overflow from the tank/pipe

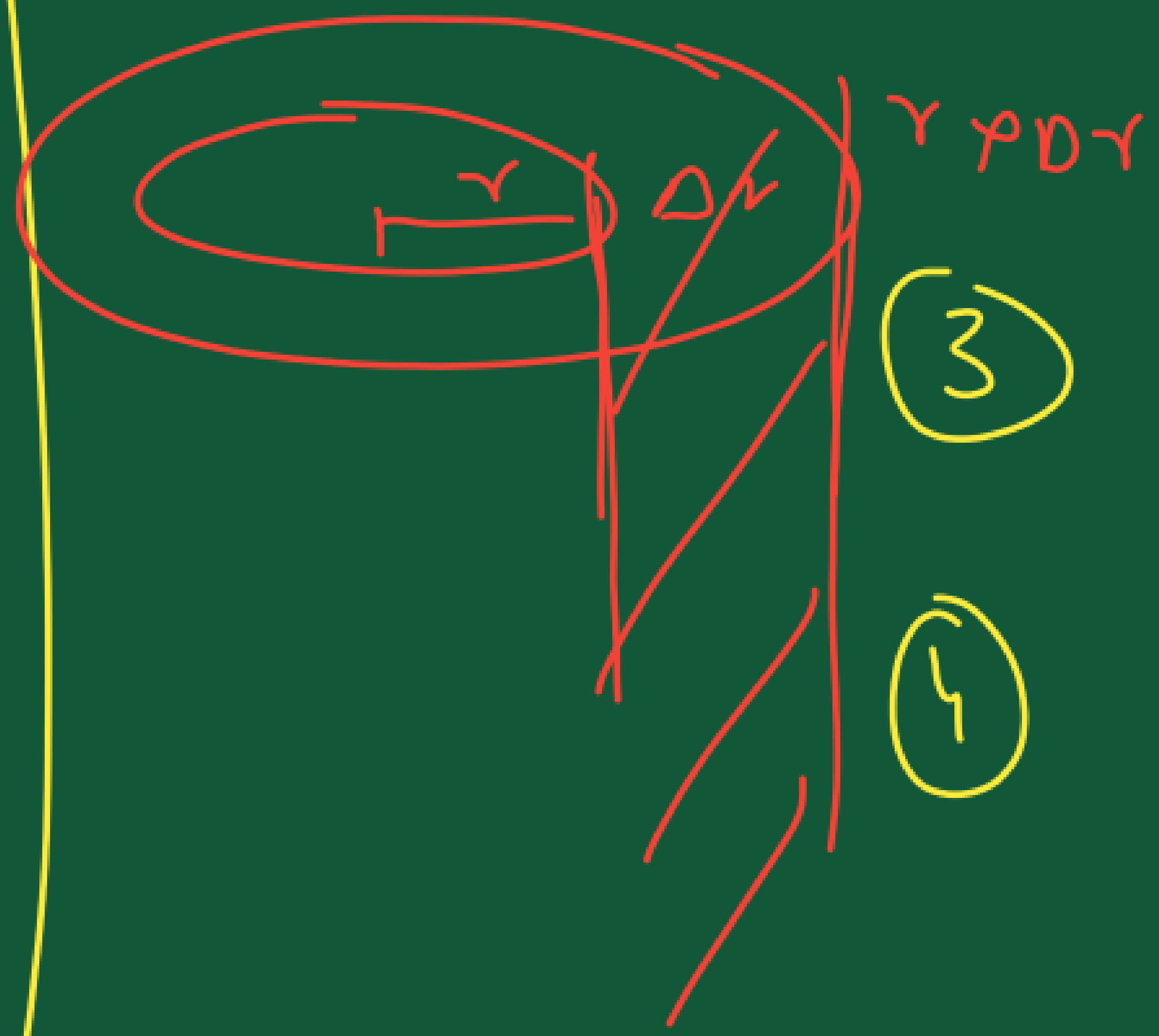
B.C

$r = R$, $\tau = \tau_{max}$ (1) $V_z = 0$

$r = R + aR$ (2) $\tau = 0$ $V_z = V_{z\,max}$

(1) $\tau_{rz} \cdot 2\pi r L \Big|_r - \Big|_{r+\Delta r}$

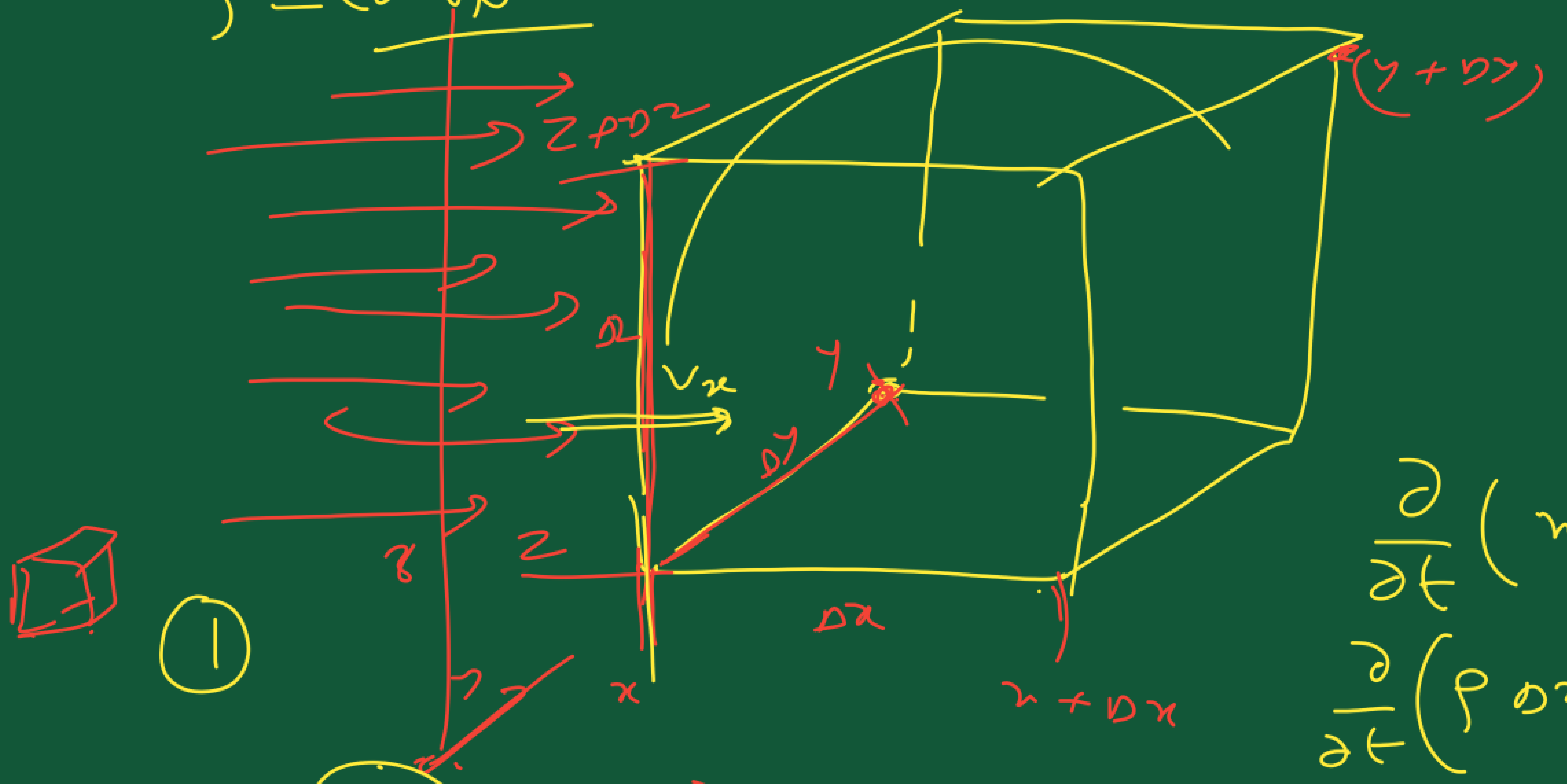
(2) $(\rho V_z) V_z \cdot 2\pi r \Delta r \Big|_{z=0} - \Big|_{z=L}$



(3) $P\, 2\pi r \Delta r \Big|_{z=0} - P \Big|_{z=L}$

(4) $\rho g_z 2\pi r \Delta r L$

$\rho = \text{Const}$



① equation of continuity

⇕

Conservation of Mass

$$\text{Accum.} = \text{in} - \text{out} + \text{gen} \quad (\text{gen} \to 0)$$

$$\frac{\partial}{\partial t}(m) = \rho \,\Delta y \Delta z\, V_x \Big|_{x} \text{ in}$$

$$\frac{\partial}{\partial t}(\rho\, \Delta x \Delta y \Delta z) = \qquad \Big|_{x+\Delta x} \text{ out}$$

$$\rho A v \qquad \rho(\Delta x \Delta z) V_y \Big|_{y} \qquad \Big|_{y+\Delta y}$$

$$\rho(\Delta x \Delta y) V_z \Big|_{z} \qquad \Big|_{z+\Delta z}$$

① in

② x

③ y

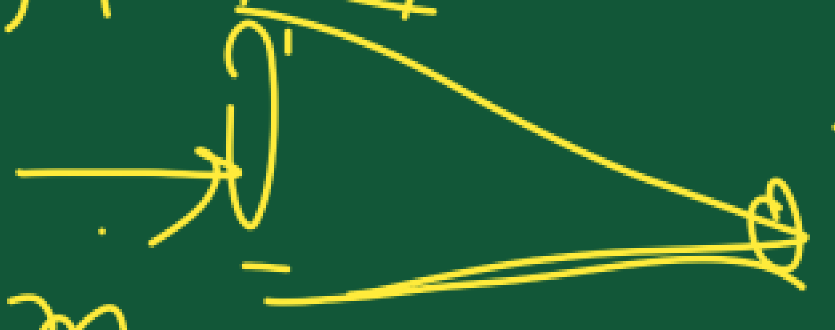
④ z

out_x $\rho A v$

$x+\Delta x$

$y+\Delta y$ $\dot{m}$

$z+\Delta z$



$\frac{Dv}{Dt} = \frac{\partial v}{\partial t} + v_x \frac{\partial v}{\partial x} + v_y \frac{\partial v}{\partial y} + v_z \frac{\partial v}{\partial z}$

$\frac{\partial}{\partial t}(\rho \Delta x \Delta y \Delta z) = \rho v_x \Delta y \Delta z \Big|_x - \rho v_x \Delta y \Delta z \Big|_{x+\Delta x}$

$\Big|_y \quad \Big|_{y+\Delta y}$

$\Big|_z \quad \Big|_{z+\Delta z}$

S.S $\frac{\partial}{\partial t} = 0$

Divide by $\Delta x \Delta y \Delta z$

$\frac{\partial}{\partial t}(\rho) = \frac{\rho v_x|_x - \rho v_x|_{x+\Delta x}}{\Delta x} + \frac{}{\Delta y} + \frac{}{\Delta z}$

const

Li
$\Delta x \to 0$
$\Delta y \to 0$
$\Delta z \to 0$

$= -\frac{\partial}{\partial x}(\rho v_x) - \frac{\partial}{\partial y}(\rho v_y) - \frac{\partial}{\partial z}(\rho v_z) \Rightarrow$

$\frac{\partial \rho}{\partial t} = -(\nabla \cdot \rho u) \Rightarrow -\left[\left(\rho \frac{\partial v_x}{\partial x} + v_x \frac{\partial \rho}{\partial x}\right) + \left(\rho \frac{\partial v_y}{\partial y} + v_y \frac{\partial \rho}{\partial y}\right) + \left(\quad \right)_z\right]$

$\frac{\partial \rho}{\partial t} + v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} \to$ RTT

$= -\rho \frac{\partial v_x}{\partial x} + \rho \frac{\partial v_y}{\partial y} + \rho \frac{\partial v_z}{\partial z}$

$=$

RTT → Reynold Transport theorem

general form

Substantial change

Total change of material property

$\boxed{\frac{D\rho}{Dt} = -\rho(\nabla \cdot u) = 0}$

EOC
eq. of con

Incompressible fluid

$\frac{D\rho}{Dt} = 0$

$(\nabla \cdot u) = 0$

0