Equation of Motion

Force

$$\frac{d(mv)}{dt}$$

$\frac{mv}{\rho}$, ρv, $v \cdot A$

balance

① $x \rightarrow$

$x \rightarrow$
$$\tau_{xx}\,\Delta y \Delta z\big|_x - \big|_{x+\Delta x}$$
$$\tau_{yx}\,\Delta x \Delta z\big|_y - \big|_{y+\Delta y}$$
$$\tau_{zx}\,\Delta y \Delta x\big|_z - \big|_{z+\Delta z}$$

y $\frac{\partial \tau_{xy}}{\partial x}$ ②
$$\tau_{xy}\,\Delta y \Delta z\big|_x - \big|_{x+\Delta x}$$
$$\tau_{yy}\,\Delta x \Delta z - \big|_{y+\Delta y}$$
$$\tau_{zy}\big|_z \Delta y \Delta x \quad \big|_{z+\Delta z}$$

z ③

② Convective part

$x \rightarrow$
$$(\rho v_x) v_x \cdot \Delta y \Delta z\big|_x - \big|_{x+\Delta x} \qquad -\frac{\partial(\rho v_x)v_x}{\partial x}$$
$$(\rho v_y) v_x\, \Delta x \Delta z\big|_y - \big|_{y+\Delta y} \qquad -\frac{\partial(\rho v_y)v_x}{\partial y}$$
$$(\rho v_z) v_x\, \Delta y \Delta x\big|_z - \big|_{z+\Delta z} \qquad -\frac{\partial(\rho v_z)v_x}{\partial z}$$

$y \rightarrow$
$$(\rho v_x) v_y - \big|_x$$
$$(\rho v_y) v_y\, \Delta x \Delta z\big|_y - \big|_{y+\Delta y}$$
$$(\rho v_z) v_y - \big|_z$$

$z \rightarrow$
$$(\rho v_x) v_z$$
$$(\rho v_y) v_z$$
$$(\rho v_z) v_z$$

x: $\left(-\frac{\partial}{\partial x}\tau_{xx} - \frac{\partial}{\partial y}\tau_{yx} - \frac{\partial}{\partial z}\tau_{zx}\right)$

y: $+\left(-\frac{\partial}{\partial x}\tau_{xy} - \frac{\partial}{\partial y}\tau_{yy} - \frac{\partial}{\partial z}\tau_{zy}\right)$

z: $+\left(-\frac{\partial}{\partial x}\tau_{xz} - \frac{\partial}{\partial y}\tau_{yz} - \frac{\partial}{\partial z}\tau_{zz}\right)$

$-\frac{\partial}{\partial x}(\rho v_x)v_x - \frac{\partial}{\partial y}(\rho v_y)v_x - \frac{\partial}{\partial z}(\rho v_z)v_x$

ρv_x^2

$+\left(-\frac{\partial}{\partial x}(\rho v_x)v_y - \frac{\partial}{\partial y}(\rho v_y)v_y - \frac{\partial}{\partial z}(\rho v_z)v_y\right)$

$+\left(-\frac{\partial}{\partial x}(\rho v_x)v_z - \frac{\partial}{\partial y}(\rho v_y)v_z - \frac{\partial}{\partial z}(\rho v_z)v_z\right)$

$\rho\frac{\partial v_x}{\partial t} + v_x\frac{\partial \rho}{\partial t}$

$= -(\nabla\cdot\tau) - (\ ?\ )$

$(\nabla\cdot\rho v_x)v_x$

$-v_x\left(\frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z)\right)$

$\frac{\partial(\rho v)}{\partial t} = -(\nabla\cdot\tau) + (\nabla\cdot\rho v v)$

Tensor

Tensor

$-(\nabla p) + \rho g$

(x) $\rightarrow \nabla p$ ... $\hat{i} + \dots$

$\frac{\partial p}{\partial x} + \frac{\partial p}{\partial y} + \frac{\partial p}{\partial z}$

$\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

(4) $\hat{i}\ \hat{j}\ \hat{k}$

(3)

x: $(\rho\,\Delta y\Delta z)|_x - (\ )|_{x+\Delta x}$

y: $\rho\,\Delta x\Delta z|_y - \ |_{y+\Delta y}$

z: $\rho\,\Delta x\Delta y|_z - \ |_{z+\Delta z}$

$-\frac{\partial p}{\partial x}$

$-\frac{\partial p}{\partial y}$

$-\frac{\partial p}{\partial z}$

$(\rho\,\Delta x\Delta y\Delta z)g_x + (\ )g_y + (\ )g_z$

$\rho g_{x,y,z}$

$$\frac{\partial}{\partial t}(mv) = \nabla \cdot \rho v\, v$$

$$\frac{\partial}{\partial t}(\rho\, \Delta x \Delta y \Delta z\, V_x) + \frac{\partial}{\partial t}(\rho\, \Delta x \Delta y \Delta z)\, V_y + \frac{\partial}{\partial t}(\rho\, \Delta x \Delta y \Delta z)\, V_z$$

$$\frac{\partial \rho V_x}{\partial t} + \frac{\partial \rho V_y}{\partial t} + \frac{\partial \rho V_z}{\partial t}$$

$$\frac{\partial (\rho v_x)}{\partial t} = \rho \frac{\partial V_x}{\partial t} + V_x \frac{\partial \rho}{\partial t}$$

$$\frac{\partial}{\partial x}(\rho V_x) V_x \rightarrow V_x \frac{\partial}{\partial x}(\rho V_x) + \rho V_x \frac{\partial V_x}{\partial x}$$

$$\frac{\partial}{\partial y}(\rho V_y) V_x \rightarrow V_x \frac{\partial}{\partial y}(\rho V_y) + \rho V_y \frac{\partial V_x}{\partial y}$$

$$\frac{\partial}{\partial z}(\rho V_z) V_x \rightarrow V_x \frac{\partial}{\partial z}(\rho V_z) + \rho V_z \frac{\partial V_x}{\partial z}$$

$$V_x \left(\frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) + \frac{\partial}{\partial z}(\rho V_z) \right) \Rightarrow EOC \Rightarrow 0$$

$$+ \left(\rho V_x \frac{\partial V_x}{\partial x} + \rho V_y \frac{\partial V_x}{\partial y} + \rho V_z \frac{\partial V_x}{\partial z} \right)$$

$\nabla \cdot \rho v \Rightarrow$ rate of loss of mass per unit volume
↓
scalar

$\nabla \cdot \rho v\, v \Rightarrow$ rate of loss of momentum " " $\frac{D\rho}{Dt}$

Rate of Momentum in x dir

$$\frac{d\, xy}{dx} \Rightarrow y \frac{dx}{dx} + x \frac{dy}{dx}$$

$$\frac{\partial (\rho V_x)}{\partial t} = (\nabla \cdot \tau)_x - \left(\rho V_x \frac{\partial V_x}{\partial x} + \rho V_y \frac{\partial V_x}{\partial y} + \rho V_z \frac{\partial V_x}{\partial z} \right) - \frac{\partial P}{\partial x} + \rho g_x$$

$$\rho\frac{\partial v_x}{\partial t} + v_x\frac{\partial \rho}{\partial t} + \left(\rho v_x \frac{\partial v_x}{\partial x} + \rho v_y \frac{\partial v_x}{\partial y} + \rho v_z \frac{\partial v_x}{\partial z}\right) = -(\nabla\cdot\tau) - \frac{\partial P}{\partial x} + \rho g_x$$

$$\rho\left[\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right] + v_x\frac{\partial \rho}{\partial t} =$$

$$\Downarrow$$

$$\rho\frac{Dv_x}{Dt} + \cancel{v_x\frac{\partial \rho}{\partial t}}^{\,0} = - - -$$

$$\tau_{xy} = -\mu\frac{\partial v_y}{\partial x}$$

$$\boxed{\rho\frac{Dv_x}{Dt} = -(\nabla\cdot\tau) - \frac{\partial P}{\partial x} + \rho g_x}$$

Assumption (1): Newtonian fluid $\tau =$

$$\tau_{xx} = -2\mu\frac{\partial v_x}{\partial x} + \frac{2}{3}\mu(\nabla\cdot v)$$

$$\tau_{xy} = \tau_{yx} = -\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$$

$$\tau_{zx} = \tau_{xz} = -\mu\left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right)$$

Stress — normal: Pres σ_{xx}

Shear τ — Parallel

$$\boxed{\tau_{xx} = \sigma_{xx}}$$

y, z

Car

$$\rho \frac{DV_x}{Dt} = -\frac{\partial P}{\partial x} + \rho g_x + \frac{\partial}{\partial x}\left[2\mu \frac{\partial V_x}{\partial x} - \frac{2}{3}\mu(\nabla \cdot v)\right] + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial V_z}{\partial x} + \frac{\partial V_x}{\partial z}\right)\right]$$

$\underbrace{(\nabla \cdot v)}_{=0}$

EOC

Assum 2 const. ρ and const μ $\Rightarrow (\nabla \cdot v) = 0$

ρ

Newton

$$= -\frac{\partial P}{\partial x} + \rho g_x + \mu\left[2\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_y}{\partial x \partial y} + \frac{\partial^2 v_z}{\partial x \partial z} + \frac{\partial^2 v_x}{\partial z^2}\right]$$

$$-\mu \nabla^2 v$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$