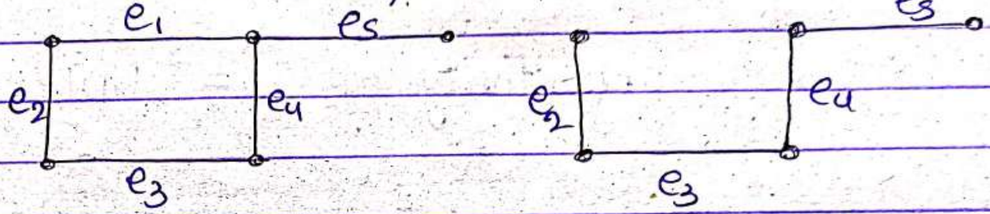
Fundamental cut set ⇒ let G be a connected graph and T be a spanning tree. let e be a branch of T. The cutset S which contain one and only one branch, namely e of T is called a fundametal cutset of G with respect to T.

Example :-



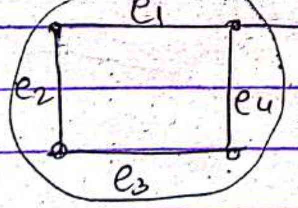
$S_1 = \{e_1, e_2\}$ is the fundament cutset of graph G w.r. to spanning tree T

$S_1 = \{e_5\}$ " " "

S_1 contain only one branch of spanning tree

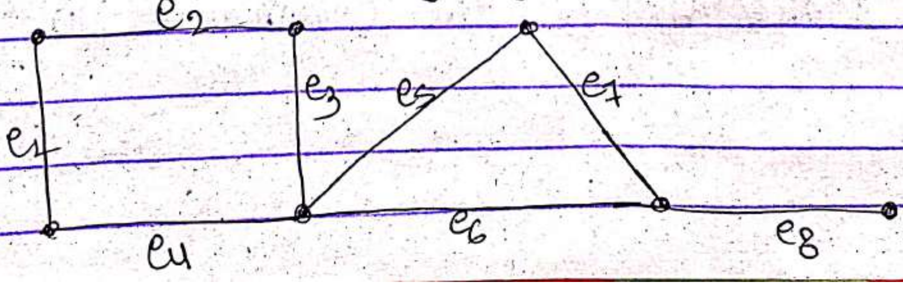
Fundamental Circuit :- A circuit formed by adding a chord to a spanning tree is called a fundamental circuit

$S = \{e_1, e_2, e_3, e_4\}$ is the fundamental circuit of G w.r. to T



By adding e_1 formed a circuit $\{e_1, e_2, e_3, e_4\}$

Qus for the following graph



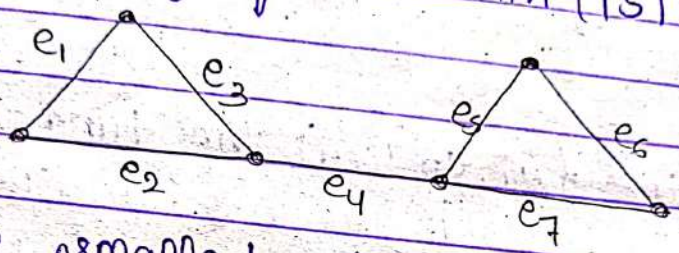
(i) Find all spanning tree of G
(ii) also find all fundamental cutset of G w.r.t T

Edge connectivity :- let G be a graph, then edge connectivity of G = No. of edges in a smallest cutset of G.

or

edge connectivity of G = Min$\{|S| : S$ is the cutset of $G\}$

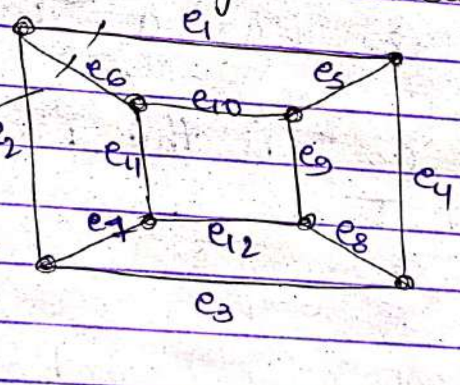
Example :



~~edge~~ smallest cutset of graph is $\{e_4\}$

so, edge connectivity of $G = 1$

Ques find edge connectivity of following graph

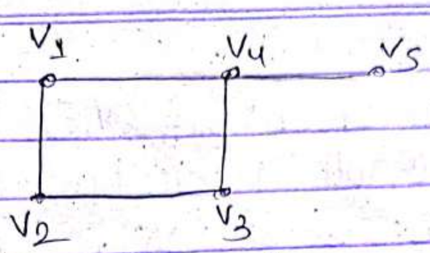


Ans - 3

Vertex connectivity of G :-

vertex connectivity of the graph G = Min$\{|S|$: where $S \subseteq V(G)\}$
$G-S$ is disconnected

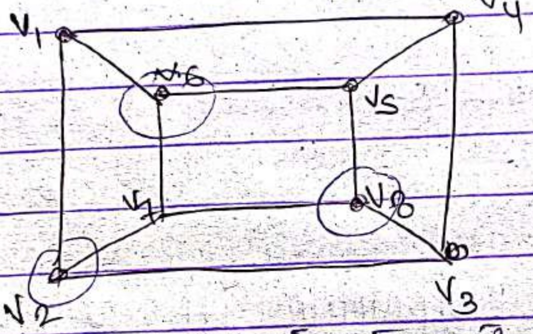
Example :-



$V = \{v_1, v_2, v_3, v_4, v_5\}$

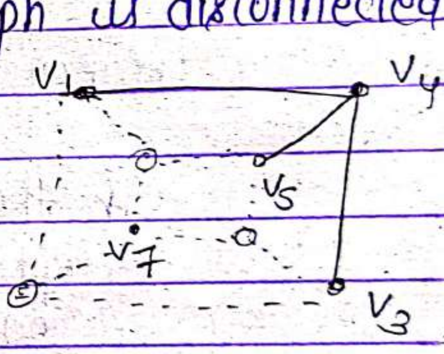
$S = \{v_4\}$

vertex connectivity of $G = \cancel{2}\ 1$



$S = \{v_6, v_8, v_2\}$

when remove v_6, v_8, v_2 then our graph is disconnected



so, vertex connectivity of $G = 3$

Theorem : let G be a connected graph the edge connectivity of $G \le \min \deg_G (v_i)$, $v_i \in V(G)$

Proof : let $v_i \in G$ then if we remove all edges incident on v_i, then G is disconnected (then v_i become is isolated vertex)

$\Rightarrow$ edge connectivity of $G \le \deg_G(v_i)$, for each $v_i \in V(G)$

$\Rightarrow$ edge connectivity of $G \le \min\limits_{v_i \in V(G)}$ of $\deg (v_i)$

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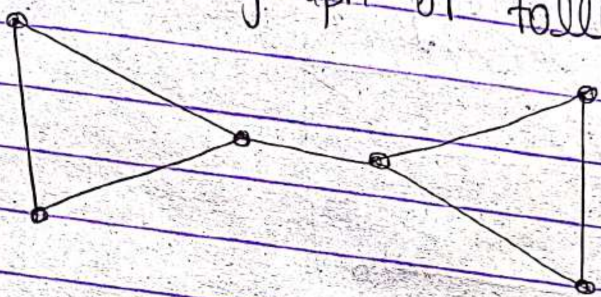
Line graph :- A graph $G = (V, E)$ is called a line graph if vertices of G can be ordered as $v_1, v_2, \ldots v_i, \ldots, v_n$ such that the vertex v_i $(2 \le i \le n-1)$ is adjacent to v_{i-1} and v_{i+1} but v_1 and v_n are non adjacent.

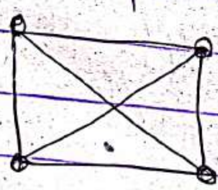
Qus find the linear graph of following graph



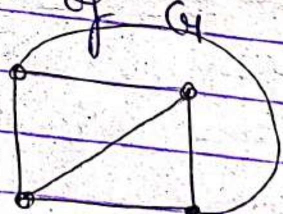
Theorem A Graph G is bipartite iff G has no cycle of odd length

Planer graph :- a graph G is called planer graph if the edges in G can be draw in the plane such that no two edges intersect except at the end vertices. this drawing is called planer drawing of G

Ex: (1)

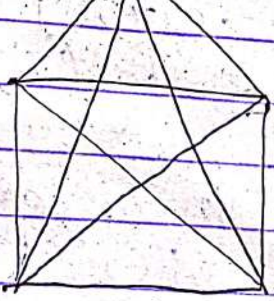


can be drawn



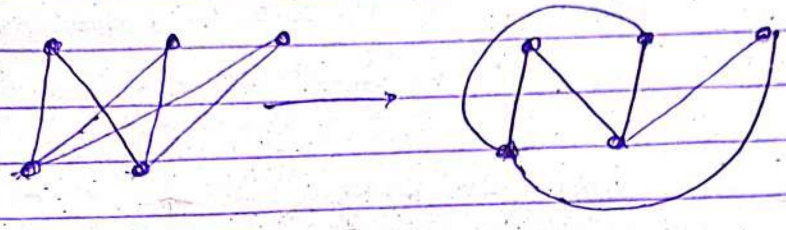
K_4 is planer graph

(2)



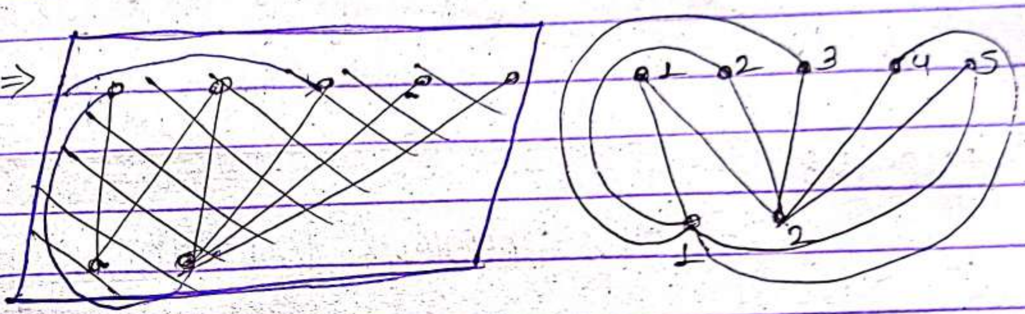
K_5 is not planer graph

(3) $K_{3,2}$ (i.e complete bipartite graph with 3 & 2 vertices



its planer graph

(4) $K_{5,2} \Rightarrow$



Note :- (1) $K(m,n)$ if one of m & n value is 2. i.e $m=2$ or $n=2$ then our G is planer graph.

(2) (K_n) if $n \geq 5$ then graph is not planer.

(3) $K(m,n)$ if $m,n \geq 3$ then our graph is not planer.

Regions :- let G be a planer graph, then a planer drawing of G divides the plane in to various subject called regions. each region has boundry comparising the edges of G.
Thus it is exactly one connected region.
It is denoted by R_i

Degree of a region R_i = No. of edges traversed by while going above the boundry of R_i and coming back to the same point.

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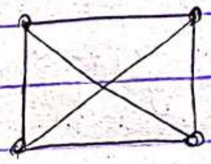
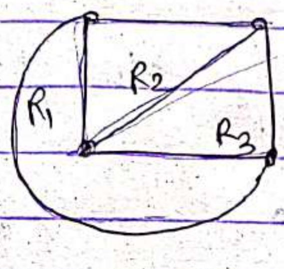
its planer graph

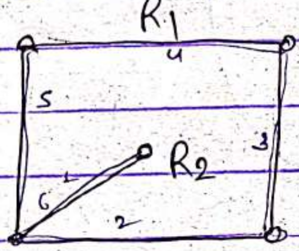
Ex-1 



Interior degree of $R_1 = 3$

" " $R_2 = 3$

" " $R_3 = 3$

2 

Outer degree of $R_1 = 4$

Inter degree of $R_2 = 6$

19/09/23

Euler's theorem for planer graph

Let G be connected planar graph and R denotes the no. of regions in a planar drawing for G

$$\underset{n}{|V|} - \underset{e}{|E|} + R = 2 \quad \text{[Euler's formula]}$$

OR

A connected planer graph with n vertices and e-edges has $\boxed{e-n+2}$ regions

Proof If G is tree then,

$|V| = n$

$|E| = n-1$

$R = 1$

then $n-(n-1)+1 = 2$

suppose, G is not tree

let, T be spanning tree of G

then $|V(T)| - |E(T)| + R = 2$

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let $e \in G - T$

put $G' = T \cup \{e\}$

then, no. of edges in $G' = |E(T)| + 1$

and G' has exactly 1 cycle, Hence

Number of region w.r.to $G' = 2$

further, $|V(G')| = |V(T)| = |V(G)|$

Hence, Number of vertices

$|V(G')| - |E(G')| + R(G')$

$= n - n + 2$

$= 2$

then in each successive addition of the edge $e \in G - G'$, no. of edges & no. of region both increas by 1 and

Number of vertices does not increases

thus eventually leads to G

for G

$$|V| - |E| + R = 2$$

Hence proved