Theorem: Every connected graph G has atleast one spanning tree

Proof: If G has no cycle, then G itself a spanning tree of G.
If G has cycles, then successively remove one edge from a cycle at a time till the resulting subgraph of G becomes acyclic.

Therefore, H is a spanning tree.
Hence every connected graph G has atleast one spanning tree

Definition :- suppose G be a connected graph then diametre G i.e;

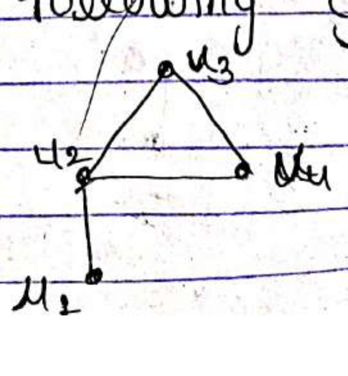
$$\boxed{dia(G) = \max_{u,v \in V(G)} d(u,v)}$$

let $u \in V(G)$, then eccentricity of u i.e;

eccentricity $$\boxed{E(u) = \max_{v \in V(G)} d(u,v)}$$

Radius of G = $Rad(G) = \min_{u \in V(G)} E(u)$

Qus Find the diameter of, radius and eccentrictity of the following graph



$d(u_1, u_2) = 1$
$d(u_1, u_3) = 2$
$d(u_1, u_4) = 2$
$d(u_2, u_3) = 1$
$d(u_2, u_4) = 1$
$d(u_3, u_4) = 1$

v_1, v_n

$$dia(G) = \max_{u,v \in G} d(u,v)$$

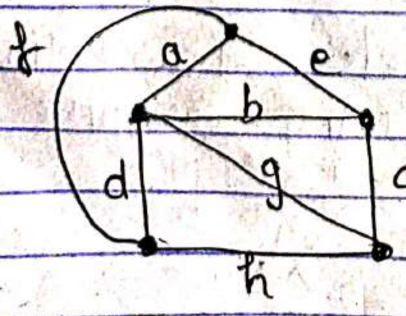
$$\boxed{dia(G) = 2}$$

eccentricity $E(u_1) = 2$
$E(u_2) = 1$
$E(u_3) = 2$
$E(u_4) = 2$

radius of G $\boxed{R(G) = \min E(u)}$
$= 1$

Ques ① find the ~~d of~~ radius ~~of~~ diameter and eccentricity of the petersen graph.

② Find all spanning tree of a graph



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Centre of Graph : a vertex of minimum ecentricity is called centre of graph

for example : 

$E(1) = 2$ $E(2) = 1$ $E(3) = 2$

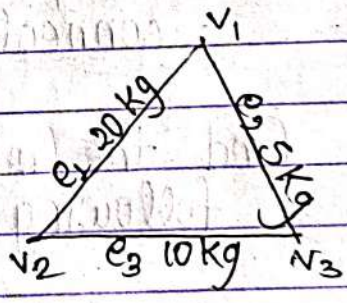
since min. eccentricity is 1 of vertex 2. is the centre of the graph

Theorem : A Tree has either 1 or 2 centree

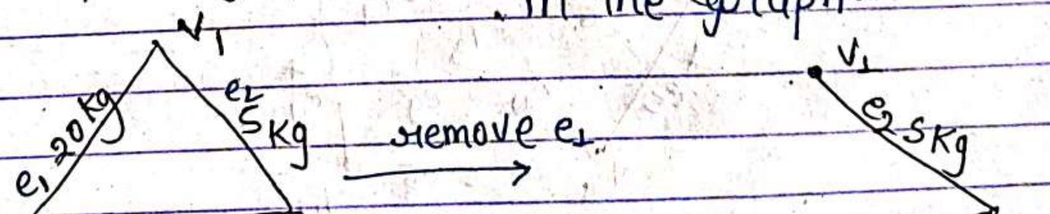
Qus If diametre of Graph $dia(G) \geq 3$, then $dia(\bar{G}) \leq 3$ complemetory grap

Weighted graph :- Let $G = (V, E)$ be a graph. a function $\phi : E \rightarrow R^{+}$ is called a weighted function of G and $G(V, E, \phi)$ is called weighted graph

$\phi(e_1) = 20$ Kg



Min. spanning tree : remove the max. weighted edges in the graph.



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Definition: a spanning tree with minimum total weight is called *minimal spanning tree*.

$$\Rightarrow \boxed{\text{Total weight of } T = \sum_{e_i \in E(T)} \phi(e_i)}$$

Algarthim for finding minimal spanning tree of a connected weighted graph.

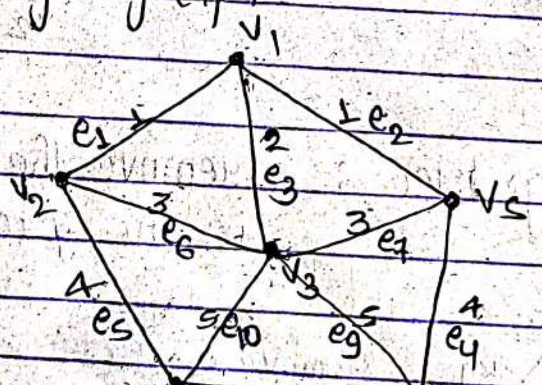
① Krushkel's Algorithm :- let $G=(V,E)$ be a weighted connected graph

Step 1. let $H=(V, E(H))$ where $E(H)=\phi$

Step 2. add $e \in E(V)-E(H)$ such that e has minimum weighted include in $E(H)$ $H=(V,E(H))$ is acyclic.

Step 3. repeat step II till $H=(V,E(H))$ become connected and stop if H has $n-1$ edges

Qus Find the minimal spanning tree of the following graph



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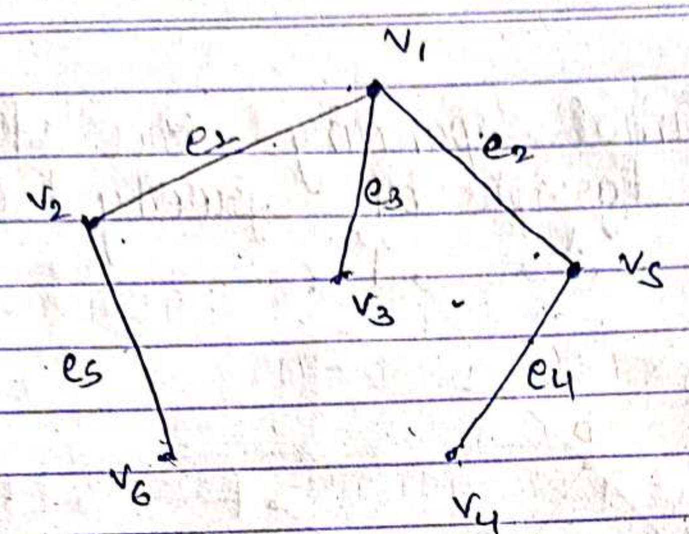

⇒ this is minimal spanning ~~tree~~ tree

total min. weight of spann. tree $= 1+1+2+4+4 = 12$

$\boxed{\text{Max. No. of spanning tree} = 2^{n-1}}$ where n - No of verti

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Greedy Algorithm for finding minimal spanning tree for a connected weighted graph

Proof - let $G = (V, E)$ be a connected weighted graph

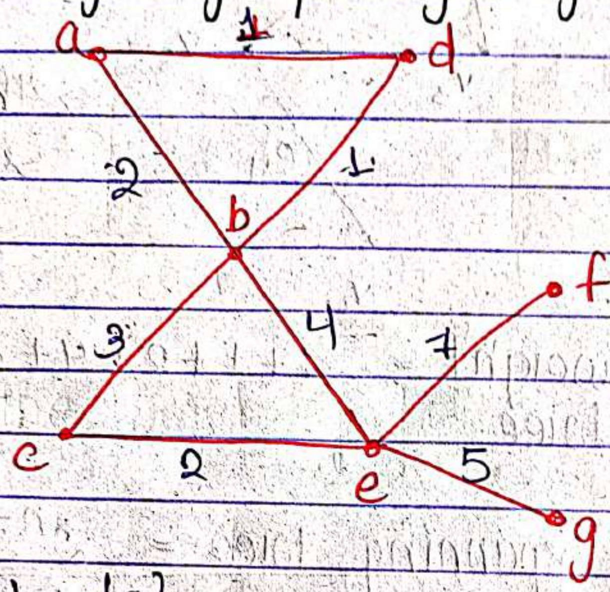
step I. choose a vertex u and let $H = (V(H), E(H))$, where $V(H) = \{u\}$ $E(H) = \phi$

Step II. for a vertex $v \in G$ such that $v \notin H$ and such that v is adjacent to a vertex in H and the edge xv where $x \in H$ such that, the weight of xv is smallest

Now, $H = (V(H) \cup \{v\}, E(H) \cup \{xv\})$

Step III. Continue the step II till we get, $H = (V(H), E(H))$ such that $V(H) = V(G)$ & H is the smallest spanning tree.

Qus Find the minimal spanning tree of the following graph by greedy algorithm



let, $V(H) = \{a\}$

then, $H = (\{a\}, \phi)$

$H = (\{a, d\}, \{ad\})$

$H = (\{a, d, b\}, \{ad, bd\})$

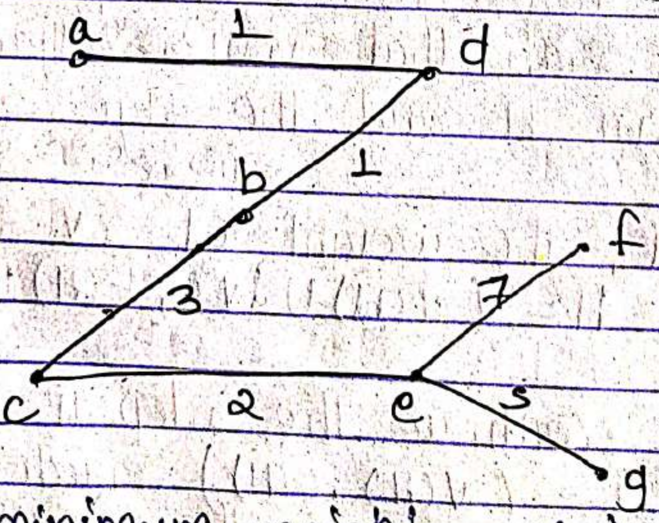
$H = (\{a, d, b, c\} \{ad, bd, bc\})$

$H = (\{a, d, b, c, e\} \{ad, bd, bc, ce\})$

$H = (\{a, d, b, c, e, g\}, \{ad, bd, bc, ce, eg\})$

$H = (\{a, d, b, c, e, g, f\}, \{ad, bd, bc, ce, eg, ef\})$

this is minimal spanning tree



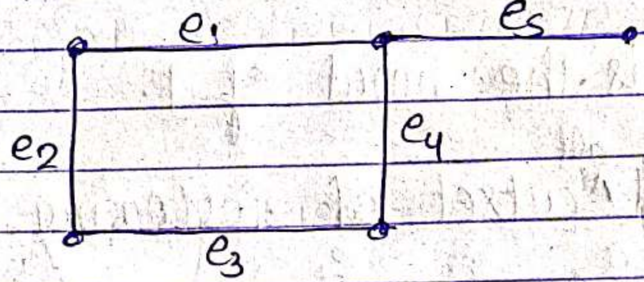
total minimum weight $w(T) = 1+1+3+2+5+7$

$= 19$

Cut Set ⇒ let $G = (V, E)$ be connected graph a subset S of edge in G is called a cutset for G, If

(i) $G - S = (V, E - S)$ is disconneted
(ii) for each proper subset E' of S such that, $G - E'$ is connected.

Example =



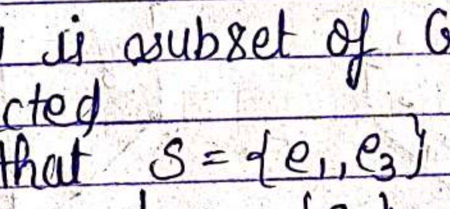
$E = \{e_1, e_2, e_3, e_4, e_5\}$

If let, $S = \{e_1, e_3\}$ is subset of G then graph G is disconnected
then we say that $S = \{e_1, e_3\}$ is the cut set of G,
and $S = \{e_5\}$

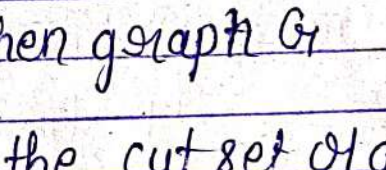
$S = \{e_1, e_2, e_3\}$ is not cut set of G.
∵ take subset of S $\{e_1, e_2\}$ then our graph is disconnected

Branch ⇒ let G be a connected graph and T be a spanning tree of G.
An edge $e \in G$ is called a branch of ~~t~~ T if $e \in T$

for example :-



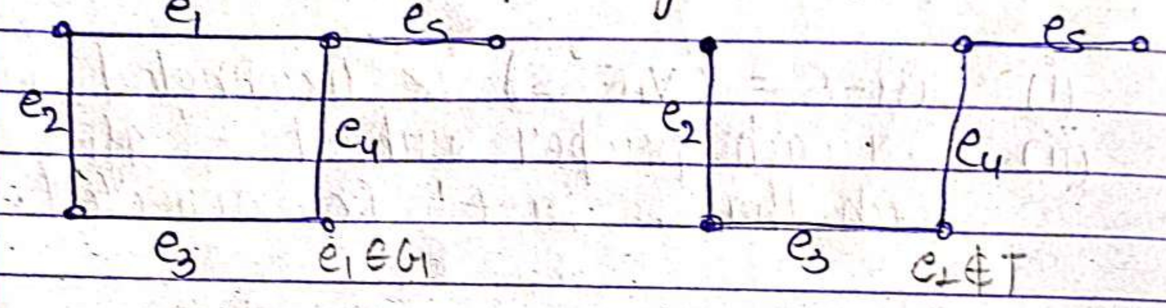
$e_2, e_3, e_4, e_5 \in G$



$e_1 \notin T$
$e_2, e_3, e_4, e_5 \in T$

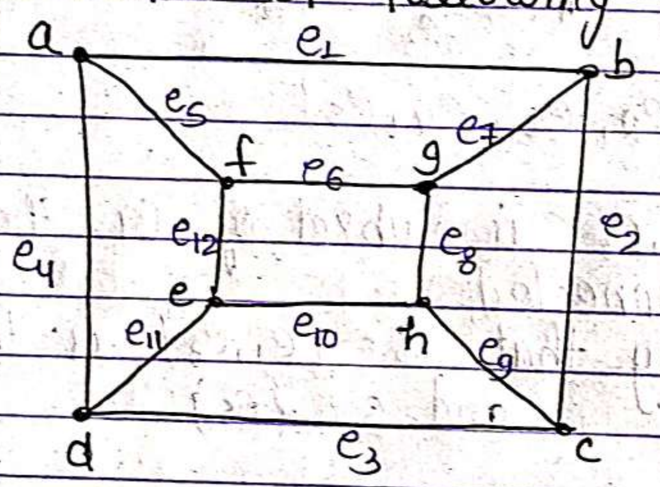
Hence e_2, e_3, e_4, e_5 are branch of T

Chord ⇒ An edge $e \in G$ is called chord of spanning tree T. if $e \notin T$



So, e_1 is the chord of T.

Qus Find all cutset of following graph



$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}\}$

$S_1 = \{e_1, e_5, e_{10}, e_3\}$ $\quad S_2 = \{e_4, 12, e_8, e_2\}$

$S_3 = \{e_1, e_5, e_{11}, e_3\}$ $\quad S_4 = \{e_1, e_7, e_9, e_3\}$

$S_5 = \{e_1, e_5, e_4\}$ $\quad S_6 = \{e_1, e_7, e_2\}$

$S_7 = \{e_4, e_{11}, e_3\}$ $\quad S_8 = \{e_2, e_9, e_3\}$

$S_9 = \{e_5, e_7, e_9, e_{11}\}$