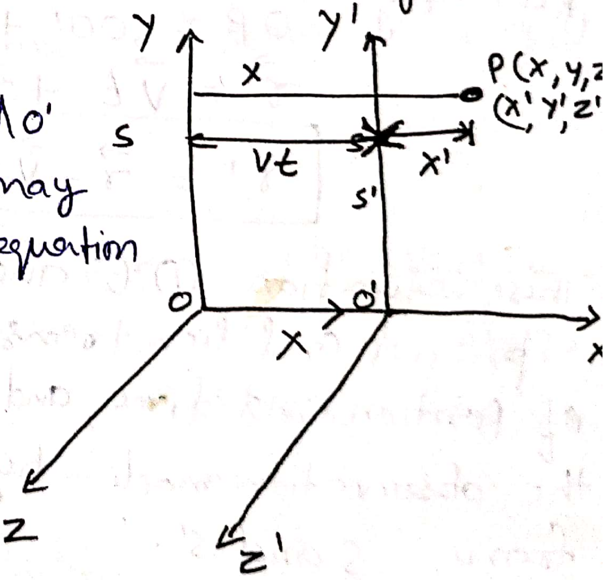
Galilean Transformation

The equation which provide the relationship between the Co-ordinates of two reference system are called Transformation equation or Galilean transformations. The Galilean transformations are used to transform the Co-ordinates of position and time from one inertial frame to the another.

→ Let us Consider two inertial frame S and S'. S is at rest and S' moving with a Constant velocity v relative to S. The position of two observers O and O', observing an event at any point 'P' Coincide with origin of two frames S and S'

→ For making relation of position and motion between these two frame there are two possibility -

1→ When the second frame move relative to first along positive direction of X-axis -



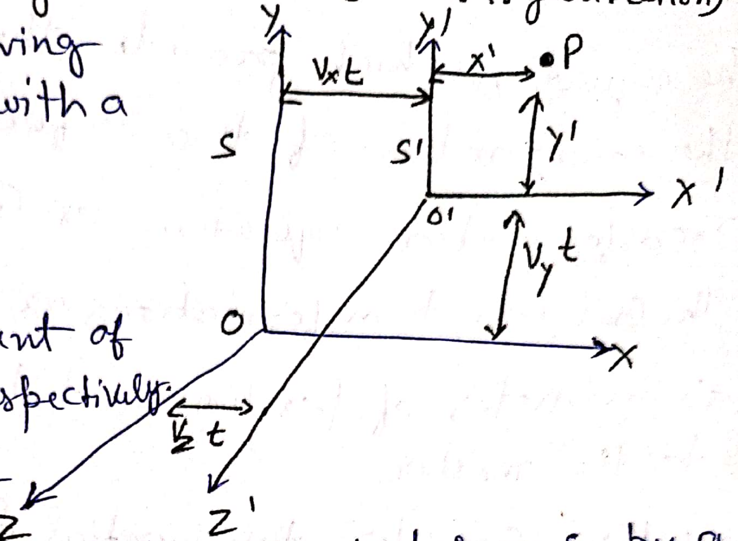
The observation of two observers O and O' of the same event happening at P may be seen to be related by the following equation

$$x' = x - vt, \quad z' = z$$
$$y' = y \quad t' = t \qquad \text{--- (1)}$$

These equation are Called Galilean Transformation

2 → When the second frame moving along a straight line relative to first along any direction. (Arbitrary direction)

– Let the second frame s' moving relative to first frame s with a velocity $\vec{V}$, such that



$$\vec{V} = \hat{i} V_x + \hat{j} V_y + \hat{k} V_z$$

Here V_x, V_y, V_z are component of V along X, Y, Z axis respectively.

If the origin of two initially then after t time the frame s' is separated from s by a distance $V_x t$, $V_y t$ and $V_z t$ along X, Y, Z axis respectively. then

$$\left.\begin{aligned} x' &= x - V_x t & z' &= z - V_z t \\ y' &= y - V_y t & t' &= t \end{aligned}\right| \quad -(2)$$

These equation are also Galilean transformation equation.

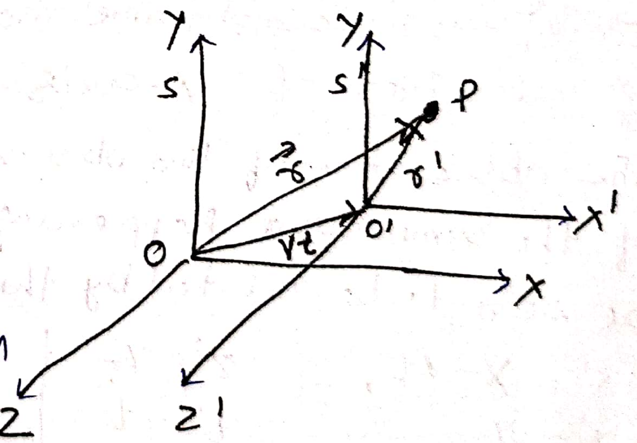
* These Galilean transformation of position may be obtained by a single equation by using vector addition law of triangle

From fig

$$\vec{OP} = \vec{OO'} + \vec{O'P}$$

$$\vec{r} = \vec{V}t + \vec{r}'$$

$$\boxed{\vec{r}' = \vec{r} - \vec{V}t} \quad -(3)$$



These equation (1), (2) and (3) represent Galilean transformation of position and time, and relate the observation made by two observers in two different inertial frame s and s'.

→ Galilean transformation of the velocity and acceleration of a particle :

Consider two inertial frame S and S', the frame S' moving with velocity $\vec{v}$ relative to S which is given as -

$$\vec{v} = \hat{i}v_x + \hat{j}v_y + \hat{k}v_z$$

Let r and r' is the position vectors of any particle at time t as observed by observers in frame S and S' respectively. Then from Galilean transformation we have

$$\left.\begin{aligned} \vec{r}' &= \vec{r} - \vec{v}t \quad (a) \\ t' &= t \quad (b) \end{aligned}\right\} - (1)$$

differentiating and keeping v is constant we get

$$\left.\begin{aligned} dr' &= dr - v\,dt \quad (a) \\ dt' &= dt \quad (b) \end{aligned}\right\} \quad (2)$$

dividing (a) by (b)

$$\frac{dr'}{dt'} = \frac{dr}{dt} - \frac{v\,dt}{dt} = \frac{dr}{dt} - v \;..(3) \qquad [\because dt' = dt]$$

Since $\frac{dr'}{dt'} = u'$, velocity of the particle relative to frame S' and $\frac{dr}{dt} = u$, velocity of the particle relative to S frame. Hence eq (3) gives

$$\boxed{u' = u - v} \quad — (4)$$

which shows the velocity measure by the observers in two frame of reference are not the same, so we can say that velocity of a body is not invariant under Galilean transformation.

→ The inverse transformation from S' to S is given by -

$$\boxed{u = u' + v} \quad — (5)$$

Relation (4) and (5) are known as Galilean law of addition of velocities.

→ Eq. (4) differentiating w.r. to t then, $\frac{du'}{dt} = \frac{du}{dt} \Rightarrow \frac{du'}{dt'} = \frac{du}{dt}$ $[\because t' = t]$

$$\boxed{a' = a} \; -(6)$$

→ So from eq (6) it is clear that the acceleration measured by the observer in different inertial frames is same. So we can say acceleration is invariant under Galilean transformation.

Galilean Invariance:- The concept of classical or Newtonian relativity is known as Galilean invariance. According to this the fundamental laws and principles are well invariant under Galilean transformation. So we can say that the basic law of physics are identical in all inertial frame of reference.

– Transformation of force from one inertial system to another

Let force in frame S is, $F = \frac{d(m\vec{u})}{dt} = m\frac{du}{dt} = ma$ —(1)

and similarly force in frame S', $F' = \frac{d(m\vec{u}')}{dt} = m\frac{du'}{dt} = ma'$ —(2)

But according to Galilean transformation of acceleration

$a = a'$, so eq (1) and (2) will equal so $\boxed{F = F'}$

which shows that force and Newton's law of motion invariance under Galilean transformation.

– Conservation of Momentum

In frame S, two particle of mass m_1 and m_2 collide, before the collision their velocities are $\vec{u}_1$ and $\vec{u}_2$ and after collision their velocities $\vec{v}_1$ and $\vec{v}_2$, then according to law of conservation of momentum

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2 \quad (1)$$

If we observe same collision from the frame S' which moving with velocities $\vec{v}$ with respect to S, then according to Galilean transformation of velocity we have -

$$\vec{u}_1' = \vec{u}_1 - \vec{v}, \quad u_2' = \vec{u}_2 - \vec{v}$$
$$v_1' = \vec{v}_1 - \vec{v}, \quad v_2' = v_2 - v \quad (2)$$

where $\vec{u}_1', \vec{u}_2'$ and $\vec{v}_1', \vec{v}_2'$ are ~~~~ respective velocities of the particles, before and after collision in frame S'. Substituting the values from (2) in eq (1)

$$m_1(\vec{u}_1' + \vec{v}) + m_2(\vec{u}_2' + \vec{v}) = m_1(\vec{v}_1' + \vec{v}) + m_2(\vec{v}_2' + \vec{v})$$

on solving $m_1\vec{u}_1' + m_2\vec{u}_2' = m_1\vec{v}_1' + m_2\vec{v}_2'$ —(3)

eq (3) represent the law of conservation of momentum in the moving frame S', so the law of conservation of momentum as observed invariant under Galilean transformation