Chapter – 03

Kinematics of fluid

describe the fluid motion and its consequences w/o consideration of the nature of forces causing motion.

This form the ground work for the studies on dynamical behaviour of fluid in consideration of the forces accompanying the motion.

Flow-field

Scalar field | vector field

Fluid motion is described by two methods:
1. Lagrangian method; 2. Eulerian method

<u>Lagrangian method</u>: In this, kinematic behaviour of each and every individual particle is studied constituting the flow.

$$\vec{S} = S(\vec{S_0}, t) \quad \text{--- (1)}$$

time

Position vector at $t=t$ (say) ~~i.e. at present~~.

Initial position at $t=t_0$

Since position vector $\vec{S}$ can be written as

$$\vec{S} = \hat{i}x + \hat{j}y + \hat{k}z$$

where $\hat{i}, \hat{j}$ & $\hat{k}$ are unit vectors and x, y, z are scalar

Now,

$$\left.\begin{aligned} x &= x(x_0, y_0, z_0, t) \\ y &= y(x_0, y_0, z_0, t) \\ z &= z(x_0, y_0, z_0, t) \end{aligned}\right\}$$

where x_0, y_0, z_0 are initial coordinates

Where x, y, z are coordinate at time 't'

Therefore, velocity ($\vec{v}$) and acceleration ($\vec{a}$) of the fluid particle can be obtained from the material derivative of the position vector of the particle wrt time.

Therefore,

$$\vec{V} = \left[\frac{d\vec{S}}{dt}\right]_{S_0} \quad ; \quad \vec{V} = \hat{i}\,u + \hat{j}\,v + \hat{k}\,w$$

or interms of scalar components,

$$u = \left[\frac{dx}{dt}\right]_{x_0, y_0, z_0} \quad ; \quad v = \left[\frac{dy}{dt}\right]_{x_0, y_0, z_0} \quad ; \quad w = \left[\frac{dz}{dt}\right]_{x_0, y_0, z_0}$$

Similarly,

$$\vec{a} = \left[\frac{d\vec{V}}{dt}\right] = \left[\frac{d^2\vec{S}}{dt^2}\right]_{S_0} \quad ; \quad \vec{a} = \hat{i}\,a_x + \hat{j}\,a_y + \hat{k}\,a_z$$

or interms of scalar components,

$$a_x = \left[\frac{d^2x}{dt^2}\right]_{x_0, y_0, z_0}$$

$$a_y = \left[\frac{d^2y}{dt^2}\right]_{x_0, y_0, z_0}$$

$$a_z = \left[\frac{d^2z}{dt^2}\right]_{x_0, y_0, z_0}$$

Eulerian method : In this, velocity and its variation wrt time at each and every location ($\vec{S}$) in the flow field.

Mathematically,

The flow field in this method,

$$\vec{V} = V(\vec{S}, t)$$

where $\vec{V} = \hat{i}\,u + \hat{j}\,v + \hat{k}\,w$

and $\vec{S} = \hat{i}\,x + \hat{j}\,y + \hat{k}\,z$

Therefore,

$$u = u(x, y, z, t)$$
$$v = v(x, y, z, t)$$
$$w = w(x, y, z, t)$$

Relation b/w Lagrangian & Eulerian method

$$\vec{V} = \left[\frac{d\vec{S}}{dt}\right]_{S_0} \qquad \vec{V} = V(\vec{S}, t)$$

$$\therefore \frac{d\vec{S}}{dt} = V(\vec{S}, t)$$

Similarly,

$$\frac{dx}{dt} = u = u(x, y, z, t)$$

$$\frac{dy}{dt} = v = v(x, y, z, t)$$

$$\frac{dz}{dt} = w = w(x, y, z, t)$$

The integration of combined relation yields the constants of integration which are to be found by putting the initial considerations or coordinates.

Material Derivative

The total differentiation $\left(\frac{D}{Dt}\right)$ is known as the material or substantial derivative wrt time.

(Material or Substantial derivative) = (Temporal or local derivative) + (Convective derivative)

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

eg.

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{Dw}{Dt} = \underbrace{\frac{\partial w}{\partial t}}_{A} + \underbrace{u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}}_{B}$$

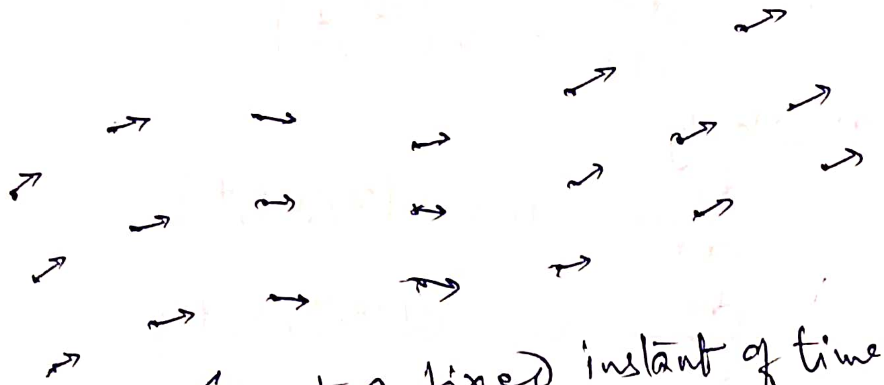
(A) Local or temporal acc^n = 0 for steady flow

(B) Convective acceleration = 0 for uniform flow

Both (A) + (B)
$\underbrace{}_{}$ $l = 0$ for steady & uniform flow.

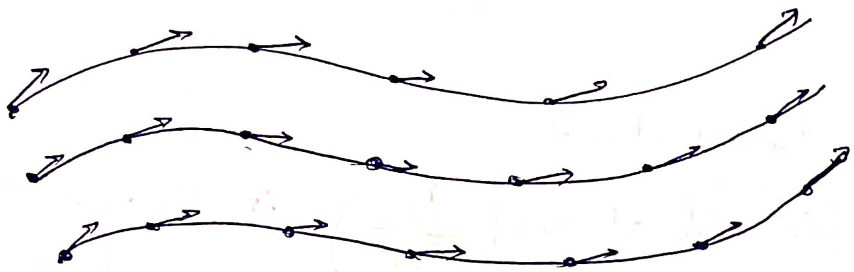
Streamline, Path lines and Streak lines

In a fluid flow, the velocity vectors are represented as



Say at a fixed instant of time 't'

Now, if we draw a space curve in such a way that it is tangent everywhere to the velocity vector, then the curve is called a streamline [Fig(B)].
[Note: Eulerian method gives a series of streamlines].



Fig(B): Streamline

$\vec{V} \times d\vec{s} = 0$ where $\vec{V} = \hat{i}u + \hat{j}v + \hat{k}w$
and $\vec{s} = \hat{i}dx + \hat{j}dy + \hat{k}dz$
then $\dfrac{dx}{u} = \dfrac{dy}{v} = \dfrac{dz}{w}$