$$lx + my + nz = -1 \quad \text{—— (1)}$$

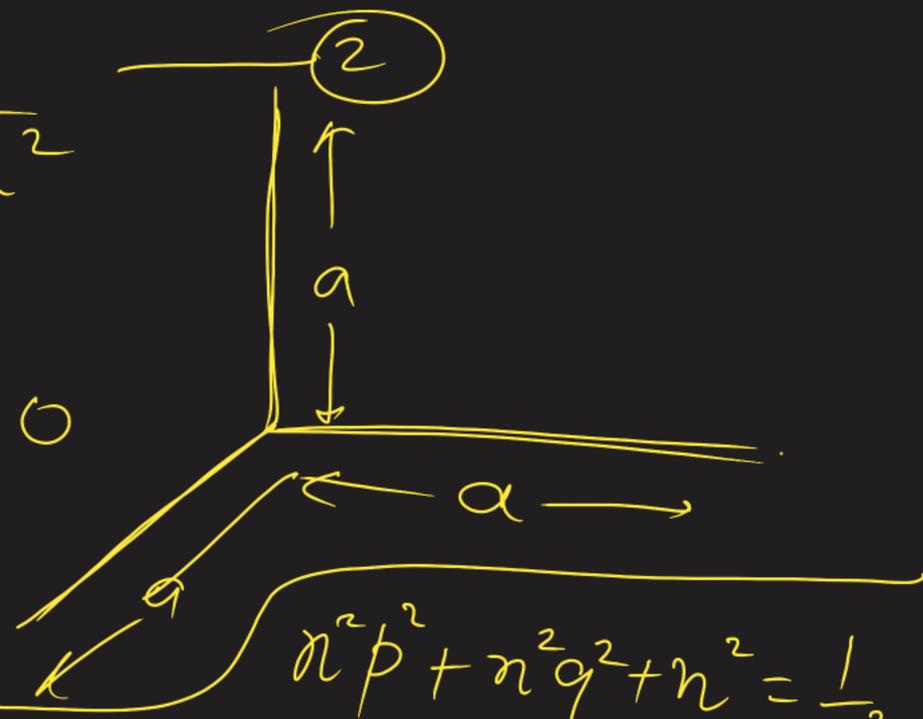
where $l^2 + m^2 + n^2 = \frac{1}{a^2}$ —— (2)

$$l + n\frac{\partial z}{\partial x} = 0$$

$$l = -np$$

$$m + nq = 0$$

$$m = -nq$$



$$n^2p^2 + n^2q^2 + n^2 = \frac{1}{a^2}$$

$$n^2(p^2+q^2+1) = \frac{1}{a^2}$$

$$n = \frac{1}{a\sqrt{p^2+q^2+1}}$$

$$\boxed{z = px + qy - a\sqrt{1+p^2+q^2}}$$

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$

$$\boxed{(x-a)^2 + (y-b)^2 + z^2 = r^2}$$

$$2(x-a) + 0 + 2zp = 0$$

$$(x-a) = -zp. \quad \text{—— (I)}$$

$$y - b = -zq \quad \text{—— (II)}$$

$$\boxed{z^2(p^2+q^2+1) = r^2}$$

Que: $z^2(1+a^3) = 8(x+ay+b)^3$ —— (I)

$(1+a^3)2zp = 24(x+ay+b)^2$ —— (II)

$(1+a^3)2zq = 24a(x+ay+b)^2$ —— (III)

(II) ÷ (III)

$\frac{p}{q} = \frac{1}{a} \Rightarrow \cancel{p=a}\ q = pa.$

$$\frac{z^{4}\cancel{(1+a^3)^{3}}}{8z^{3}(1+a^3)^{3}p^3} = \frac{64\cancel{(x+ay+b)^6}}{(24)^3\cancel{(x+ay+b)^6}}$$

$$a = \frac{q}{p}$$

$$\Rightarrow \frac{z}{8(1+a^3)p^3} = \frac{\cancel{8}\times\cancel{8}}{\underset{3}{\cancel{24}}\times\underset{3}{\cancel{24}}\times\underset{3}{\cancel{24}}}$$

$$\Rightarrow 27z = (1+a^3)p^3$$

$$27z = \left(1+\left(\frac{q}{p}\right)^3\right)p^3$$

$$\boxed{27z = p^3+q^3}$$

Que: $z = xy + F(x^2+y^2)$

$$p = y + F'(x^2+y^2) \cdot 2x \quad \text{—— (I)}$$

$$q = x + F'(x^2+y^2) \cdot 2y \quad \text{—— (II)}$$

$$F'(x^2+y^2) = \frac{p-y}{2x} \qquad F'(x^2+y^2) = \frac{q-y}{2y}$$

$$\Rightarrow \boxed{py - qx = y^2 - x^2}$$

Que: $z = F\left(\frac{xy}{z}\right)$

Que: $z = y^2 + 2F\left(\frac{1}{x} + \log y\right)$

w.r.to x,

$\therefore \quad p = 0 + 2f'x\left(-\frac{1}{x^2}\right) \quad \text{—} (1) \times x^2$

$q = 2y + 2f'x\left(\frac{1}{y}\right) \quad \text{—} (2) \times y$

$\Rightarrow$ on adding eq (1) $\times x^2$ + (2) $\times y$ $\Rightarrow$ $\boxed{px^2 + qy = 2y^2}$

$f(x) = x^2$
$\sin x$

$$f(x, y, z, p, q) = 0 \quad \text{—— (1)}$$

$$z = F(x, y, a, b)$$

$$M = \begin{bmatrix} F_a & F_{xa} & F_{ya} \\ F_b & F_{xb} & F_{yb} \end{bmatrix}.$$

$$F_a = \frac{\partial F}{\partial a} \qquad F_{xa} = \frac{\partial^2 F}{\partial x \partial a}.$$

$$F_b = \frac{\partial F}{\partial b} \qquad F_{ya} = \frac{\partial^2 F}{\partial y \partial a}.$$

$$P(x,y,z)p + Q(x,y,z)q = R(x,y,z) \quad \text{--- (1)}$$

$$\Rightarrow \quad \frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}.$$

<u>auxiliary</u>

$$\left\{ \begin{array}{l} \phi(u,v) = 0 \\ \text{or} \quad v = \phi(u) \\ \text{or} \quad u = \phi(v) \end{array} \right\}.$$