Interest and Investment Costs

Book : *Plant Design and Economics for Chemical Engineers*, M.S. Peters and K. D. Timmerhaus
Chapter 7 (4th Edition)

Time value of money

- Money can be used to earn money by investment. An initial amount of money that is invested increases in value with time. This effect is known as the "time value of money".
- By virtue of its capacity to earn (earning capacity), an amount of money available at the present time is worth more or equivalent to a greater amount in the future.
- For eg., Rs. 1000 invested at 10% compound interest is worth Rs. 2594 after 10 years. This value of at a future time (Rs. 2594) is called the "***future worth***" or "***future value***" of the money.

- It is often necessary to determine the amount of money that must be available in the present time in order to have a certain amount accumulated at some definite time in the future
- Thus, the principal that must be deposited at a given interest rate to yield the desired amount at some future date is known as the "***present worth***" or "***present value***" of a future amount
- For eg., Rs. 1000 is the present worth of the future amount of Rs. 2594 at 10% compound interest
- Thus, Rs. 1000 and Rs. 2594 (after 10 years) are said to be ***equivalent*** at 10% rate of interest compounded annually.

Time value of money

- When calculating compound interest, the final amount or *future worth of a present value* P is written as

$$S = P(1+i)^n$$

- Therefore, the present worth of a future amount S can be written as

$$P = \frac{S}{(1+i)^n}$$

- The factor $(1+i)^n$ is called the discrete single payment present worth factor
- If the interest is compounded continuously, the present worth is

$$P = \frac{S}{e^{nr}}$$

- The calculation of the future worth of a present amount of money is known as "***compounding***" while calculation of the present worth of a future amount is called "***discounting***"
- In business terminology, the difference between the indicated future value and the present worth is known as the "***discount***"

Problem

It is desired to have Rs.90000 available 12 years from now. If Rs. 50000 is available for the investment at the present time, what discrete annual rate of compound interest be necessary to give the desired amount?

P = Rs. 50000 S = Rs. 90000 n = 12

$$S = P(1+i)^n$$

$$90000 = 50000(1+i)^{12}$$

i = 0.0502

or Annual rate of interest = 5.02%

Problem

A bond has a maturity value of Rs. 100000 and is paying discrete compound interest at an effective annual interest rate of 3%. Determine the following at a time 4 years before the bond reaches the maturity value:

(i) Present worth
(ii) Discount
(iii) Discrete compound rate of effective interest which will be received by a purchaser if the bond was obtained at Rs. 700000
(iv) Repeat part (i) where the nominal bond interest is 3% compounded continuously

(i) Present worth

$$P = \frac{S}{(1+i)^n} = \frac{100000}{(1+0.03)^4} = Rs. 88849$$

(ii) Discount

$$Discount = Future\ Value\ - Present\ worth = 100000 - 88849 = Rs. 11151$$

(iii) Discrete compound rate of effective interest can be estimated from $S = P(1+i)^n$

$$P = Rs. 70000 \qquad S = Rs. 100000$$

$$S = P(1+i)^n$$

$$100000 = 70000(1+i)^4$$

i = 0.093265 or 9.3265%

(iv) Present worth when nominal bond interest is 3% compounded continuously

$$P = \frac{S}{e^{nr}} = \frac{100000}{e^{4X0.03}} = Rs. 88692$$

Annuity

- A annuity is a series of equal payments made at equal time intervals
- This is similar to a recurring deposit in a bank
- Such deposits can be used to payoff a debt, accumulate a desired amount of capital etc. .
- Engineers often encounter annuities in depreciation calculations where the decrease in value of equipment with time is accounted for by an annuity factor
- ***Ordinary annuity*** is when payments are made at the end of each interest period
- ***Annuity due*** is when payments are made at the beginning of each period instead of the end
- ***Deferred annuity*** is when the first payment is due after a definite number of years
- ***Annuity term*** is the time from the beginning of the first payment period to the end of the last payment period
- The ***amount of an annuity*** is the sum of all the payments plus interest if allowed to accumulate at a definite rate of interest from the time of initial payment to the end of the annuity term.

Annuity

- Suppose periodic payments = R
- Interest rate (as fraction) = i
- Length of an annuity (annuity term) = n
- Annuity value (amount of annuity) = S
- If payment is made after 1 year, then this amount will earn interest for (n-1) years
- If payment is in the beginning of the year, then the interest is earned for n years
- If the first case is considered,

$$R(1+i)^{n-1} + R(1+i)^{n-2} + R(1+i)^{n-3} + \dots\dots\dots. + R = S$$

- Multiplying by $(1+i)$ we get,

$$R(1+i)^{n} + R(1+i)^{n-1} + R(1+i)^{n-2} + \dots\dots\dots. + R(1+i) = S(1+i)$$

- Subtracting the first equation from the second,

$$S(1+i) - S = R(1+i)^n - R$$

$$Si = R(1+i)^n - R = R[(1+i)^n - 1]$$

$$\boxed{R = \frac{Si}{(1+i)^n - 1}}$$

Present worth of an Annuity

The present worth of an annuity is defined as the principal that must be invested in the present time at compound interest rate i to yield a total amount at the end of the annuity term equal to the amount of annuity

$$S = P(1+i)^n$$

Also, $S = \frac{R[(1+i)^n - 1]}{i}$

Since, $\frac{R[(1+i)^n - 1]}{i} = P(1+i)^n$

$$P = \frac{R[(1+i)^n - 1]}{i(1+i)^n}$$

Problem : A cash flow of $ 12000 per year is received at the end of each year (uniform periodic payments) for 7 consecutive years. The rate of interest is 9% per year compounded annually. What is the present worth of such a cash flow? [GATE 2014]

Present worth of an annuity, $P = \frac{R[(1+i)^n - 1]}{i(1+i)^n}$

$$P = \frac{12000[(1+0.09)^7 - 1]}{0.09(1+0.09)^7} = \$\ 60395.43$$