Interest and Investment Costs

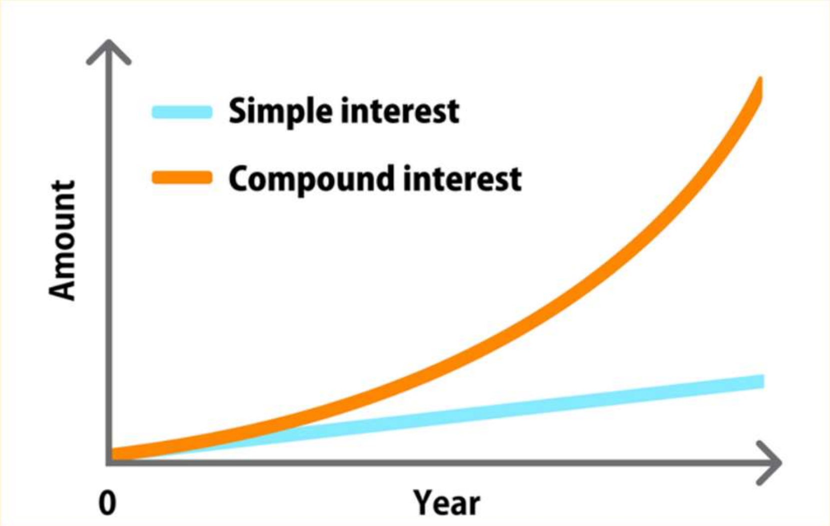
Book : *Plant Design and Economics for Chemical Engineers*, M.S. Peters and K. D. Timmerhaus
Chapter 7 (4th Edition)

- **Interest**, as defined by engineers, **is the compensation paid for borrowed capital.**
- The rate at which interest will be paid is usually fixed at the time the money is borrowed, and a guarantee is made to return the capital at some set time in the future or according to a agreed-upon pay-off schedule.

- **Interest is usually of two kinds** –

  (i) Simple interest

  (ii) Compound interest



Simple Interest

- This is the simplest form of interest where compensation is paid at a constant interest rate based only on the original principal.
- The interest earned does not earn any interest and the principal (original capital borrowed) never changes
- If time for borrowing is less than one year, simple interest can be used.

If P = principal amount
i = interest rate expressed as a fraction
n = number of years (number of interest periods)
I = amount of simple interest

$$I = Pin$$

If S= total amount (principal + interest) = future value

$$S = P + Pin$$

$$S = P(1 + in)$$

Simple Interest

- The time unit used to determine the number of interest periods is usually one year, and the interest rate (i) is expressed on a yearly basis
- When the interest period is less than one year, simple interest is estimated assuming the year consists of 12 30 day months (360 days) – this is ***ordinary simple interest***
- If the number of days in a year is considered as 365, then the interest calculated is called ***exact simple interest***

Ordinary simple interest = $Pi\left(\frac{d}{360}\right)$

Exact simple interest = $Pi\left(\frac{d}{36}\right)$

d = number of days in the interest period

- In case of simple interest, it makes no difference if the interest is paid at the end of each time unit or after a number of time units or even at the end of the designated period (loan period)
- In all cases the, the amount of total money to be paid is the same.

Compound Interest

- Calculation of compound interest considers that money has a time value
- If the interest is paid at the end of each time unit, it can be put to use by the receiver to get returns – this is taken into account when calculating compound interest
- If interest is not paid at the end of each time unit, the amount due is added to the principal, and interest is charges on this new principal in the next time unit.
- If the time for calculation is above one year, compound interest is used

If P = principal amount

Interest at the end of 1st year = Pi

Amount (sum) at the end of 1st year = $P + Pi = P(1+i)$

Interest at the end of 2nd year = $P(1+i)i$

Amount (sum) at the end of 2nd year = $P(1+i) + P(1+i)i = P(1+i)(1+i) = P(1+i)^2$

Interest at the end of 3rd year = $P(1+i)^2 i$

Amount (sum) at the end of 3rd year = $P(1+i) + P(1+i)^2 i = P(1+i)^2(1+i) = P(1+i)^3$

Therefore, the total amount (S) of principal and compound interest due after n interest periods is

$$S = P(1+i)^n$$

$(1+i)^n$ = compound interest factor

Nominal and Effective Interest Rates

- In common industrial practice, the fixed interest rate i is based on one year and the discrete interest period is one year
- Suppose, the interest rate is 3% per period and the interest is compounded at half year periods, then this would be reported as "*interest rate of 6% compounded semi-annually*"
- The interest rate reported in this form is called the ***nominal interest rate***
- Now, if the above interest rate is based on the original principal and time unit of one year, then it is called the ***effective interest rate***

- When we talk about nominal interest rate (as stated above), the annual return on the principal would not be exactly 6%, but some what larger due to the compounding effect at the end of semi-annual (6 month) period

For eg., If Rs. 1000 Is invested at a nominal interest rate of 20% compounded annually, the amount after one year would be = $P(1+i)^n = 1000(1+0.2)^1 = 1200$

If this is compounded semi annually, the amount after one year would be

$$= P\left(1+\frac{i}{m}\right)^{nm} = 1000\left(1+\frac{0.2}{2}\right)^2 = 1210$$

Nominal and Effective Interest Rates

- All nominal interest rates should always include a qualifying statement indicating the compounding period.
- In common engineering practice, it is always preferable to deal with effective interest rates, rather than nominal interest rates
- A relation can be found between the effective and nominal interest rate

If P = principal amount

r = nominal interest rate expressed as a fraction

m = number of compounding periods per year

$\frac{r}{m}$ = rate of interest per period

The principal P is compounded m times at a rate of $\frac{r}{m}$

Therefore, total amount (S) at the end of one year is

$$S = P\left(1 + \frac{r}{m}\right)^m$$

If the effective interest rate is = i_{eff}

Then, total amount (S) at the end of one year is $S = P(1 + i_{eff})$

Nominal and Effective Interest Rates

- Therefore, $P\left(1+\frac{r}{m}\right)^{m} = P(1+i_{eff})$

or $$i_{eff} = \left(1+\frac{r}{m}\right)^{m} - 1$$

- Now, if the sum has to be calculated after n years with compounding done m times a year at a nominal interest rate of r we have,

$$S = P\left(1+\frac{r}{m}\right)^{nm}$$

$\frac{r}{m}$ = rate of interest per period
nm = number of compounding periods in n years

- In the above case, the payments are charged at discrete intervals and the intervals have a finite length of time. If these time intervals become infinitely small, the interest is said to be ***compounded continuously***
- The concept of continuous interest is that the cost or income due to interest flows regularly

Continuous Compounding Interest Rates

- If the interest is compounded continuously, the number of compounding periods = ∞ (infinity)

$$S = P\ \mathcal{L}t_{m\to\infty}\left(1+\frac{r}{m}\right)^{nm}$$

$$S = P\ \mathcal{L}t_{m\to\infty}\left(1+\frac{r}{m}\right)^{\frac{m}{r}nr}$$

$$\boxed{S = Pe^{nr}}$$

$$\because\quad \mathcal{L}t_{x\to\infty}(1+x)^{\frac{1}{x}} = e$$

- If the effective interest rate is = i_{eff} , then for one year

$$Pe^{r} = P\left(1+i_{eff}\right)$$

or $$\boxed{i_{eff} = e^{r} - 1}$$

and $$r = \ln\ (1+i_{eff})$$

For n years, $$nr = n\ln\ (1+i_{eff}) = \ln\left(1+i_{eff}\right)^{n}$$

$$e^{nr} = \left(1+i_{eff}\right)^{n}$$

- Therefore, $$\boxed{S = Pe^{nr} = P\left(1+i_{eff}\right)^{n}}$$

Problem

An industry borrows Rs.50,00,000 to meet a financial obligation. The money is borrowed at a monthly interest rate of 2%. Determine :

(a) The total amount after 2 years if interest is calculated as simple interest and no intermediate payments are made

(b) The total amount after 2 years if interest is calculated as compound interest and no intermediate payments are made

(c) Nominal interest rate when interest is compounded monthly

(d) Effective interest rate when interest is compounded monthly

(e) The total amount after 2 years if interest is compounded continuously and the effective interest rate under these conditions

Length of interest period = 1 month

Number of interest periods in 2 years = 24

(a) For simple interest, $S = P(1 + in)$

$$S = 50{,}00{,}000(1 + 0.02 \times 24)$$

$$S = Rs.\,74{,}00{,}000$$

(b) For compound interest, $S = P(1+i)^n$

$$S = 50{,}00{,}000(1+0.02)^{24}$$

$$S = Rs.\,8042186.25$$

(c) Nominal Interest rate = 0.02 X 12 = 24% per year

(d) Number of interest periods per year = m = 12

Nominal interest rate = r = 24%

Therefore, $i_{eff} = \left(1+\frac{r}{m}\right)^m - 1 = \left(1+\frac{0.24}{12}\right)^{12} - 1 = 0.2682$

Effective interest rate = 26.82%

(e) For continuous compounding, $S = Pe^{nr}$

$$S = 50{,}00{,}000 \times\times\ e^{0.24X2} = Rs.\,80.80{,}372$$

(a) Effective interest rate, $i_{eff} = e^r - 1$

$$= e^{0.24} - 1 = 0.2712$$

Effective interest rate = 27.12%