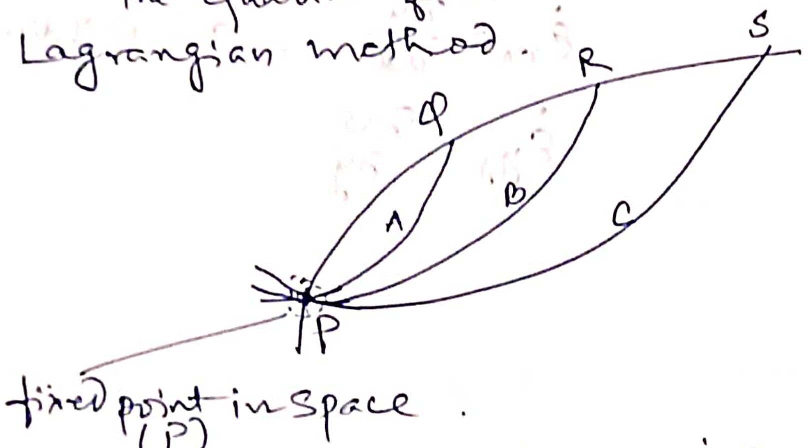
Path lines: These are outcomes of the Lagrangian method.
A path line is the trajectory of a fluid particle of fixed ~~point~~ identity i.e. $\vec{S}_1 = S(\vec{S_0}, t)$
Path lines are drawn over a span of time.

Note: Both pathlines and streamlines are identical ~~to~~ ~~stream~~ in a steady flow as the Eulerian and Lagrangian versions becomes the same.

Streaklines: A streaklines at any instant of time is the locus of the temporary locations of all particles that have passed though a fixed point in the flow field.
The equation of streakline can be derived using Lagrangian method.

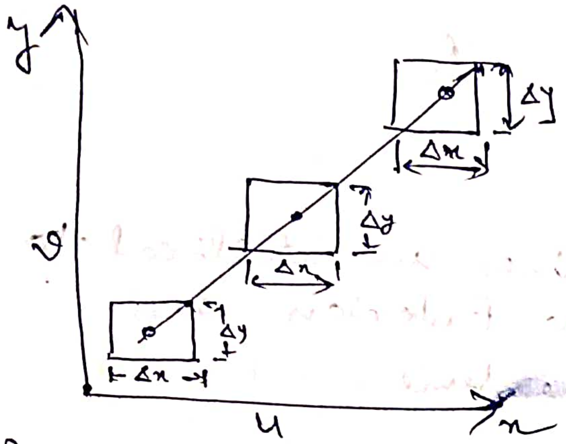


Three different particles arrive at diff^n times will traverse diff^n path says PAQ, PBR & PCS.
Say at any time instant 't', these particle arrives at points Q, R and S. Therefore, curve joining the points Q, R, S and point 'P' will be the streak line.

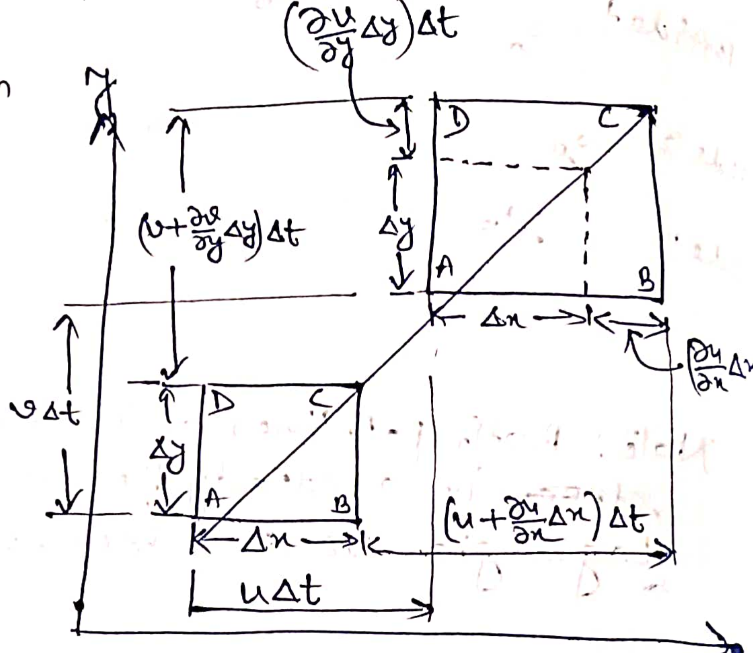
Translation, Rate of Deformation and Rotation

Movement of a fluid element in space are of three types:

(1) Translation } w/o deformation only rigid body displacement
(2) Rotation }
(3) Rate of deformation



fig(1): Pure translation
(u = Constant
v = Constant)



fig(2): Translation with Continuous Linear deformation
$$\begin{bmatrix} u = u(x) \\ v = v(y) \end{bmatrix}$$

Linear strain rate Component in the x direction

$$\dot{\epsilon}_{xx} = \frac{\partial u}{\partial x}$$, and in y direction

$$\dot{\epsilon}_{yy} = \frac{\partial v}{\partial y}$$

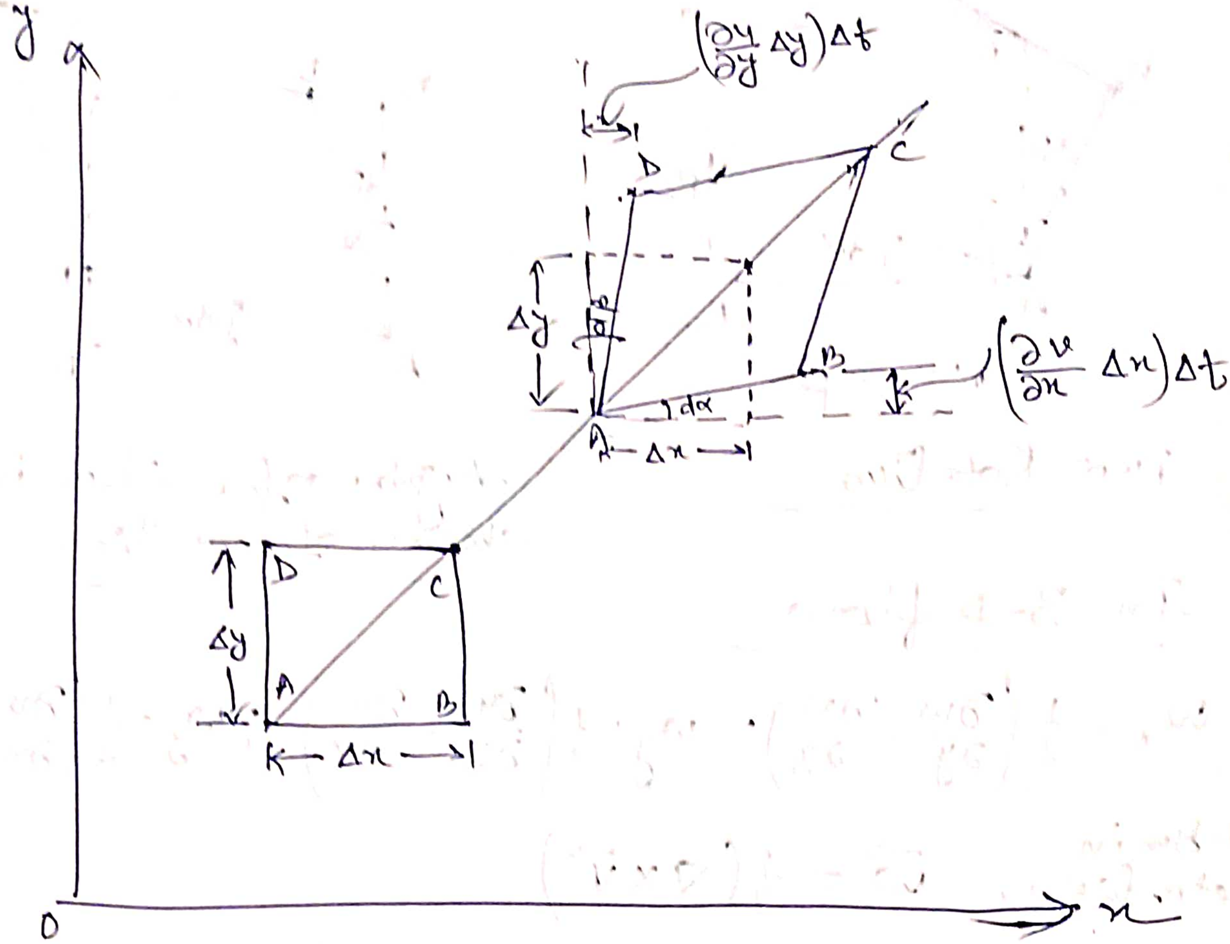


fig (3): Translation with simultaneous linear and angular deformation rates

$$\left[\begin{array}{l} u = u(x,y) \\ v = v(x,y) \end{array}\right]$$

The Rate of angular deformation ($\dot{\gamma}_{xy}$)

$$\dot{\gamma}_{xy} = \left(\frac{d\alpha}{dt} + \frac{d\beta}{dt}\right) = \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

Now, the rotation of point A about z axis,

$$\omega_z = \frac{1}{2}\left(\frac{d\alpha}{dt} - \frac{d\beta}{dt}\right)$$

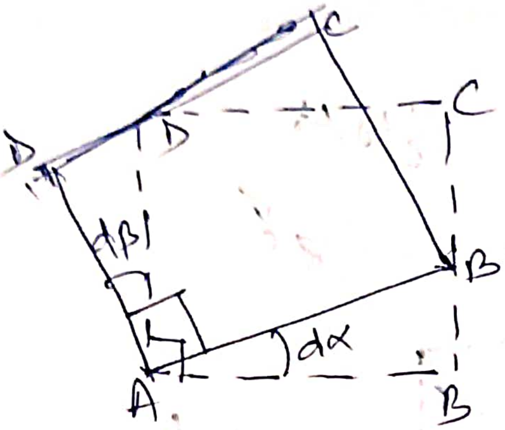
taking anticlockwise as (+)ve !

$$= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Special Case: Pure rotation (~~$\dot{\gamma}_{xy} = 0$~~) ($\omega_z = 0$)

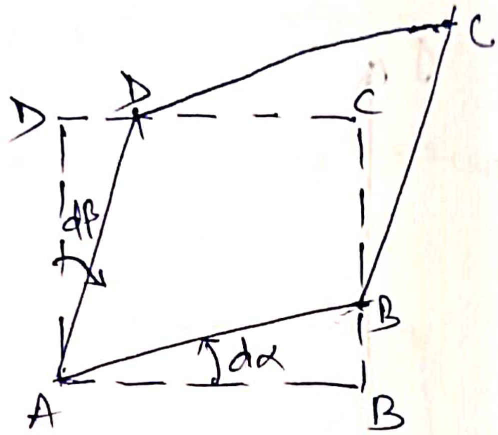
$$\omega_z = \frac{\partial v}{\partial x} = +\frac{\partial u}{\partial y}$$

$$\dot{\gamma}_{xy} = 2\frac{\partial v}{\partial x} = 2\frac{\partial u}{\partial y}$$



$d\alpha = -d\beta$

Pure Rotation



Angular deformation in absence of rotation

In 3-D flow

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right);\quad \omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right);\quad \omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Rotation in vector form, $\vec{\omega} = \frac{1}{2}(\nabla \times \vec{V})$

for irrotational flow, $\vec{\omega} = \frac{1}{2}(\nabla \times \vec{V}) = 0$

Vorticity (Ω)

The vorticity is defined as a vector which is equal to two times (or twice) the rotation vector.

$$\vec{\Omega} = 2\vec{\omega} = \nabla \times \vec{V}$$

Vortex line : If an imaginary line is drawn in the fluid so that tangent to it at each point is in the direction of the vorticity vector $\vec{\Omega}$ at that point, the line is called Vortex line.

$$\vec{\Omega} \times d\vec{s} = 0$$

[Note: Vorticity is an antisymmetric tensor]

$$\therefore \frac{dx}{\Omega_x} = \frac{dy}{\Omega_y} = \frac{dz}{\Omega_z}$$