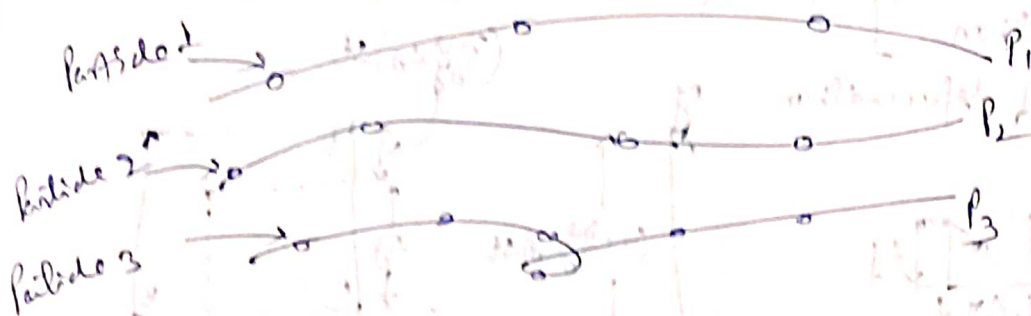


Pathlines: These are outcomes of the Lagrangian method.

A path line is the trajectory of a fluid particle of fixed point identity i.e. $\vec{S}_1 = S(\vec{S}_0, t)$.

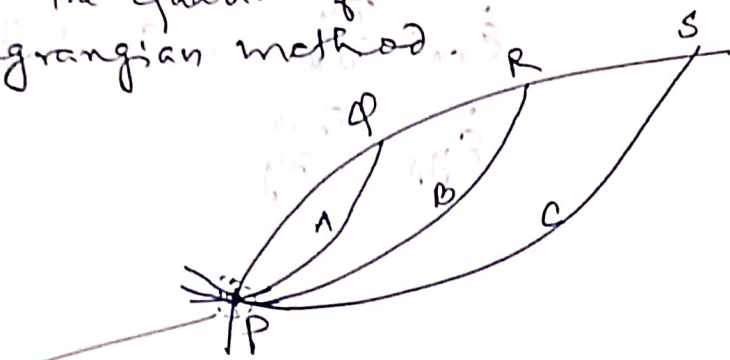
Path lines are drawn over a span of time.



Note: Both pathlines and streamlines are identical to stream in a steady flow as the Eulerian and Lagrangian versions becomes the same.

Streaklines: A streakline at any instant of time is the locus of the temporary locations of all particles that have passed through a fixed point in the flow field.

The equation of streakline can be derived using Lagrangian method.



fixed point in space (P).

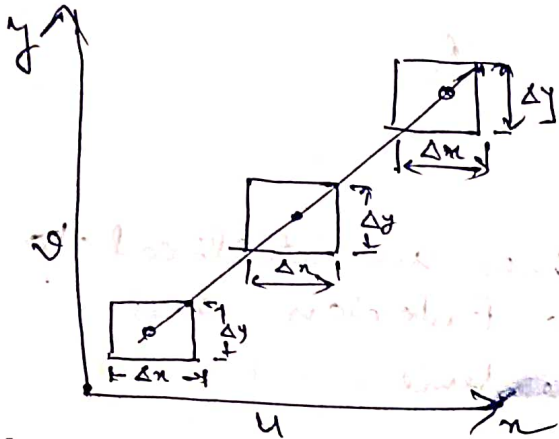
Three different particles arrive at diffⁿ times will traverse diffⁿ paths say PAQ, PBR & PCS.

Say at any time instant 't', - these particles arrive at points Q, R and S. Therefore, Curve joining the points Q, R, S and Point 'P' will be the streakline.

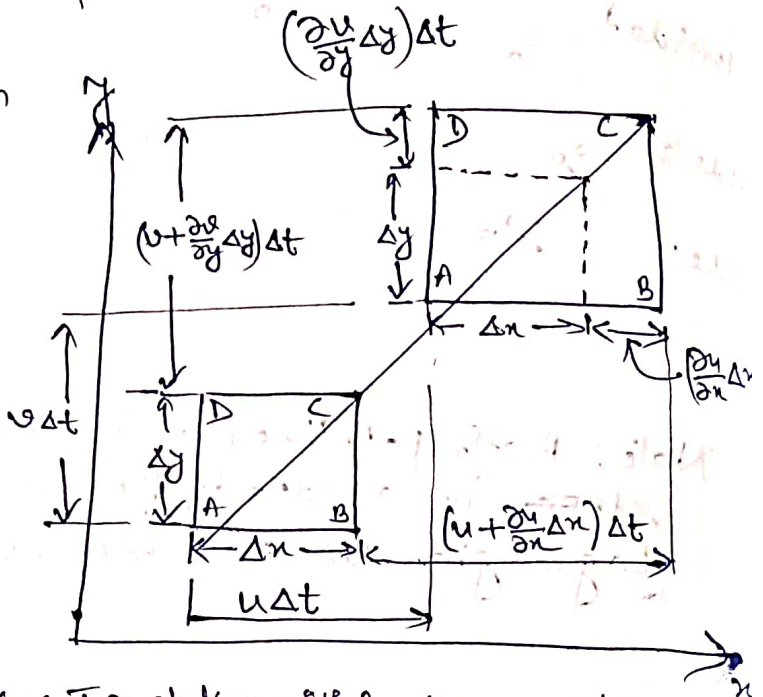
Translation, Rate of Deformation and Rotation

Movement of a fluid element in space are of three types:

- (1) Translation } w/o deformation only rigid body displacement
- (2) Rotation }
- (3) Rate of deformation



Fig(1): Pure translation
 $(u = \text{constant})$
 $(v = \text{constant})$



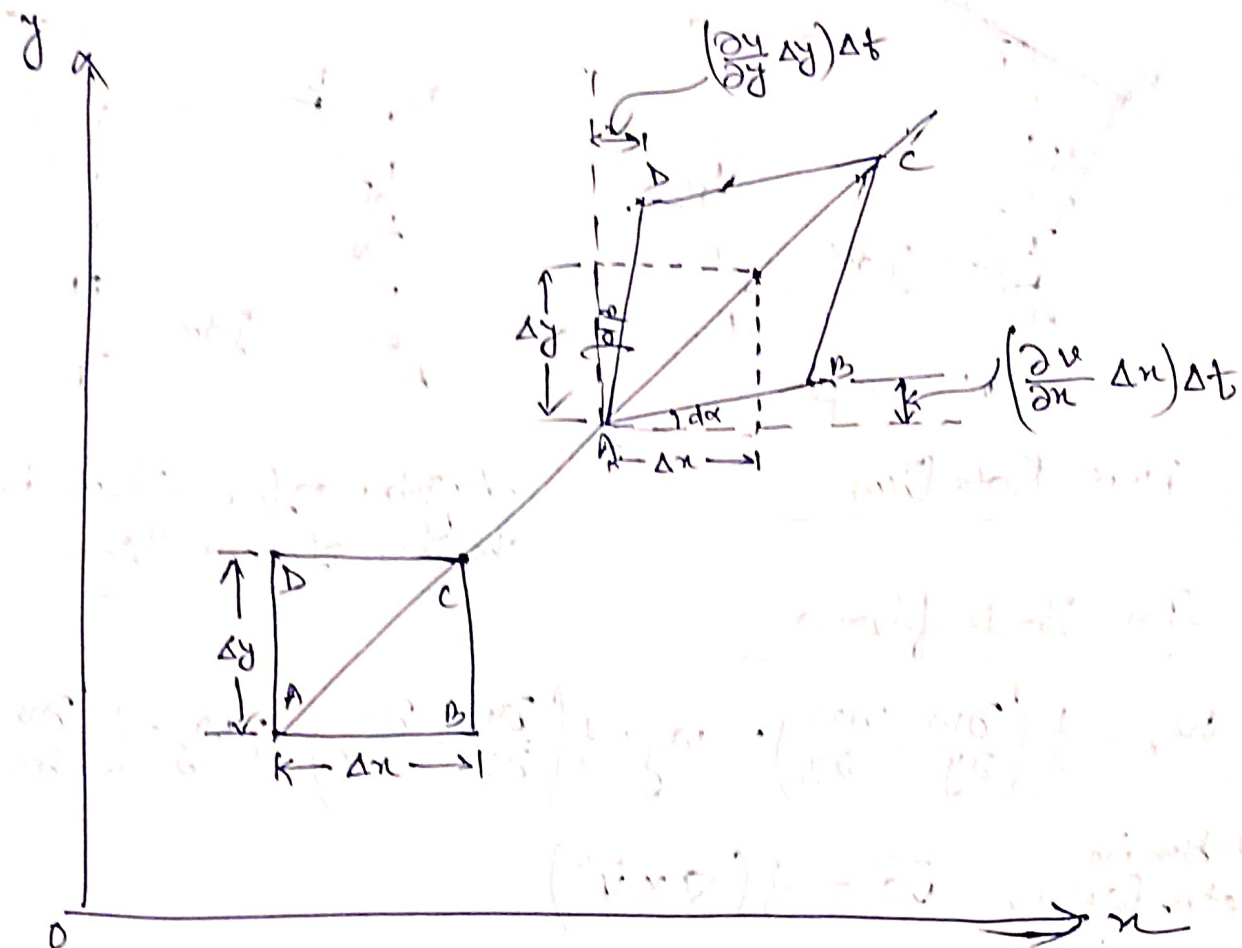
Fig(2): Translation with continuous linear deformation
 $[u = u(x)]$
 $[v = v(y)]$

Linear strain rate component in the x direction

$$\dot{\epsilon}_{xx} = \frac{\partial u}{\partial x}, \text{ and in y direction}$$

$$\dot{\epsilon}_{yy} = \frac{\partial v}{\partial y}$$

Now we will try to derive the expression for the strain rate component in the x direction. Consider a fluid element of length Δx and width Δy. The element is moving with velocity u in the x-direction and v in the y-direction. The strain rate component in the x direction is given by the change in length of the element over time. The change in length is given by the difference in the velocity of the two ends of the element. The velocity of the right end is u + ∂u/∂x Δx and the velocity of the left end is u. The change in length is (u + ∂u/∂x Δx - u) Δt = ∂u/∂x Δx Δt. The strain rate is the change in length divided by the original length Δx, which gives ∂u/∂x.



Fig(3): Translation with simultaneous linear and angular deformation rates

$$\left[\begin{array}{l} u = u(x, y) \\ v = v(x, y) \end{array} \right]$$

The Rate of angular deformation ($\dot{\gamma}_{xy}$)

$$\dot{\gamma}_{xy} = \left(\frac{d\alpha}{dt} + \frac{d\beta}{dt} \right) = \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)$$

Now, the rotation of point A about z axis,

$$\omega_z = \frac{1}{2} \left(\frac{d\alpha}{dt} - \frac{d\beta}{dt} \right)$$

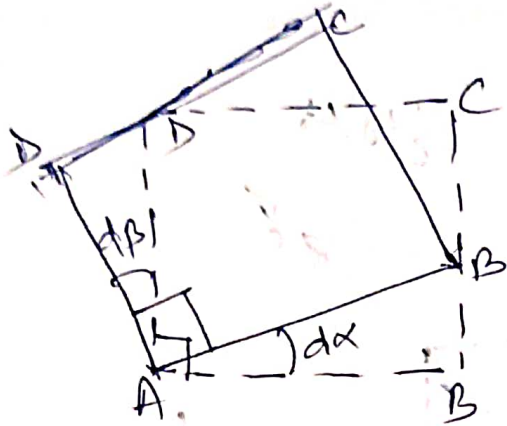
(taking anticlockwise as (+)ve)

$$= \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

Special case: Pure rotation ($\dot{\gamma}_{xy} = 0$) ($\omega_z \neq 0$)

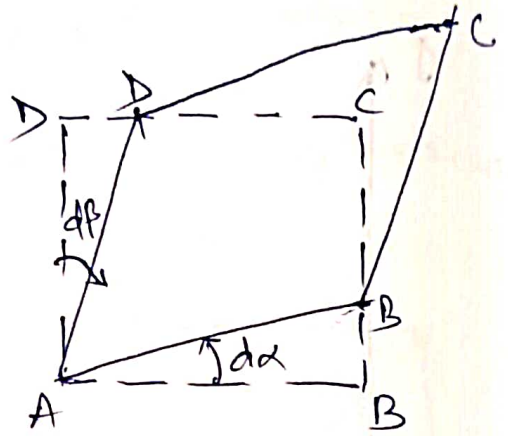
$$\omega_z = \frac{\partial v}{\partial x} = + \frac{\partial u}{\partial y}$$

$$\dot{\gamma}_{xy} = 2 \frac{\partial v}{\partial x} = 2 \frac{\partial u}{\partial y}$$



$$dx = -dy$$

Pure Rotation



Angular deformation in absence of rotation

In 3-D flow

$$\omega_x = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right); \quad \omega_y = \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right); \quad \omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

Rotation in vector form, $\vec{\omega} = \frac{1}{2} (\nabla \times \vec{v})$

for irrotational flow, $\vec{\omega} = \frac{1}{2} (\nabla \times \vec{v}) = 0$

Vorticity ($\vec{\Omega}$)

The vorticity is defined as a vector which is equal to two times (or twice) the rotational vector.

$$\vec{\Omega} = 2\vec{\omega} = \nabla \times \vec{v}$$

Vortex line: If an imaginary line is drawn in the fluid so that tangent to it at each point is in the direction of the vorticity vector $\vec{\Omega}$ at that point, the line is called vortex line.

$$\vec{\Omega} \times d\vec{s} = 0$$

$$\therefore \frac{dx}{\Omega_x} = \frac{dy}{\Omega_y} = \frac{dz}{\Omega_z}$$

[Note: Vorticity is an antisymmetric tensor]