$$\frac{r\tau_{rz}|_r - r\tau_{rz}|_{r+\Delta r}}{r\Delta r} + \cancel{\rho v_z v_z|_{z=0}} - \cancel{\rho v_z v_z|_{z=L}} + \frac{P|_{z=0} - P|_{z=L}}{L} + \rho g_z = 0$$

$$\lim_{\Delta r \to 0} \frac{r\tau_{rz}|_r - r\tau_{rz}|_{r+\Delta r}}{\Delta r} + r\left(\frac{P|_{z=0} - P|_{z=L}}{L} + \rho g\right) = 0$$

$$\Delta r \to 0 \quad -\frac{d}{dr}\left(r\tau_{rz}\right) + r\left(\frac{P_{z=0} - P_{z=L} + \rho g L}{L}\right) = 0$$

$$\boxed{\mathcal{P} = P - \rho g h}$$

$z=0 \quad \mathcal{P}_0 = P_{z=0} - \rho g \cdot 0$

$z=L \quad \mathcal{P}_L = P_{z=L} - \rho g L$

$$\mathcal{P}_0 - (P_{z=L} - \rho g L)$$

$$r\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{L}\right)$$

$$-\frac{d}{dr}\left(r\tau_{rz}\right) + \frac{r(\mathcal{P}_0 - \mathcal{P}_L)}{L} = 0$$

$$\frac{d}{dr}\left(r\tau_{rz}\right) = \frac{r(\mathcal{P}_0 - \mathcal{P}_L)}{L}$$

$$\tau_{rz} = \frac{r}{2L}\left(\mathcal{P}_0 - \mathcal{P}_L\right) + \frac{C_1}{r}$$

$$\rightarrow \tau_{rz} = r\left(\frac{P_0 - P_L}{2L}\right) + \frac{C_1}{r} \rightarrow \infty$$

$r = 0 \quad \tau_{rz} = 0 \rightarrow$ finite value

$r = R \quad \tau_{rz} = \tau_{rz}\big|_{max}$

$\rightarrow C_1 = 0$

$$\tau_{rz} = \frac{r}{2L}\left(P_0 - P_L\right)$$

A   I.F $\rightarrow v$

$$\int \frac{dV_z}{dr} = -\frac{r}{\mu 2L}\left(P_0 - P_L\right) dr$$

$$V_z = -\frac{(P_0 - P_L)}{2\mu L}\frac{r^2}{2} + C_2$$

$r = 0 \quad V_z = V_{max}$

$r = R \quad V_z = 0$

$$C_2 = \frac{P_0 - P_L}{2\mu}\frac{R^2}{2}$$

$$V_z = \frac{P_0 - P_L}{4\mu L}\left[-r^2 + R^2\right]$$

$$= \frac{(P_0 - P_L)R^2}{4\mu L}\left[-\left(\frac{r}{R}\right)^2 + 1\right]$$

$$Q = \int \left(\frac{P_0 - P_L}{4\mu L}\right) R^2 \left[1 - \left(\frac{r}{R}\right)^2\right] 2\pi r\, dr$$

$$= 2\pi \frac{P_0 - P_L}{4\mu L} R^2 \left[\frac{r^2}{2} - \frac{1}{R^2} \times \frac{r^4}{4}\right]_0^R$$

$$\left[\left(\frac{R^2}{2} - \frac{R^2}{4}\right) - (0)\right]$$

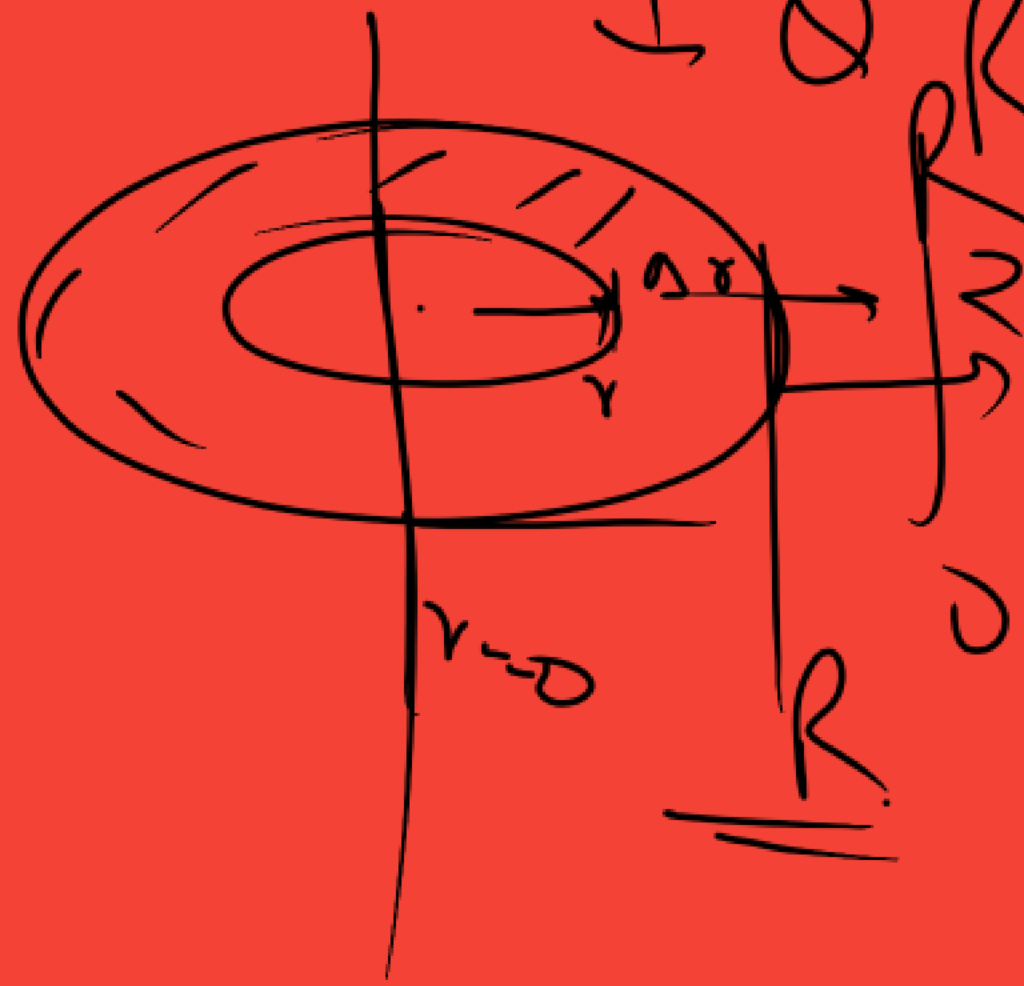
$$\left[\frac{R^2}{4}\right]$$

ΔP

$$Q = \frac{\pi (P_0 - P_L) R^4}{8 \mu L}$$

$(V_z)_{avg}$ = velocity $= \dfrac{Q}{A}$

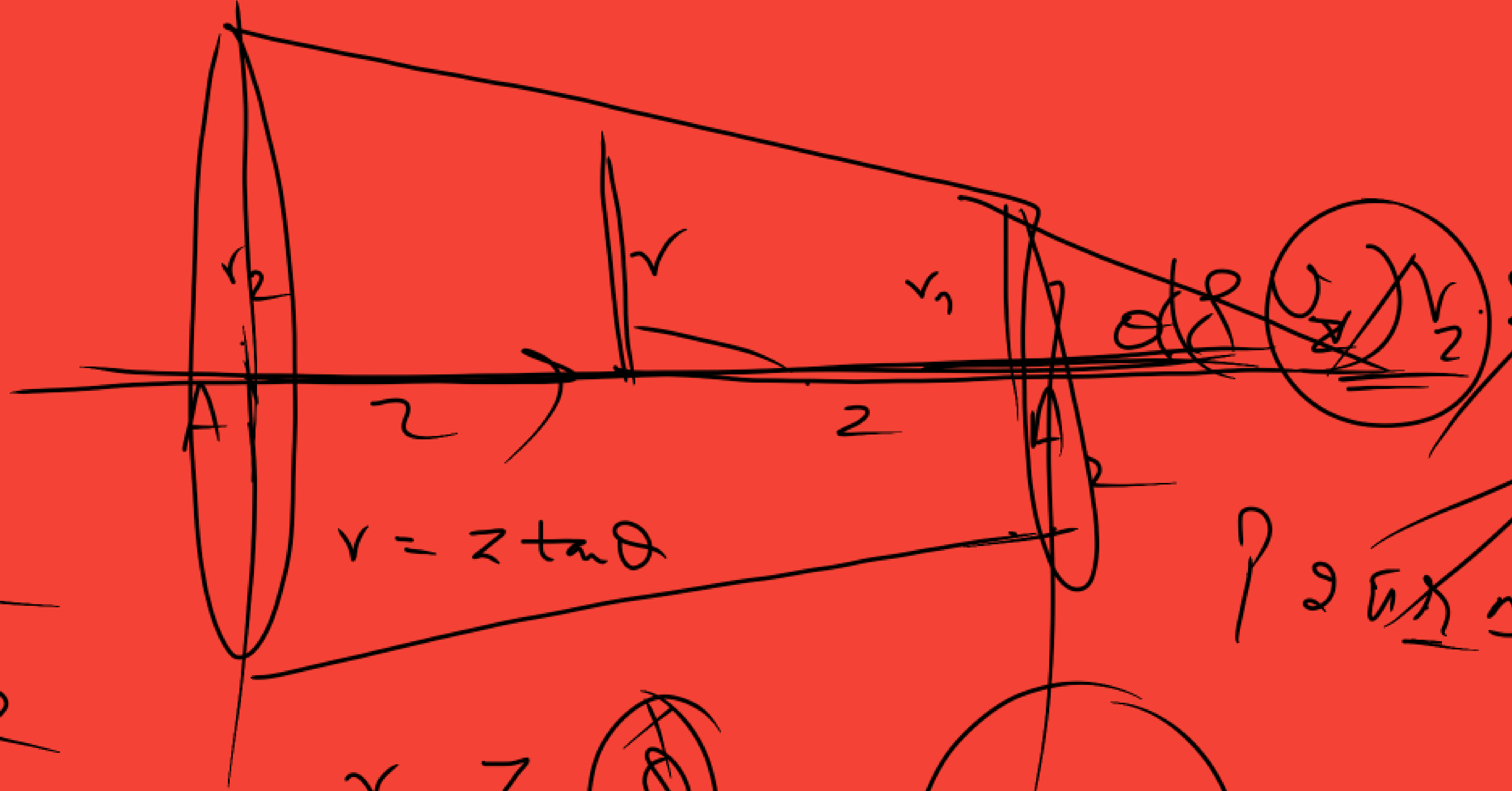
$\downarrow Q$



$\int_0^R 2\pi r\, dr$

$2\pi \dfrac{R^2}{2}$

$= \pi R^2$



$r = z \tan\theta$

r, z

$r = f(z)$

$dQ = (V_z)\, 2\pi r\, dr$

$\int_{z=0}^{z=L}$

$\int 2\pi r\, dr$

$z=0$

$z=L$

$$\tau_{rz} = \frac{\cancel{r}\,R}{2L}\left(P_0 - P_L\right)$$

$$= \frac{R}{\cancel{2}L}\left(P_0 - P_L\right) \times \cancel{2}\pi R \cancel{L}$$

$$\boxed{F = \pi R^2 \left(P_0 - P_L\right)}$$

Force $\rightarrow$ Stress F/A

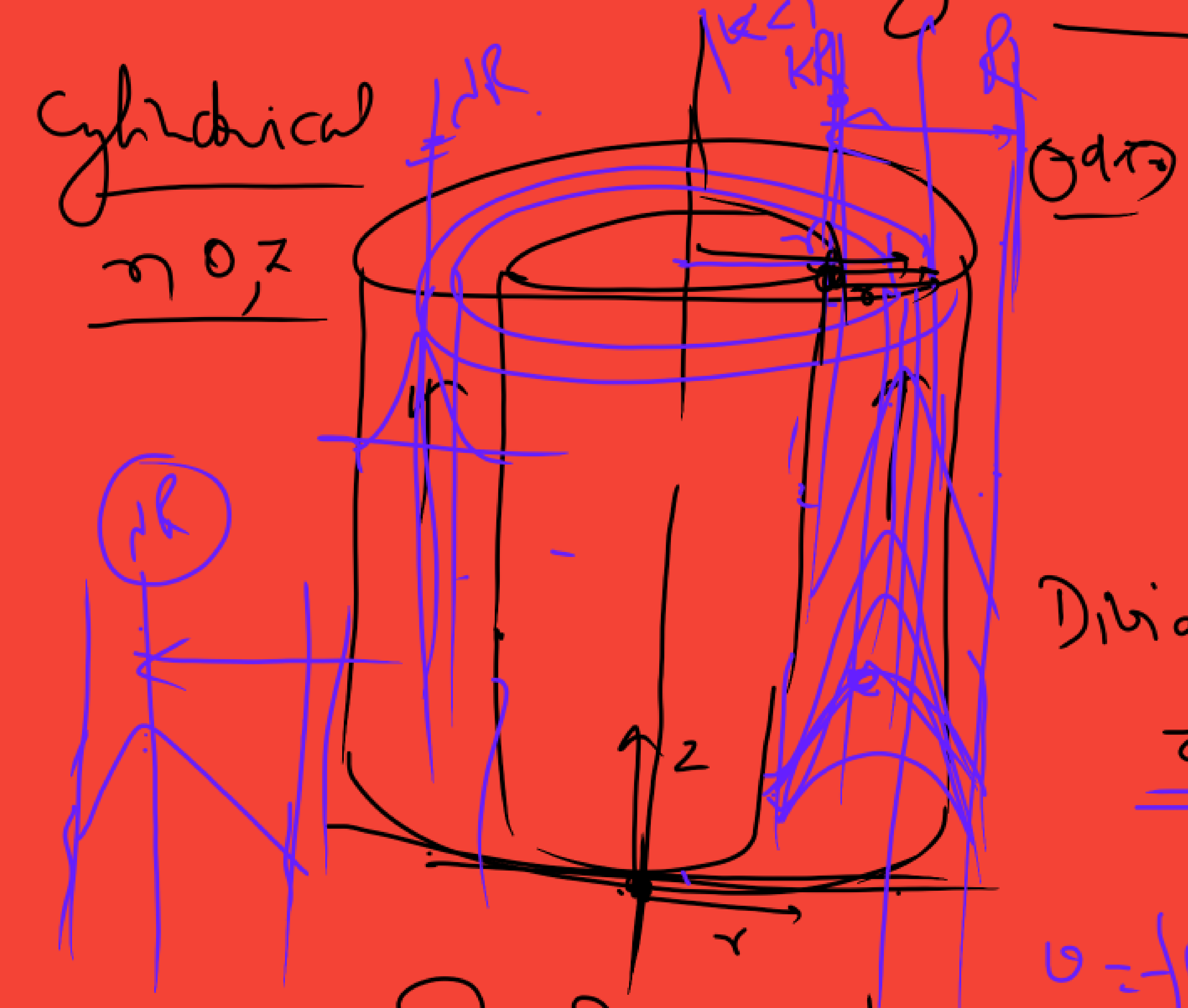
$\tau \times A$

$$\tau_{rz}\big|_{r=R} \times 2\pi R L$$

Flow through Annulus

Cylindrical

r, θ, z



$\tau_{rz} 2\pi r L|_r - |_{r+\Delta r}$

$(\rho v_z) v_z 2\pi r \Delta r|_{z=0} - |_{z=L}$

$P 2\pi r \Delta r|_{z=0} - |_{z=L}$

$\rho g_z 2\pi r \Delta r L$

Divide $2\pi r \Delta r L$

$\tau_{rz} = \pm r \left(\frac{P_0 - P_L}{2L}\right) + \frac{C_1}{r}$

$v = f(r^2)$ $\quad r = \lambda R \Rightarrow \tau_{rz} = 0$

$f(r^3)$ $\quad 0 = \pm \frac{\lambda R}{2L}(P_0 - P_L) + \frac{C_1}{\lambda R} \Rightarrow C_1 = -\frac{\lambda^2 R^2}{2L}(P_0 - P_L)$

$\mathcal{P} = P - \rho g h$

$r = \lambda R$

B.C. $\tau_{rz} = 0$, $v_z = v_{max}$

$\frac{(k+1)}{2}R$ $\quad r = kR$ $\quad \tau_{rz} = \tau_{max}$, $v_z = 0$

$r = R$ $\quad \tau_{rz} = \tau_{max}$, $v_z = 0$

$\tau_{rz} = -\mu \frac{dv_z}{dr}$

$\tau_{rz} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L\mu}\right)\left[r - \frac{\lambda^2 R^2}{r}\right]$ $\quad dr$

$\left[\frac{r^2}{2} - \lambda^2 R^2 \ln r\right]_{kR}^{R} + C_2$

$v_z = \frac{\mathcal{P}_0 - \mathcal{P}_L}{2L\mu}\left[\frac{R^2}{2} - \lambda^2 R^2 \ln R - \left(\frac{k^2 R^2}{2} - \lambda^2 R^2 \ln kR\right)\right]$

$\left(\frac{\Delta P}{4L\mu}\right)R^2 \left[(1 - k^2) + 2\lambda^2 [\ln k]\right] = 0$

A

$\lambda^2 = -\frac{(1-k^2)}{2 \ln k}$

$\lambda^2 = \frac{(k^2 - 1)}{2 \ln k}$

v_z kR R C_2