Optimum Design and Design Strategy

ok : *Plant Design and Economics for Chemical Engineers*, M.S. Peters and K. D. Timmerh

Chapter 11 (4th Edition)

Optimum conditions in cyclic operations

- Many processes are carried out by the use of cyclic operations which involve periodic shutdowns for discharging, cleanout or reactivation, eg., batch reactor
- In a semi-continuous cyclic operation, the product is delivered continuously while the unit is in operation but the rate of delivery decreases with time
- Thus, in a batch or cyclic operation, the variable of total time required per cycle must be considered when determining optimum conditions

Basis: Time for one cycle

$$(I) \quad Total\ annual\ cost = \frac{operating\ and\ shutdown\ costs}{cycle} \times \frac{cycles}{year} + annual\ fixed\ costs$$

$$(II) \quad Annual\ production = \left(\frac{production}{cycle}\right)\left(\frac{cycles}{year}\right)$$

$$(III) \quad \frac{Cycles}{year} = \frac{operating + shutdown\ time\ used/year}{operating + shutdown\ time/cycle}$$

xample: Determination of conditions for minimum total cost in a batch operatio

n organic chemical is being produced by a batch operation in which no product is obtained until the batch is finished. Each ycle consists of the operating time necessary to complete the reaction plus a total time of 1.4 h for discharging and chargin he operating time per cycle is equal to $1.5P_b^{0.25}$ h, where P_b is the kilograms of product produced per batch.

he operating costs during the operating period are \$20 per hour, and the costs during the discharge-charge period are \$15 er hour. The annual fixed costs for the equipment vary with the size of the batch as follows:

$$C_F = 340P_b^{0.8} \text{ dollars per batch}$$

nventory and storage charges may be neglected. If necessary, the plant can be operated 24 h per day for 300 days per year. he annual production is 1 million kg of product. At this capacity, raw-material and miscellaneous costs, other than those lready mentioned, amount to \$260,000 per year. Determine the cycle time for conditions of minimum total cost per year.

$$\frac{ycles}{year} = \frac{Annual\ production}{Production/_{cycle}} = \frac{1{,}000{,}000}{P_b}$$

$$ycle\ time = Operating\ time + Shutdown\ time = 1.5P_b^{0.25} + 1.4$$

$$\frac{Operating + Shutdown\ costs)}{cycle} = \$\left[\left(1.5P_b^{0.25}\right)20 + (1.4)15\right]$$

$$nnual\ fixed\ costs = \$\left(340P_b^{0.8} + 260{,}000\right)$$

$$otal\ annual\ costs\ = C_T = \left[\left(1.5P_b^{0.25}\right)20 + (1.4)15\right]\left(\frac{1{,}000{,}000}{P_b}\right) + \left(340P_b^{0.8} + 260{,}000\right)$$

$$= \left[\left(30P_b^{0.25}\right) + 21\right]\left(\frac{1{,}000{,}000}{P_b}\right) + \left(340P_b^{0.8} + 260{,}000\right)$$

t minimum C_T, $\frac{dC_T}{dP_b} = 0$

ow, $C_T = 30 \times 10^6 P_b^{-0.75} + \frac{21\times10^6}{P_b} + 340P_b^{0.8} + 260{,}000$

$$\frac{dC_T}{dP_b} = -0.75 \times 30 \times 10^6 P_b^{-1.75} - \frac{21\times10^6}{P_b^2} + 340 \times 0.8\, P_b^{-0.2} = 0$$

$$272\, P_b^{-0.2} = 22.5 \times 10^6 P_b^{-1.75} - \frac{21 \times 10^6}{P_b^2}$$

$$272\, P_b^{1.8} = 22.5 \times 10^6 P_b^{0.25} - 21 \times 10^6$$

olving by trial and error,

$_b$ = 1000	LHS = 683.23 X 10^5	RHS = 1475.267 X 10^5
$_b$ = 1630	LHS = 1646.3 X 10^5	RHS = 1639.65 X 10^5
$_b$ = 1627	LHS = 1640.84 X 10^5	RHS = 1638.99 X 10^5

ince LHS is approximately equal to RHS, Optimum P_b = 1627 kg per batch r conditions of minimum annual cost

$$ycle\ time = 1.5P_b^{0.25} + 1.4 = 1.5(1627)^{0.25} + 1.4 = 10.927 \sim 11\ h$$

$$otal\ time\ used\ per\ year = \left(\frac{cycles}{year}\right)(cycle\ time) = \frac{1{,}000{,}000}{1627} \times 11 = 6760.9\ h$$

$$otal\ available\ time\ per\ year = 300\ \times 24 = 7200\ h$$

ot all the available operating and shutdown time would be used

cale formation in evaporators (Optimum operating time)

uring evaporation operations, solids often deposit on the heat transfer surfaces forming a scale

he continuous scale formation causes a gradual increase in the resistance to heat flow and onsequently a decrease in the rate of heat transfer and rate of evaporation

nder these conditions, the evaporator must be shut down and cleaned after an optimum perating time, and then the cycle is repeated

/hen scale formation occurs, if θ_b is the operating time, the overall heat transfer coefficient, U may be relate
o the time of operation by the following expression,

$$\frac{1}{U^2} = a\theta_b + d \qquad a, d = \text{constants for any operation}$$

$$U = \frac{1}{\sqrt{a\theta_b + d}}$$

n a given evaporator, area and driving force are not a function of time but U is.

Q = total heat transferred during the operating time θ_b

A = heat transfer area

ΔT = temperature driving force

he rate of heat transfer at any instant is $$\frac{dQ}{d\theta_b} = UA\Delta T = \frac{A\Delta T}{\sqrt{a\theta_b + d}}$$

$$\int_0^Q dQ = A\Delta T \int_0^{\theta_b} \frac{d\theta_b}{\sqrt{a\theta_b + d}}$$

$$Q = \frac{A\Delta T}{a}\left[\frac{(a\theta_b + d)^{1/2}}{1/2}\right]_0^{\theta_b}$$

$$\boxed{Q = \frac{2A\Delta T}{a}\left[\sqrt{a\theta_b + d} - \sqrt{d}\right]}$$

Heat transfer rate per cycle

ycle time for maximum amount of heat transfer

ach cycle of evaporator operation consists of an operating (or boiling) time of θ_b h and if the time per cycle or emptying, cleaning and recharging is θ_c, then

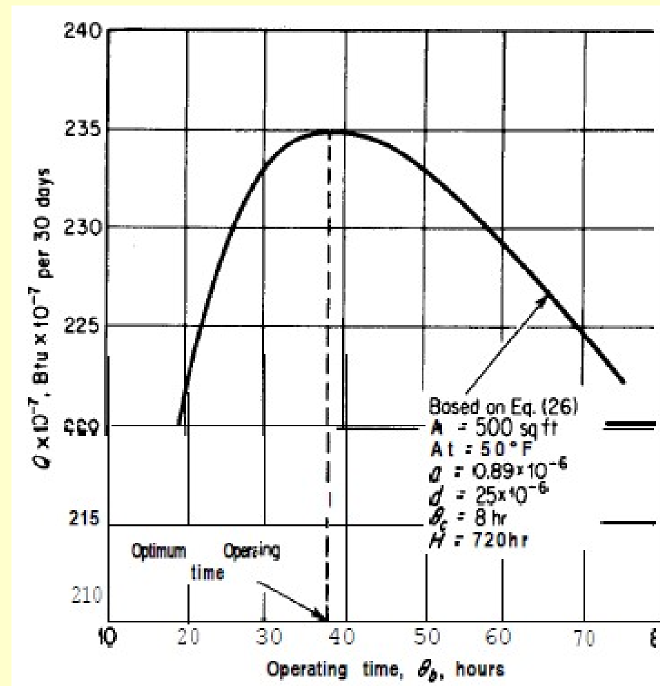
otal time per cycle = $\theta_b + \theta_c$

o of operating hours annually = H

herefore, $Number\ of\ cycles/year = \frac{H}{\theta_b + \theta_c}$

$$otal\ annual\ heat\ transfer\ rate\ per\ year = Q_H = \frac{H}{\theta_b + \theta_c} \times Q = \frac{H}{\theta_b + \theta_c} \times \frac{2A\Delta T}{a}\left[\sqrt{a\theta_b + d} - \sqrt{d}\right]$$

lot of total heat transferred (Q_H) versus θ_b (the only variable) shows a maximum at an optimum value of θ_b



he optimum operating time (θ_b) can be determined analytically from

$$\frac{dQ_H}{d\theta_b} = 0$$

$$r,\quad \frac{d\left[\frac{2A\Delta TH}{(\theta_b + \theta_c)a}\left[\sqrt{a\theta_b + d} - \sqrt{d}\right]\right]}{d\theta_b} = 0$$

$$r,\quad \frac{2A\Delta TH}{a}\left[\frac{(\theta_b + \theta_c)\left\{\frac{a}{2}(a\theta_b + d)^{-1/2}\right\} - \left\{\sqrt{a\theta_b + d} - \sqrt{d}\right\}}{(\theta_b + \theta_c)^2}\right] = 0$$

$$\frac{a(\theta_b + \theta_c)}{2\sqrt{a\theta_b + d}} = \sqrt{a\theta_b + d} - \sqrt{d}$$

$$a(\theta_b + \theta_c) = 2(a\theta_b + d) - 2\sqrt{ad\theta_b + d^2}$$

$$2\sqrt{d(a\theta_b + d)} = 2(a\theta_b + d) - a(\theta_b + \theta_c) = (a\theta_b + 2d - a\theta_c)$$

$$4[d(a\theta_b + d)] = (a\theta_b + 2d - a\theta_c)^2$$

$$4da\theta_b + 4d^2 = a^2\theta_b{}^2 + 4d^2 + a^2\theta_c{}^2 + 4da\theta_b - 4da\theta_c - 2a^2\theta_b\theta_c$$

$$a^2\theta_b{}^2 + a^2\theta_c{}^2 = 4da\theta_c + 2a^2\theta_b\theta_c$$

$$(a\theta_b - a\theta_c)^2 = 4da\theta_c$$

$$a(\theta_b - \theta_c) = 2\sqrt{ad\theta_c}$$

$$\theta_b = \frac{2}{a}\sqrt{ad\theta_c} + \theta_c$$

$$\boxed{\theta_b = 2\sqrt{\frac{d\theta_c}{a}} + \theta_c}$$

This operating time ensures maximum amount of heat transfer

ycle time for minimum cost per unit of heat transfer

ow if the objective function is cycle time for minimum cost per unit of heat transfer, then the total variable osts for H h of operating and cleaning time is given by,

$$otal\ variable\ cost\ for\ H\ h\ of\ operating\ and\ cleaning\ time\ = C_{T,for\ H\ h}\ = (C_c + S_b\theta_b)\,\frac{H}{\theta_b+\theta_c}$$

here, C_c = cost for one cleaning

$_b$= direct labour cost per hour during operation

$\frac{H}{_b+\theta_c}$ = number of cycles during H h

eplacing H from the expression, $Q_H = \frac{H}{\theta_b+\theta_c} \times \frac{2A\Delta T}{a}\left[\sqrt{a\theta_b + d} - \sqrt{d}\right]$ in the expression for C_T we get,

$$C_{T,for\ H\ h}\ = \frac{aQ_H(C_c + S_b\theta_b)}{2A\Delta T\left[\sqrt{a\theta_b + d} - \sqrt{d}\right]}$$

he optimum value of θ_b for minimum total cost may be obtained by plotting C_T versus θ_b or by setting $\frac{C_T}{\theta_b} = 0$ and solving for θ_b,

$$\boxed{\theta_b\ per\ cycle\ for\ minimum\ total\ cost = \frac{C_c}{S_b} + \frac{2}{aS_b}\sqrt{adC_cS_b}}$$

Optimum flowrate of cooling water in condenser

n a condenser with water as the cooling medium, the cooling water may be circulated at a high rat
vith a small change in cooling water temperature or at a low rate with a large change in cooling wa
emperature

he temperature of the water affects the temperature difference driving force for heat transfer

Jse of an increased amount of water will cause a decrease in necessary amount of heat transfer ar
nd a decrease in the original investment and fixed charges

On the other hand, the cost of water will increase if more water is used

herefore, an economic analysis has to be done between high water rate – low surface area and lov
vater rate – high surface area

uch an analysis shows that an optimum flowrate of cooling water occurs at the point of minimum
otal cost for cooling water and equipment fixed charges

, $q\ (Btu/h)$ = heat removed from condenser vapour
$t'\ (^oF)$ = condensation temperature of vapour (constant)
$t_1\ (^oF)$ = inlet temperature of cooling water
$t_2\ (^oF)$ = temperature of cooling water leaving the condenser
$W\ (lb/h)$ = flow rate of cooling water

$_p\ (Btu/lb\ ^oF)$ = heat capacity of cooling water

$(Btu/h\ ft^2\ ^oF)$ = constant overall heat transfer coefficient determined at optimum conditions

(ft^2) = area of heat transfer

$t_{lm}\ (^oF)$ = log mean temperature difference driving force over condenser

$_y\ (h/year)$ = hours the condenser is operated per year

$_w\ (\$/lb)$ = cooling water cost, which is proportional to cooling water supplied

$_n\ (\$/ft^2)$ = installed cost of heat exchanger per square foot of heat transfer area

$_F$ = annual fixed charges including maintenance, expressed as a fraction of initial cost for completely installec quipment

$$q = WC_P(t_2 - t_1) = UA\Delta t_m = \frac{UA(t_2-t_1)}{ln\frac{(t'-t_1)}{(t'-t_2)}} \qquad \text{.............. (1)}$$

Therefore, $$W = \frac{q}{C_P(t_2-t_1)} \qquad \text{.............. (2)}$$

esign conditions set values of q and t_1, C_P is known

lowrate of cooling water is fixed if temperature of water leaving condenser (t_2) is fixed

) Annual cost for cooling water is = WH_yC_W

$$WH_yC_W = \frac{qH_yC_W}{C_P(t_2 - t_1)} \qquad \ldots\ldots\ldots\ldots\ldots \qquad (3)$$

I) Annual fixed charges for condenser = AK_FC_A

Total variable cost (annual) = $C_T = \frac{qH_yC_W}{C_P(t_2 - t_1)} + AK_FC_A$ .... (4)

he value of A is replaced from equation (1)

$$C_T = \frac{qH_yC_W}{C_P(t_2 - t_1)} + \frac{qK_FC_A ln\left[\frac{(t' - t_1)}{(t' - t_2)}\right]}{U(t_2 - t_1)}$$

nly variable in the above equation is t_2

o get optimum exit temperature, $\frac{dC_T}{dt_2} = 0$

he final equation obtained is,

$$\frac{(t' - t_1)}{(t' - t_{2,opt})} - 1 + ln\left[\frac{(t' - t_{2,opt})}{(t' - t_1)}\right] = \frac{UH_yC_W}{C_PK_FC_A}$$

$_{2,opt}$ can be obtained from trial and error method or it can be found from the graphical method

Optimum reflux ratio

The design of a distillation column is based on the degree of separation required for a feed supplied at a known composition, flow rate and temperature

The design engineer needs to determine the reflux ratio and then the size of the column to get the required specification

As the reflux ratio is increased, the number of theoretical stages required for a given separation decreases

An increase in the reflux ratio may result in lower fixed charges for the distillation column but larger costs for reboiler heat supply and condenser coolant

The optimum reflux ratio occurs at the point where the sum of fixed charges and operating costs is a minimum.
Usually it falls in the range of 1.1 tp 1.3 times minimum reflux ratio