Interest and Investment Costs

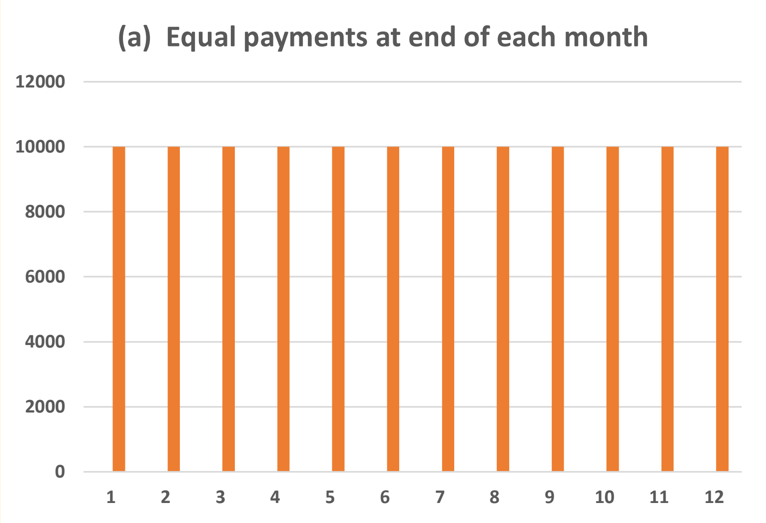
Book : ***Plant Design and Economics for Chemical Engineers*****, M.S. Peters and K. D. Timmerhaus**
Chapter 7 (4th Edition)

Cash Flow Patterns

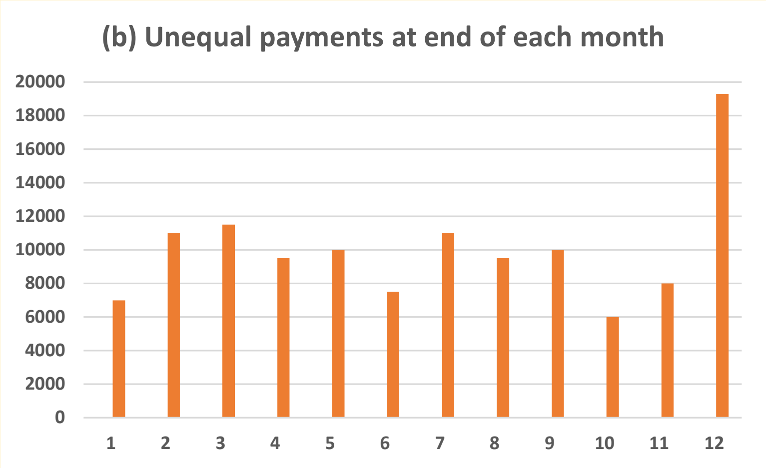
- Cash flow is the amount of funds that enter the corporate treasury as a result of the activities of the project
- The annual cash flow is equal to the net (after tax) profit plus the allowed depreciation charges for a year.
- Since the cash flow occurs over the lifetime of a project, it is necessary to convert them to equivalent values. This is done by discounting future cash flows or by compounding earlier cash flows to a particular point of time.
- While it is essential that all cash flows be converted to the same time, the time selected is not critical.

Discrete Cash Flows

- Cash flow patterns can be equal over each period or variable.
- For eg., (a) equal discrete cash flow occurring once per month at the end of the month for a period of one year or (b) they may be unequal end of month cash flow for one year



- These cash flows are equivalent at a discount rate of 10% per year. This means that they have the same worth at a particular time when calculated at the discounted rate, whether the worth is calculated at time zero, at 12 months or any other time
- This can be verified by calculating the present and future worth of the cash flow at 10% interest



Equal monthly cash flow

End of month n	Cash flow \$	Present worth (\$ at 0 month) $P = \frac{S}{(1+i)^n}$	Future worth (\$ at 0 month) $S = P(1+i)^n$
1	10000	9917	10956
2	10000	9835	10865
3	10000	9754	10775
4	10000	9673	10686
5	10000	9594	10598
6	10000	9514	10511
7	10000	9436	10424
8	10000	9358	10338
9	10000	9280	10252
10	10000	9204	10167
11	10000	9128	10083
12	10000	9053	10000
		113745	**125656**

i = 0.1/12
(since i is 10% per year)

[9917 = $\frac{10000}{(1+0.1/12)^1}$]

[9835= $\frac{10000}{(1+0.1/12)^2}$]

[10956 = $10000(1+0.1/12)^{12-1}$]

[10865= $10000(1+0.1/12)^{12-2}$]

Unequal monthly cash flow

End of month n	Cash flow $	Present worth ($ at 0 month) $P = \frac{S}{(1+i)^n}$	Future worth ($ at 0 month) $S = P(1+i)^n$
1	7000	6942	7669
2	11000	10819	11952
3	11500	11217	12392
4	9500	9190	10152
5	10000	9594	10598
6	7500	7136	7883
7	11000	10379	11466
8	9500	8890	9821
9	10000	9280	10252
10	6000	5522	6100
11	8000	7302	8067
12	19304	17474	19304
		113745	**125656**

i = 0.1/12
(since i is 10% per year)

[6942 = $\frac{7000}{(1+0.1/12)^1}$]

[10819 = $\frac{11000}{(1+0.1/12)^2}$]

[7669 =$7000(1+0.1/12)^{12-1}$]

[11952 =$11000(1+0.1/12)^{12-2}$]

Continuous Cash Flows

- A continuous cash flow is one in which receipts and expenditures occur continuously over time. Therefore, in this case, the cash flow is invested continuously as it is received
- When interest is compounded continuously and $\bar{P}$ is the continuous constant rate of cash flow per period (usually 1 year), the future worth of cash flow is

$$F \;\; or \;\; S = \bar{P}\left(\frac{e^{r}-1}{r}\right)$$

- The present worth of a 1 year, continuous, constant cash flow starting at the end of year j-1 and ending at the end of year j is given by

$$P = \bar{P}\left(\frac{e^{r}-1}{r}\right)e^{-rj}$$

- For cash flows occurring over a period of N years, starting at time zero, the future worth factor is

$$F \;\; or \;\; S = \bar{P}\left(\frac{e^{rN}-1}{r}\right)$$

- The present worth factor of such a cash flow is

$$P = \bar{P}\left(\frac{e^{rN}-1}{r}\right)e^{-rN}$$

Problem : A cash flow consisting of Rs. 10000 per year is received in one discrete amount at the end of each year for 10 consecutive years. The rate of interest is 10% per year compounded annually. Determine the present worth of such a cash flow at time zero, and the future worth at the end of 10 years of cash flow.

Present worth of an annuity, $P = \frac{R[(1+i)^n - 1]}{i(1+i)^n}$

$$P = \frac{10000[(1+0.1)^{10} - 1]}{0.1(1+0.1)^{10}} = Rs. 61446$$

Future worth of this amount is $S = P(1+i)^n$

$$S = 61446(1+0.1)^{10} = Rs. 157375$$

Thus, Rs. 10000 invested over 10 years is equivalent to Rs. 61446 invested at time zero and is equivalent to Rs. 159375 received 10 years later

Problem : For a project having a life of 10 years, the following cash flow pattern is expected.

End of years	Net cash flow (Rs)
0	50,00,000
1-10	20,00,000
10	-1,50,00,000

If the expected interest rate is 20%, what is your recommendation about implementing the project?

At the end of 10 years, Rs. 50,00, 000 becomes, $S = P(1+i)^n$ = 5000000 $(1+0.2)^{10} = Rs. 3{,}09{,}58{,}682$

For years 1 to 10, annuity is Rs. 20,00,000 at the end of each year

Total sum of annuity is, $S = \frac{R[(1+i)^n - 1]}{i} = \frac{2000000[(1+0.2)^{10} - 1]}{0.2} = Rs. 5{,}19{,}17{,}364$

Total inflow after 10 years = $Rs. 3{,}09{,}58{,}682$ + $Rs. 5{,}19{,}17{,}364 = Rs. 8{,}28{,}76{,}046$

Total outflow is = $Rs. 1{,}50{,}00{,}000$

Net cash flow (outflow < inflow) is positive. Therefor, the project can be implemented.

Capitalized cost

- A ***perpetuity*** is an annuity in which the periodic payments continue indefinitely
- This type of annuity is of particular interest to engineers, for they might desire to determine a total cost for a piece of equipment under conditions which permit the asset to be replaced perpetually without considering inflation or deflation.
- Suppose the original cost of a certain piece of equipment is Rs. 12,000

  The scrap value at the end of the useful life is Rs. 2000 and the useful life period is 10 years

  Therefore, it would be necessary to supply Rs. 10,000 every 10 years in order to replace the equipment. Therefore, a certain amount of money needs to be invested (P) such that after every 10 years, there is enough money generated from the interest to buy the equipment (the principal remains intact)

  Replacement cost = Rs.12000 – Rs. 2000 = Rs. 10000

  Interest gained by P in 10 years =$P(1+i)^n - P$

  This is equal to replacement cost of Rs. 10000

$$P(1+i)^n - P = P(1+0.06)^{10} - P = 10000$$
$$P = 12645$$

If Rs. 12645 is invested for 10 years, the amount becomes = Rs. 22645

Rs. 10000 is used to replace the equipment and the principal, Rs. 12645 remains the same and is reinvested.

Capitalized cost

- Capitalized cost of an equipment is the sum of original cost of the equipment plus the amount invested so that the equipment can be replaced at the end of the service life perpetually (present value of the renewable perpetuity)

- If P is the amount invested at i % interest, then after n years the amount S is =

$$S = P(1+i)^n$$

$C_R = C_v - C_S$
$= \text{original cost} - \text{salvage value}$

- After n years, amount S minus the principal P must be the replacement cost (C_R)

$$S - P = C_R$$

$$S - P = C_R = P(1+i)^n - P = P[(1+i)^n - 1]$$

$$\therefore \quad P = \frac{C_R}{[(1+i)^n - 1]}$$

Therefore, Capitalized cost, $\kappa = C_v(original\ cost) + P$

$$\kappa = C_v + \frac{C_R}{[(1+i)^n - 1]}$$

Capitalized cost

- Capitalized costs are used to compare different equipments
- The lower capitalized cost, means lower capital needed to run the equipment if the plant runs perpetually
- This is, however, just a concept. No one wants to invest the extra amount (P) in the bank as the money can earn more interest if invested elsewhere. Moreover, no company runs for infinite time.

Problem: A heat exchanger has been designed for use in a chemical process. A standard type of heat exchanger with negligible scrap value costs \$ 1000 and will have an useful life of 6 years. Another proposed heat exchanger of equivalent design capacity costs \$ 1700 but has an useful life of 10 years and a scrap value of \$ 200. Assuming a compound interest rate of 5% per year, determine which exchanger is cheaper.

Heat Exchanger 1

C_v = \$ 1000 C_v = \$ 0
C_R = \$ 1000 i = 5% n = 6 years

$$\kappa = C_v + \frac{C_R}{[(1+i)^n - 1]}$$

$$\kappa = 1000 + \frac{1000}{[(1+0.05)^6 - 1]}$$

$$= \$\ 3940.35$$

Heat Exchanger 1 is cheaper

Heat Exchanger 2

C_v = \$ 1700 C_v = \$ 200
C_R = \$ 1700 -\$200 =\$ 1500 i = 5% n = 6 years

$$\kappa = C_v + \frac{C_R}{[(1+i)^n - 1]}$$

$$\kappa = 1700 + \frac{1500}{[(1+0.05)^{10} - 1]}$$

$$= \$\ 4085.14$$

Cost of capital

- There are several possible sources of capital for business ventures – loans, bonds, stocks and corporate funds
- Corporate funds, primarily from undistributed profits and depreciation accumulations are usually a major source for capital for established businesses.
- Loans or borrowed funds are often used to supply all or part of the corporate investment. The interest paid on the loan is one of the costs of making a product
- Stocks and bonds are two ways to raise funds for a project.
  - **Stocks** are simply shares of individual companies. The company splits itself into shares and then sells a portion of the shares in the open market in a process known as initial public offering (IPO).
  - A person who buys a stock is actually a share holder in the company. Stock are also called equities. In case of stocks, the return is not guaranteed. A person gains when the company is doing well and makes a profit while he loses if the company is doing poorly.
  - However, when a company issues a **bond**, it is issuing a debt with the agreement to pay interest for the use of the money
  - At the end of the bond maturity period, the investor is paid back his original principal. The interest is paid to the investor all through the years till maturity. These are less risky than stocks but earn less return