is not designed properly then there is a possibility for instability
- The feedback inherently couples different art of a system
- The whole feedback process with control system could be complex
- The feedback control won't take any action till there is a change in the controlled variable
- The feedback control won't be able to do the predictive control action in order to compensate for the effects of known or measurable disturbances.
- In certain cases, the controlled variable can't be measured online
- The feedback system can't handle large time constant or long time delays, in case if there is a large and frequent disturbance then the process would operate continuously at a transient state and it would never achieve the desired state

Sensitivity

Sensitivity of the overall gain of negative feedback closed loop control system (**T**) to the variation in open loop gain (**G**) is defined as

$$S_G^T = \frac{\frac{\partial T}{T}}{\frac{\partial G}{G}} = \frac{Percentage\ change\ in\ T}{Percentage\ change\ in\ G}$$

Where, **∂T** is the incremental change in T due to incremental change in G.

sensitivity of the overall gain of closed loop control system

$$S_G^T = \frac{1}{1+GH}$$

- ✓ If the value of (1+GH) is less than 1, then sensitivity increases. In this case, 'GH' value is negative because the gain of feedback path is negative.
- ✓ If the value of (1+GH) is greater than 1, then sensitivity decreases. In this case, 'GH' value is positive because the gain of feedback path is positive.

In general, 'G' and 'H' are functions of frequency. So, feedback will increase the sensitivity of the system gain in one frequency range and decrease in the other frequency range. Therefore, we have to choose the values of 'GH' in such a way that the system is insensitive or less sensitive to parameter variations

Effect of Feedback on Stability

- A system is said to be stable, if its output is under control. Otherwise, it is said to be unstable.
- if the denominator value is zero (i.e., GH = -1), then the output of the control system will be infinite. So, the control system becomes unstable.

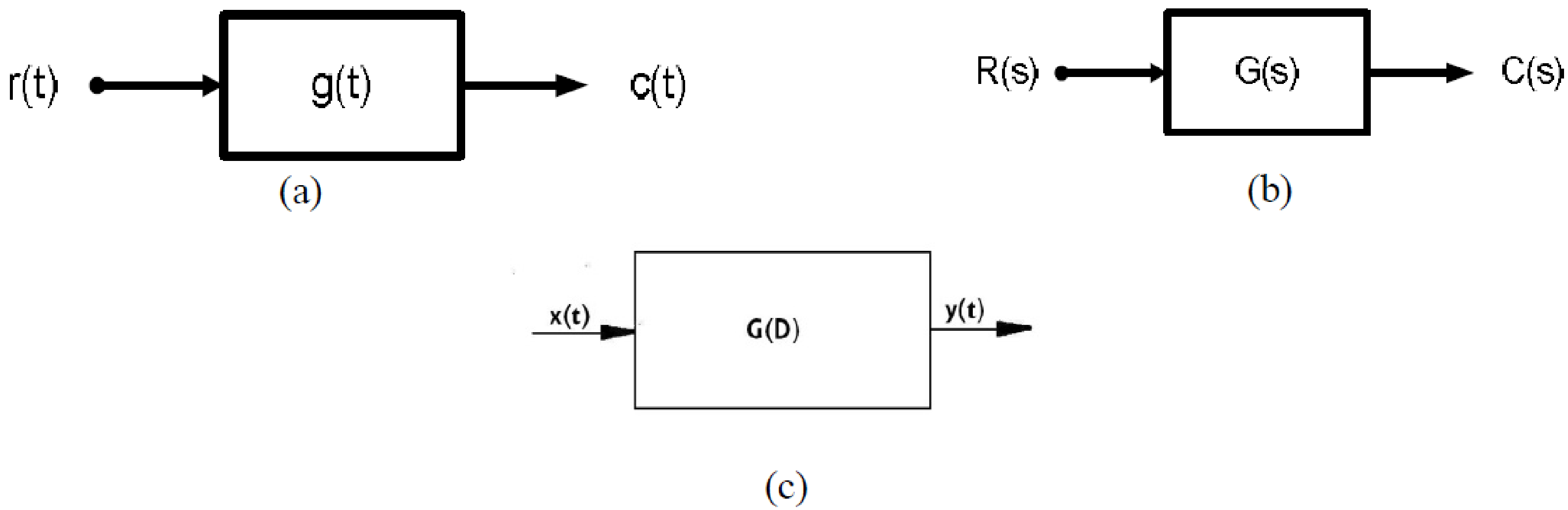
Therefore, we have to properly choose the feedback in order to make the control system stable.

Laplace transform

Laplace transforms convert differential equations into algebraic equations. They are related to frequency response.

Transfer Function

Definition: It is the ratio of Laplace transform of output signal to Laplace transform of input signal assuming all the initial conditions to be zero.



(a) A system in time domain, (b) a system in frequency domainand (c) transfer function with differential operator

G(s) is the transfer function of the system. It can be mathematically represented as follows.

$$G(s) = \left. \frac{C(s)}{R(s)} \right|_{zero \quad initial \quad condition}$$

Properties of Transfer function:

- ❑ Zero initial condition
- ❑ It is same as Laplace transform of its impulse response
- ❑ Replacing ‘s’ by *d/dt* in the transfer function, the differential equation can be obtained
- ❑ Poles and zeros can be obtained from the transfer function
- ❑ Stability can be known
- ❑ Can be applicable to linear system only

Advantages of Transfer function:

- ❖ It is a mathematical model and gain of the system
- ❖ Replacing ‘s’ by *d/dt* in the transfer function, the differential equation can be obtained
- ❖ Poles and zeros can be obtained from the transfer function
- ❖ Stability can be known
- ❖ Impulse response can be found

Disadvantages of Transfer function:

- ✓ Applicable only to linear system
- ✓ Not applicable if initial condition cannot be neglected
- ✓ It gives no information about the actual structure of a physical system

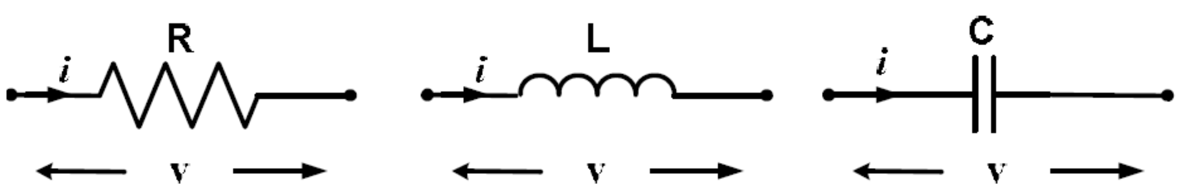
Components of an electrical system: There are three basic elements in an electrical system, i.e. (a) resistor (R), (b) inductor(L) and (c) capacitor (C).

Electrical systems are of two types,
(i) voltage source electrical system and (ii) current source electrical system.

Voltage source electrical system: If i is the current through a resistor(Fig. and v is the voltage drop in it, then $v = Ri$.

If i is the current through an inductor (Fig. and v is the voltage developed in it, then $v = L\frac{di}{dt}$.

If i is the current through a capacitor(Fig. and v is the voltage developed in it, then $v = \frac{1}{C}\int i dt$.



Current source electrical system:

If i is the current through a resistor and v is the voltage drop in it, then $i = \frac{v}{R}$.

If i is the current through an inductor and v is the voltage developed in it, then $i = \frac{1}{L}\int v dt$.

If i is the current through a capacitor and v is the voltage developed in it, then $i = C\frac{dv}{dt}$.

No.	Function	Time-domain $x(t)= \mathcal{L}^{-1}\{X(s)\}$	Laplace domain $X(s)= \mathcal{L}\{x(t)\}$
1	Delay	$\delta(t-\tau)$	$e^{-\tau s}$
2	Unit impulse	$\delta(t)$	1
3	Unit step	$u(t)$	$\frac{1}{s}$
4	Ramp	t	$\frac{1}{s^2}$
5	Exponential decay	$e^{-\alpha t}$	$\frac{1}{s+\alpha}$
6	Exponential approach	$\left(1-e^{-\alpha t}\right)$	$\frac{\alpha}{s(s+\alpha)}$
7	Sine	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
8	Cosine	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
9	Hyperbolic sine	$\sinh \alpha t$	$\frac{\alpha}{s^2-\alpha^2}$
10	Hyperbolic cosine	$\cosh \alpha t$	$\frac{s}{s^2-\alpha^2}$
11	Exponentially decaying sine wave	$e^{-\alpha t}\sin \omega t$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
12	Exponentially decaying cosine wave	$e^{-\alpha t}\cos \omega t$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$

Mechanical system

Mechanical systems are of two types,

(i) Translational mechanical system and

(ii) Rotational mechanical system.

The motion takes place along a straight line is known as translational motion.

The rotational motion of a body can be defined as the motion of a body about a fixed axis.

Translational mechanical system

The model of mechanical translational system can be obtained by using three basic,

(a) Mass

(b) spring

(c) Damper (dashpot)

The weight of the mechanical system is represented by the element of mass

The elastic deformation of the body can be represented by spring

The friction existing in rotating mechanical system can be represented by dashpot

When a force is applied to a translational mechanical system it is opposed by opposing forces due to mass, friction and elasticity of the system .

Force acting on a mechanical body are governed by Newton's second law of motion

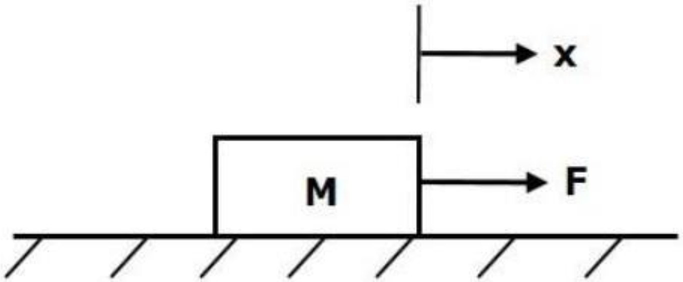
- Mass is the property of a body, which stores **kinetic energy**.
- If a force is applied on a body having mass **M**, then it is opposed by an opposing force due to mass.
- This opposing force is proportional to the acceleration of the body. Assume elasticity and friction are negligible.

Mass: A mass is denoted by M. If a force *f* is applied on it and it displays distance *x*, then

$$F_m \propto a$$

$$=> F_m = Ma = M\frac{d^2x}{dt^2}$$

$$F = F_m = M\frac{d^2x}{dt^2}$$

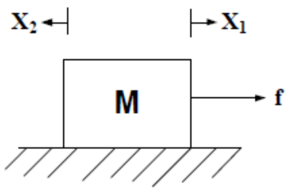


Where,

- $\boldsymbol{F}$ is the applied force
- F_m is the opposing force due to mass
- M is the mass
- a is acceleration
- x is displacement

If a force *f* is applied on a **mass M** and it displays distance *x1*in the direction of *f* and distance *x2* in the opposite direction, then

$$f = M\left(\frac{d^2x_1}{dt^2} - \frac{d^2x_2}{dt^2}\right)$$



Force applied on a mass with displacement two directions

(b) **Spring:** A spring is denoted by K. If a force f is applied on it and it displays distance x, then $f = K x$ as shown

Spring is an element, which stores **potential energy**.

- If a force is applied on spring **K**, then it is opposed by an opposing force due to elasticity of spring.
- This opposing force is proportional to the displacement of the spring. Assume mass and friction are negligible

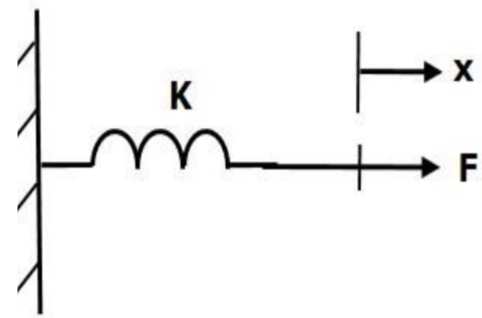
Where,

- $\boldsymbol{F}$ is the applied force
- F_k is the opposing force due to elasticity of spring
- $\boldsymbol{K}$ is spring constant
- $\boldsymbol{x}$ is displacement

$$F_k \propto x$$

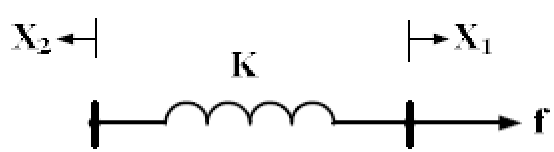
$$=> F_k = Kx$$

$$F = F_k = Kx$$



If a force f is applied on a spring K and it displays distance x_1 in the direction of f and distance x_2 in the opposite direction, then

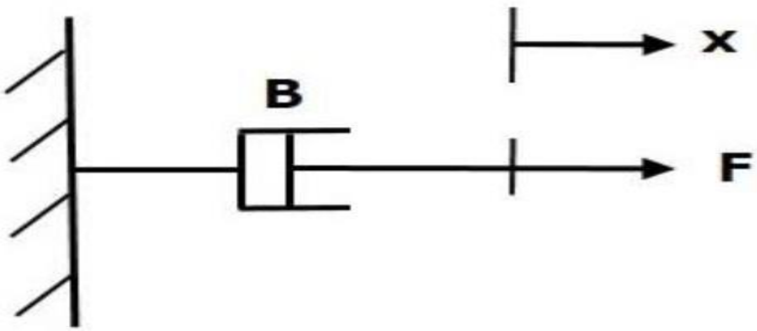
$f = K\,(x_1 - x_2)$ as shown in Fig



Force applied on a spring with displacement in two directions

Dashpot

• If a force is applied on dashpot **B**, then it is opposed by an opposing force due to **friction** of the dashpot.
• This opposing force is proportional to the velocity of the body. Assume mass and elasticity are negligible.



$$F_b \propto v$$

$$\Rightarrow F_b = Bv = B\frac{dx}{dt}$$

$$F = F_b = B\frac{dx}{dt}$$

Where,

• F_b is the opposing force due to friction of dashpot
• $\boldsymbol{B}$ is the frictional coefficient
• $\boldsymbol{v}$ is velocity
• $\boldsymbol{x}$ is displacement

Rotational Mechanical Systems

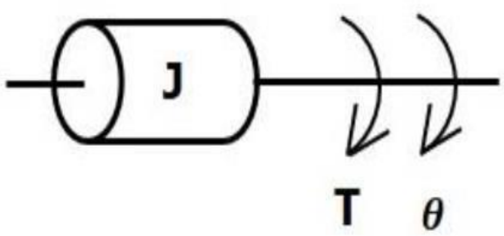
- Rotational mechanical systems move about a **fixed axis**.
- These systems mainly consist of three basic elements. Those are **moment of inertia**, **torsional spring** and **dashpot**.

If a torque is applied to a rotational mechanical system, then it is opposed by opposing torques due to moment of inertia, elasticity and friction of the system. Since the applied torque and the opposing torques are in opposite directions, the algebraic sum of torques acting on the system is zero.

Moment of Inertia

In translational mechanical system, mass stores kinetic energy. Similarly, in rotational mechanical system, moment of inertia stores **kinetic energy**.

If a torque is applied on a body having moment of inertia **J**, then it is opposed by an opposing torque due to the moment of inertia. This opposing torque is proportional to angular acceleration of the body. Assume elasticity and friction are negligible.



$$T_j \propto \alpha$$

$$=> T_j = J\alpha = J\frac{d^2\theta}{dt^2}$$

$$T = T_j = J\frac{d^2\theta}{dt^2}$$

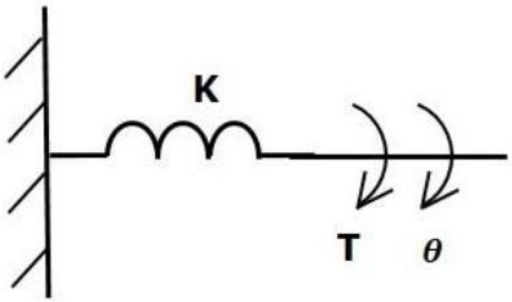
Where,

- $\boldsymbol{T}$ is the applied torque
- Tj is the opposing torque due to moment of inertia
- $\boldsymbol{J}$ is moment of inertia
- **α** is angular acceleration
- **θ** is angular displacement.

Torsional Spring

• In translational mechanical system, spring stores potential energy. Similarly, in rotational mechanical system, torsional spring stores **potential energy**.

If a torque is applied on torsional spring **K**, then it is opposed by an opposing torque due to the elasticity of torsional spring. This opposing torque is proportional to the angular displacement of the torsional spring. Assume that the moment of inertia and friction are negligible.



$$T_k \propto \theta$$

$$=> T_k = K\theta$$

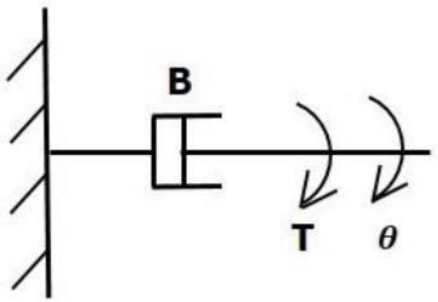
$$T = T_k = K\theta$$

Where,

- $\boldsymbol{T}$ is the applied torque
- T_k is the opposing torque due to elasticity of torsional spring
- $\boldsymbol{K}$ is the torsional spring constant
- $\boldsymbol{\theta}$ is angular displacement

Dashpot

• If a torque is applied on dashpot **B**, then it is opposed by an opposing torque due to the **rotational friction** of the dashpot.
• This opposing torque is proportional to the angular velocity of the body.
Assume the moment of inertia and elasticity are negligible



$$T_b \propto \omega$$

$$=> T_b = B\omega = B\frac{d\theta}{dt}$$

$$T = T_b = B\frac{d\theta}{dt}$$

Where,
• T_b is the opposing torque due to the rotational friction of the dashpot
• $\boldsymbol{B}$ is the rotational friction coefficient
• $\boldsymbol{\omega}$ is the angular velocity
• $\boldsymbol{\theta}$ is the angular displacement

Rotational mechanical system

There are three basic elements in a Rotational mechanical system, i.e. (a) inertia, (b) spring and (c) damper.

(a) **Inertia:** A body with aninertia is denoted by J. If a torque$T$ is applied on it and it displays distanceΘ, then $T = J\dfrac{d^2\theta}{dt^2}$. If a torque$T$ is applied on a body with inertia J and it displays distance Θ_1 in the direction of T and distance Θ_2 in the opposite direction, then $T = J\left(\dfrac{d^2\theta_1}{dt^2} - \dfrac{d^2\theta_2}{dt^2}\right)$.

(b) **Spring:** A spring is denoted by K. If a torque$T$ is applied on it and it displays distanceΘ, then $T = K\theta$. If a torque$T$ is applied on a body with inertia J and it displays distance Θ_1 in the direction of T and distance Θ_2 in the opposite direction, then $T = K(\theta_1 - \theta_2)$.

(c) **Damper:** A damper is denoted by D. If a torque$T$ is applied on it and it displays distanceΘ, then $T = D\dfrac{d\theta}{dt}$. If a torque$T$ is applied on a body with inertia J and it

displays distance Θ_1 in the direction of T and distance Θ_2 in the opposite direction, then $T = D\left(\dfrac{d\theta_1}{dt} - \dfrac{d\theta_2}{dt}\right)$.

Determine the transfer function of mechanical translational system

1.consider each mass separately
2. draw the free body diagram
3. write the differential equations
4. take the Laplace transform of differential equations
5. rearrange the s-domain equation to eliminate the unwanted variables and obtain the ratio between output variable and input variable

Mechanical rotational systems

- ❖ The model of mechanical rotational systems can be obtained by using three elements moment of inertia [J]of mass, dash-pot with rotational frictional Coefficient [B]and torsional spring with stiffness [K]

- ✓ The weight of the rotational mechanical system is represented by the moment of inertia of the mass
- ✓ The elastic deformation of the body can be represented by a spring
- ✓ The friction existing in rotational mechanical system can be represented by dashpot

Determine the transfer function of mechanical rotational system

1.consider each moment of inertia separately
2. draw the free body diagram
3. write the differential equations
4. take the Laplace transform of differential equations
5. rearrange the s-domain equation to eliminate the unwanted variables and obtain the ratio between output variable and input variable

ELECTRICAL ANALOGOUS OF MECHANICAL SYSTEMS

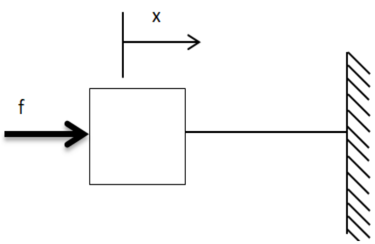
Systems remain analogous as long as the differential equations governing the systems or transfer functions are in ideal form.
Since the electrical systems are two types of inputs either voltage or current source, there are two types of analogies -force voltage analogy/ torque voltage analogy and force current analogy/ torque current analogy.

Force / torque voltage analogy -Each junction in the mechanical system response to a closed loop which consists of electrical excitation sources and passive elements analogous to the mechanical driving source and passive elements connected to the junction
Force / torque current analogy -Each junction in the mechanical system corresponds to a node which joins electrical excitation sources and passive elements analogous to the mechanical driving sources and passive elements connected to the junction

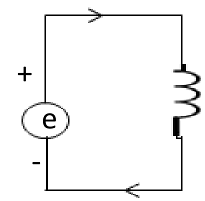
Force Voltage Analogy

Mechanical system
I/P : Force
O/P : Velocity

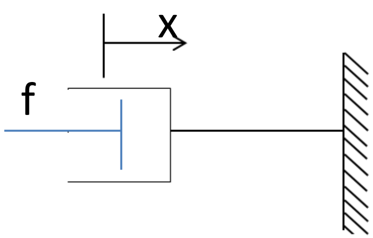
Electrical system
I/P : Voltage Source
O/P : Current through element



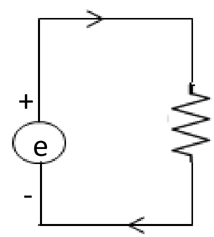
$$f = M\frac{dx^2}{dt} = M\frac{dv}{dt}$$



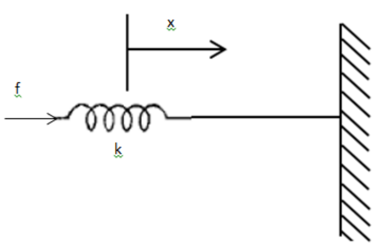
$$e = L\frac{di}{dt}$$



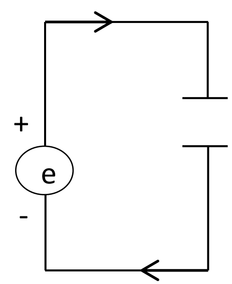
$$f = B\frac{dx}{dt} = Bv$$



$$e = i\,R$$



$$f = kx = k\int v\,dt$$



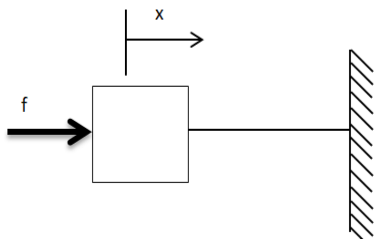
$$e = \frac{1}{C}\int i dt$$

Force Current Analogy

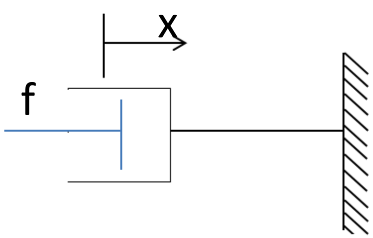
Mechanical system

I/P : Force

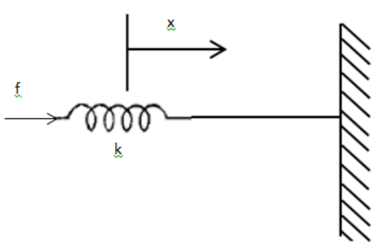
O/P : Velocity



$$f = M \frac{d^2x}{dt^2} = M\frac{dv}{dt}$$



$$f = B\,\frac{dx}{dt} = Bv$$

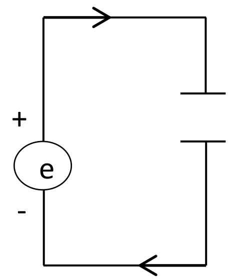


$$f = kx = k\int v\,dt$$

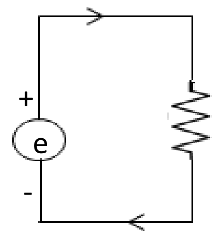
Electrical system

I/P : Current Source

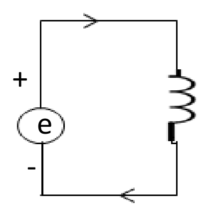
O/P : Current through element



$$i = C\,\frac{dv}{dt}$$



$$i = \frac{V}{R}$$



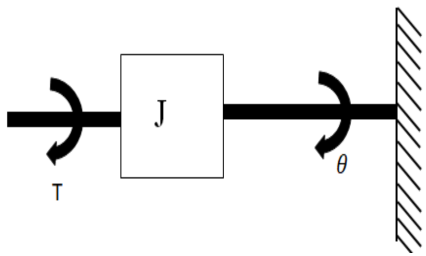
$$i = \frac{1}{L}\int V dt$$

Torque Voltage Analogy

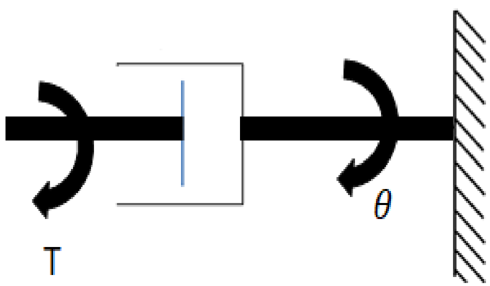
Mechanical system

I/P : Torque

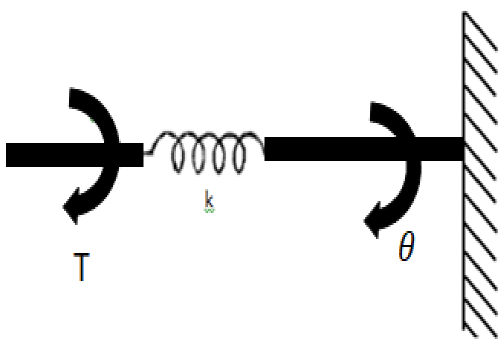
O/P : Angular Velocity



$$T = J \frac{d^2\theta}{dt^2} = J\frac{\omega}{dt}$$



$$T = B \frac{d\theta}{dt} = B\omega$$

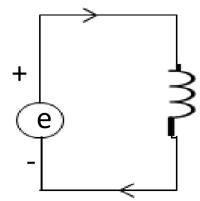


$$T = k\theta = k\int \omega \, dt$$

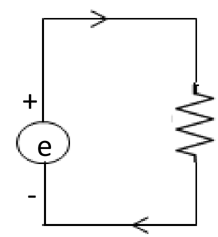
Electrical system

I/P : Voltage

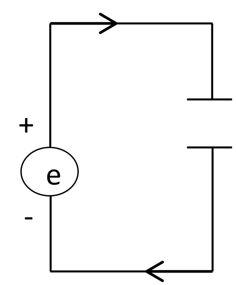
O/P : Current through element



$$e = L\frac{di}{dt}$$



$$e = iR$$

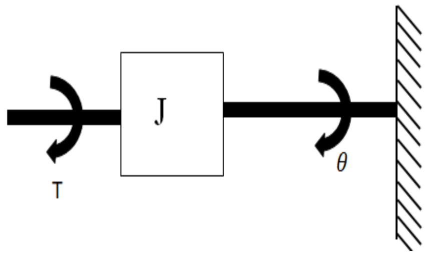


$$e = \frac{1}{C}\int i dt$$

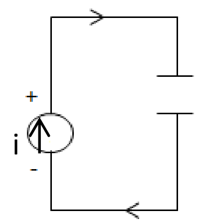
Torque Current Analogy

Mechanical system
I/P : Torque
O/P : Angular Velocity

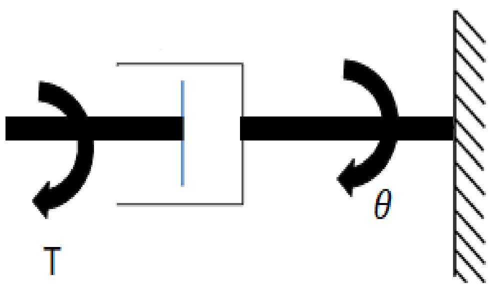
Electrical system
I/P : Current Source
O/P : Voltage across the element



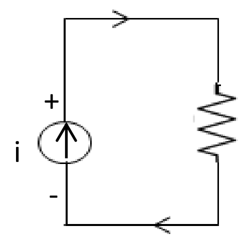
$$T = J\,\frac{d^2\theta}{dt^2} = J\frac{\omega}{dt}$$



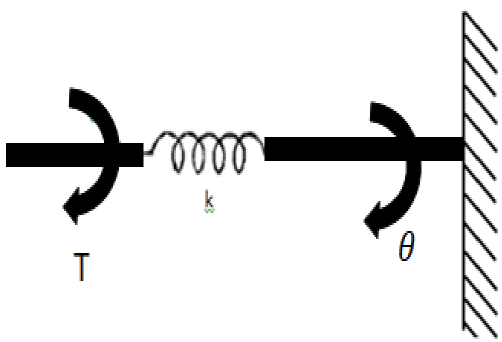
$$i = C\,\frac{dV}{dt}$$



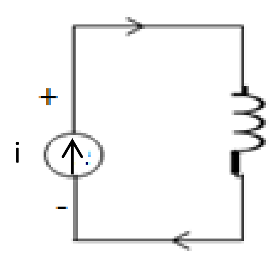
$$T = B\,\frac{d\theta}{dt} = B\omega$$



$$i = V\ R$$



$$T = k\theta = k\int \omega\, dt$$



$$i = \frac{1}{L}\int V dt$$

Force-voltage Analogy

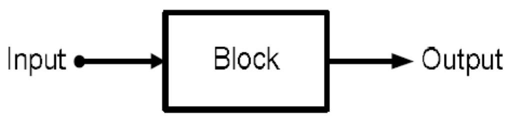
Translational	Rotational	Electrical
Force (f)	Torque (T)	Voltage (v)
Mass (M)	Inertia (J)	Inductance (L)
Damper (D)	Damper (D)	Resistance (R)
Spring (K)	Spring (K)	Elastance (1/C)
Displacement (x)	Displacement (Θ)	Charge (q)
Velocity (u) = $\dot{x}$	Velocity (u) = $\dot{\theta}$	Current (i) = $\dot{q}$

Force-current analogy

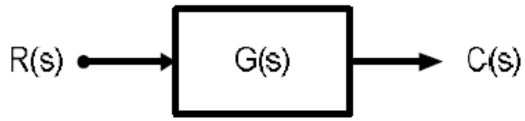
Translational	Rotational	Electrical
Force (f)	Torque (T)	Current (i)
Mass (M)	Inertia (J)	Capacitance (C)
Damper (D)	Damper (D)	Conductance (1/R)
Spring (K)	Spring (K)	Reciprocal of Inductance (1/L)
Displacement (x)	Displacement (Θ)	Flux linkage (ψ)
Velocity (u) = $\dot{x}$	Velocity (u) = $\dot{\theta}$	Voltage (v) = $\dot{\psi}$

BLOCK DIAGRAM

A block diagram of a system is a pictorial representation of the functions performed by each component and of the flow of signals.
The elements of a block diagram are block, branch point (Take off pint)and summing point (Adder).
Block It is the pictorial representation of the cause-and-response relationship between input and output of a physical system.

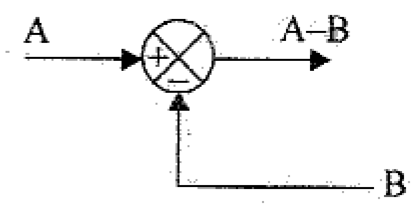


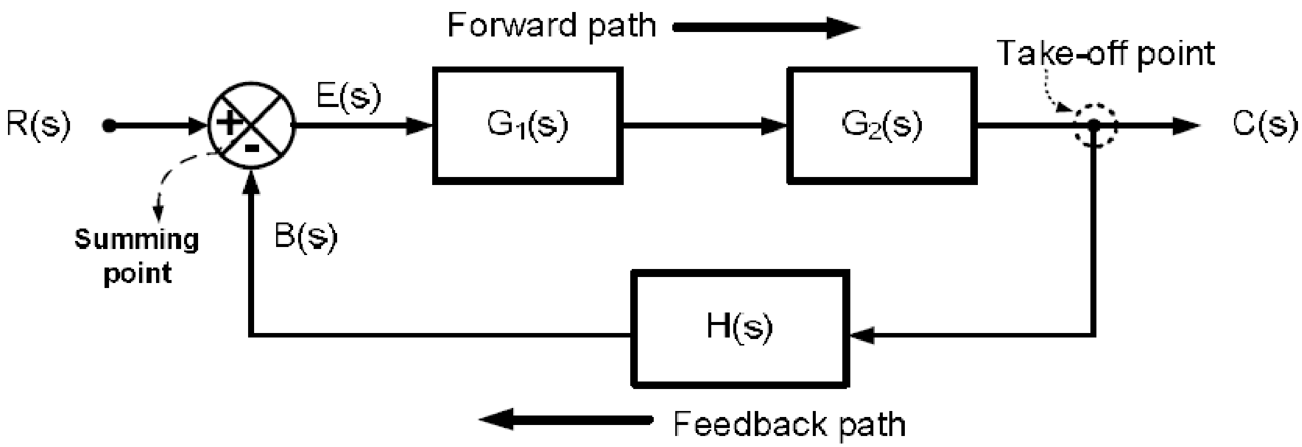
(a)



(b)

Summing point – It is used to add two or more signals in the system '+' or '-' sign at each arrowhead indicates whether the signal is to be added or subtracted.





A block diagram representation of a system showing its different components

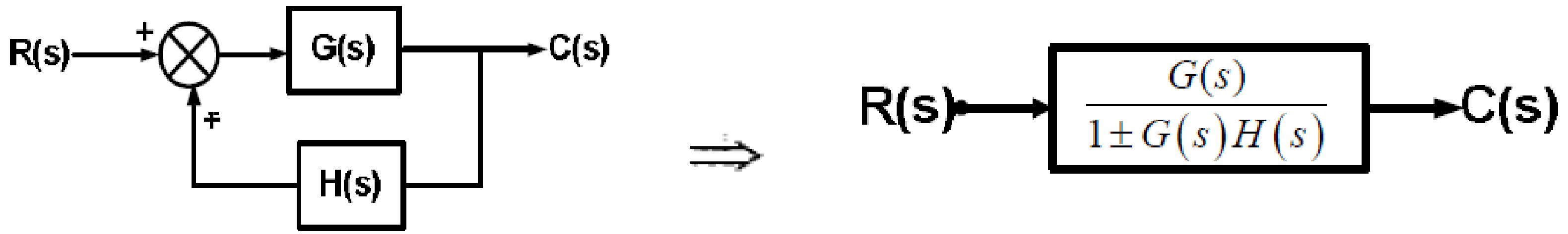
Take-off point: It is the component of a block diagram model at which a signal can be taken directly and supplied to one or more points as shown in fig.

Forward path: It is the direction of signal flow from input towards output.

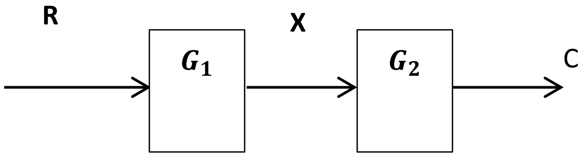
Feedback path: It is the direction of signal flow from output towards input

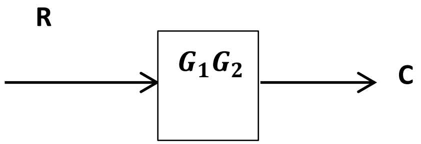
BLOCK DIAGRAM Reduction Rule

1. Representation of closed system

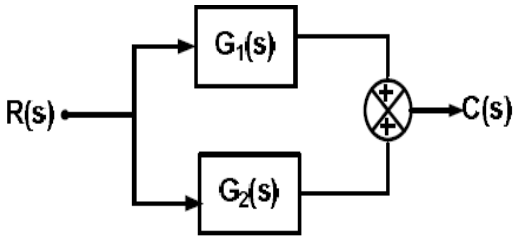


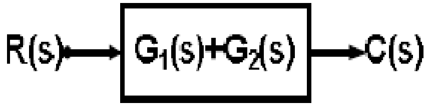
2. Blocks are connected in series / Cascade



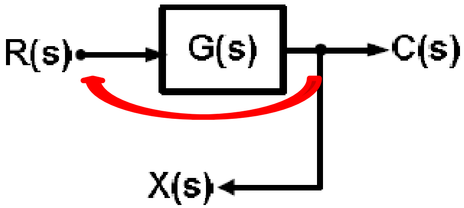


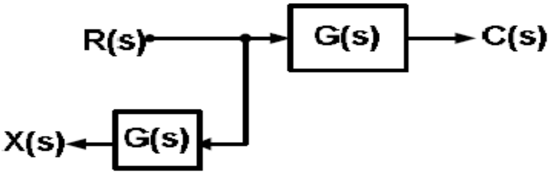
3. Blocks are connected in Parallel



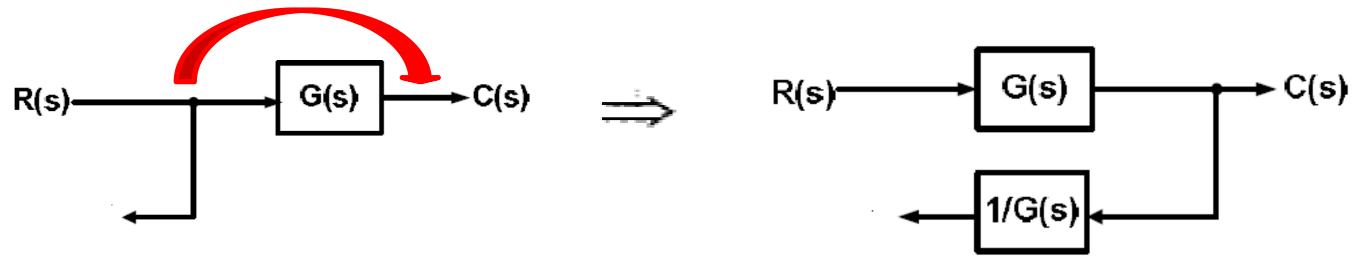


4. Move take off (branch) point before a block

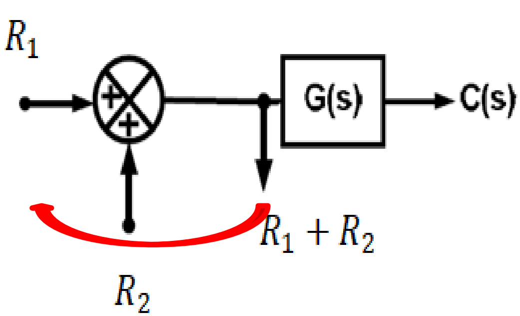




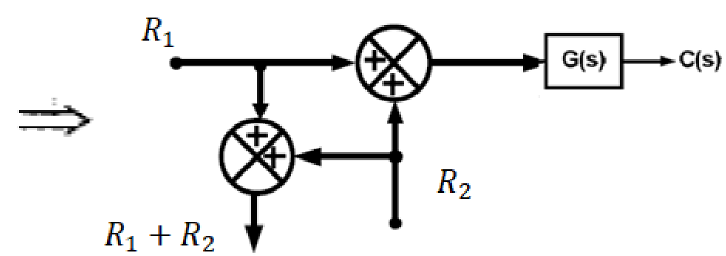
5. Move take off point after a block



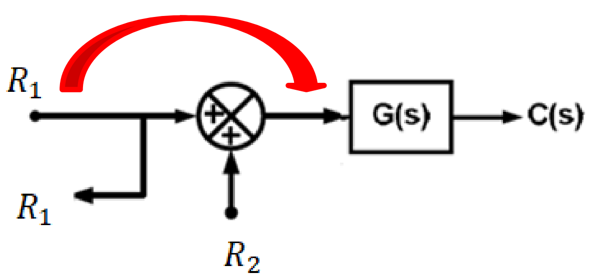
6. Move take off point before a summing point



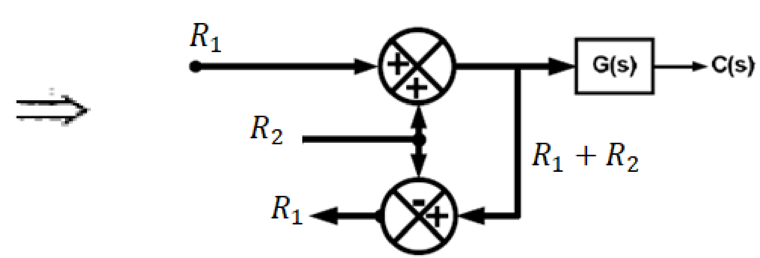
$C = (R_1 + R_2)G$



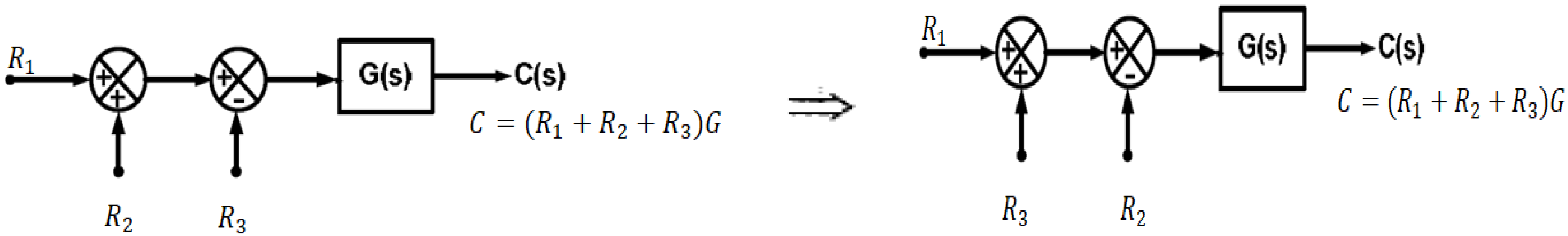
7. Move take off point after a summing point



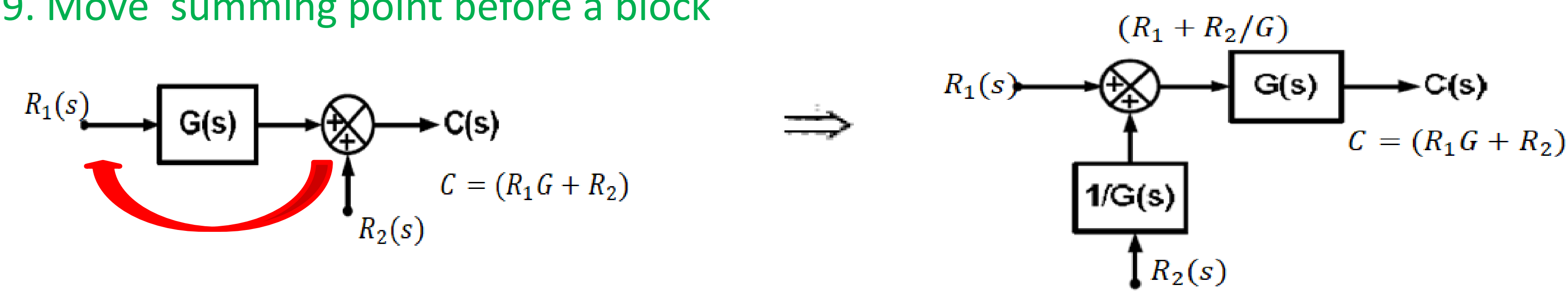
$C = (R_1 + R_2)G$



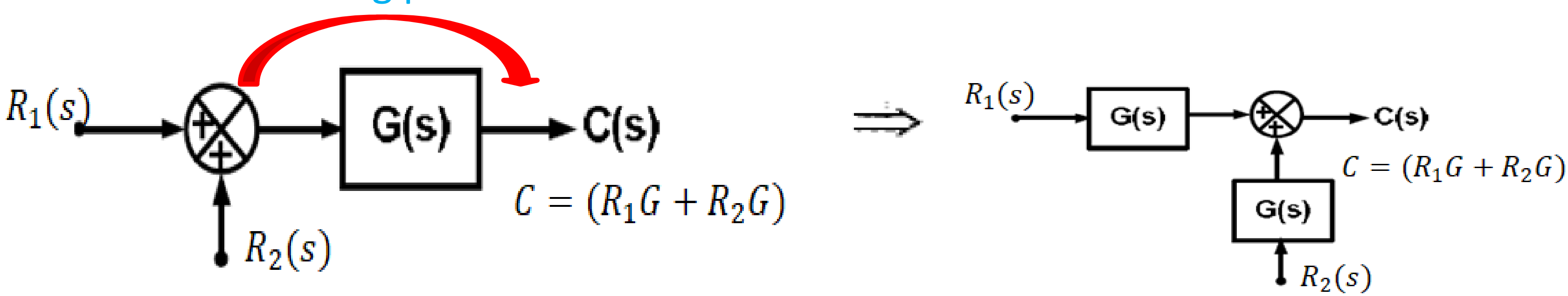
8. Rearrangement of summing (adder) point



9. Move summing point before a block



10. Move summing point after a block



Procedure for reduction of Block Diagram model: