Flow Through Pipes_Part1

Flow through pipes is an important engineering problem in fluid mechanics. Almost in all our daily operations, we come across pipe flow. For example, the household water supply, sewage flows *etc*. The pipe flow is also used for the transportation of chemicals and petroleum products in different chemical and oil industries.

Here, we will discuss this important type of problem, *i.e.*, the flow of fluids through pipes. We consider the flow of real fluids *i.e.* the fluids possessing viscosity. Thus, flow of all real fluids is termed as viscous flows. For real fluids, the property viscosity is characterized by the shear stresses or the frictional forces between the fluid layers and fluid to solid surface. Now the question is what causes the flow of real fluids? You need to understand that in case of a real fluid flow, the mechanical energy at the upstream section is more than at the downstream section. That means fluid flows by virtue of the energy gradient. So, energy is the potential that causes the flow of real fluids in pipes or any other flowing devices.

Pipe flow can be stated as the closed conduit flow of fluid under certain pressure. A typical pipe flow is shown in Fig. 1 for pipe completely full of flowing fluid. The velocity is the maximum at the center of the pipe and is zero at the solid surface. If we consider the Newton's law of shear stress, then

$$\tau \propto \frac{\partial u}{\partial y} \tag{1}$$

For Newtonian fluids,

$$\tau = \mu \frac{\partial u}{\partial y}, \tag{2}$$

where μ is the coefficient of dynamic viscosity.

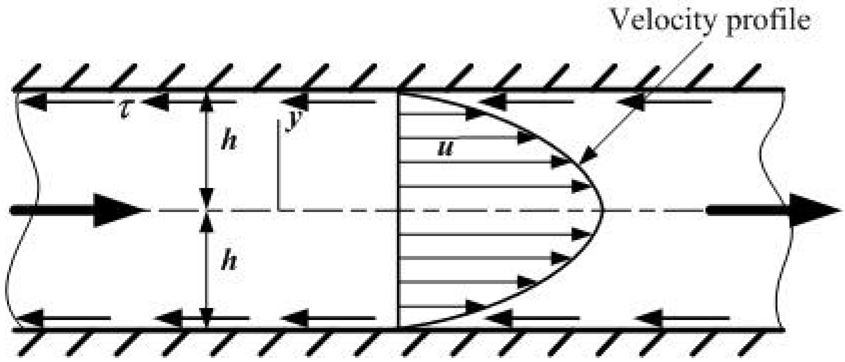

Fig.1 Typical flow through pipe

Types of flow through pipes:

Flow through pipes can be classified as **laminar or turbulent**. The non-dimensional number, Reynolds number, Re is used to determine the type of flow through pipes. The Reynolds number is given by

$$\mathrm{Re} = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho V D}{\mu}, \tag{3}$$

where ρ is the density of the fluid, V is the average velocity of flow, D is the hydraulic diameter and μ is the coefficient of dynamic viscosity.

In 1883 **Osborne Reynolds**, a British engineering professor, conducted some experiments by injecting dye into the middle of the stream of a pipe flow to observe whether the flow is laminar or turbulent. The Reynolds experimental observations are shown schematically in Fig. 2.

We have observed that flow gradually transits from laminar to turbulent depending on the velocity of flow. High velocity causes turbulence and at sufficiently low velocity, the flow is laminar. In laminar flow, the stream lines moves parallel to each other. The fluid particles move in planes which are gliding over each other. When the velocity of flow exceeds some threshold value for a given fluid in a pipe, the flow becomes turbulent. Turbulent flow is the flow, where the flow becomes irregularly fluctuating with time; flow becomes unsteady.

Reynolds number shows the nature of flow in a pipe.

Reynolds Number	**Condition of flow**
Re < 2000	Laminar
2000 < Re < 4000	Transitional
Re > 4000	Turbulent

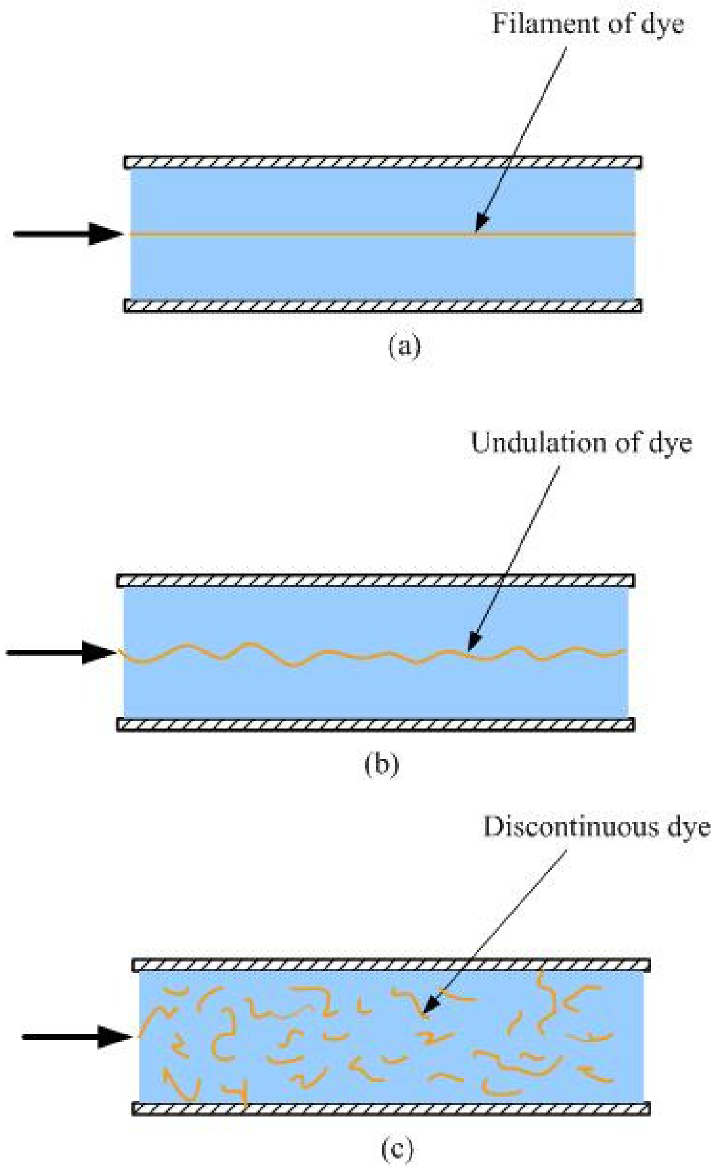

Fig. 2 Reynolds' schematic sketches of pipe flow transition: (a) Laminar viscous flow at extremely small velocity, (b) Transitional flow as the velocity increases and (c) Turbulent flow at high velocity

Laminar Flow in pipe:

Let us consider a fully developed laminar flow through a long, straight and constant diameter horizontal pipe. Consider that *D* is the diameter and *R* is the radius of the pipe. Taking a fluid element of length *L* and radius *r*, the free body diagram is shown in Fig. 3.

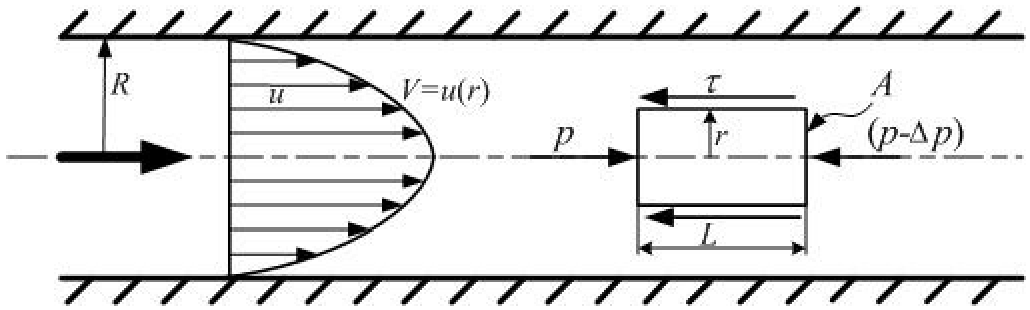

Fig. 3 Laminar flow through pipe

The force balance of the fluid element provides

$$pA-\left(p-\Delta p\right)A=\tau SL, \tag{4}$$

where τ is the shear stress, p is the pressure, Δp is the pressure drop, A is the cross-sectional area of the element and S is the perimeter.

From Eq. (4),

$$p\pi r^2-\left(p-\Delta p\right)\pi r^2=\tau\left(2\pi r\right)L,$$

$$\Delta pr=2\tau L,$$

$$\Delta p=\frac{2\tau L}{r}. \tag{5}$$

Eq. (5) provides the pressure drop in the fluid element. Shear stress distribution throughout the pipe is a linear function of the radial coordinate. Thus,

$$\tau=\frac{2\tau_w r}{D}, \tag{6}$$

where τ_w is the wall shear stress. Substituting Eq. (6) in Eq. (5) we obtain the pressure drop in the pipe as

$$\Delta p=\frac{4\tau_w L}{D}. \tag{7}$$

The pressure drop is directly proportional to the shear stress. Physically, it is because of the shear stress there is a pressure drop in the pipe. A small shear stress can produce a large pressure difference if the pipe is relatively long, *i.e.*, L/D>>1.

Shear stress can be given by (for pipe)

$$\tau=-\mu\frac{\mathrm{d}u}{\mathrm{d}r},$$

$$\frac{\mathrm{d}u}{\mathrm{d}r} = -\frac{\tau}{\mu}. \tag{8}$$

Substituting τ from Eq. (5) in Eq. (8), we obtain

$$\frac{\mathrm{d}u}{\mathrm{d}r} = -\left(\frac{\Delta p}{2L\mu}\right)r. \tag{9}$$

Integration of Eq. (9) provides

$$\int \mathrm{d}u = -\left(\frac{\Delta p}{2L\mu}\right)\int r\mathrm{d}r,$$

$$u = -\left(\frac{\Delta p}{2L\mu}\right)\frac{r^2}{2} + C, \tag{10}$$

where C is an integration constant. Using the boundary condition: at r=D/2, u=0 from Eq. (10) we obtain the value of C as

$$C = \frac{\Delta p}{16\mu L}D^2. \tag{11}$$

Using Eq. (11) in Eq. (10), the velocity distribution as a function of radial coordinate is obtained as

$$u(r) = \left(\frac{\Delta p D^2}{16\mu L}\right)\left\{1 - \left(\frac{2r}{D}\right)^2\right\}. \tag{12}$$

The velocity is maximum at the centre *i.e*. at r=0. Thus, Eq. (12) provides

$$V_c = V_{\max} = \left(\frac{\Delta p D^2}{16\mu L}\right), \tag{13}$$

where V_c is the centerline velocity (maximum velicity). Thus, Eq. (12) can be written as

$$u(r) = V_c\left\{1 - \left(\frac{2r}{D}\right)^2\right\}. \tag{14}$$

Using Eq. (7), Eq. (12) can also be written as

$$u(r) = \frac{\tau_w D}{4\mu}\left\{1 - \left(\frac{r}{R}\right)^2\right\}. \tag{15}$$

Discharge through the pipeline (Q):

$$Q = \int u\mathrm{d}A,$$

$$Q = \int_{r=0}^{r=R} u(r) 2\pi r \mathrm{d}r,$$

$$= 2\pi V_c \int_0^R \left\{ 1 - \left(\frac{r}{R} \right)^2 \right\} r \mathrm{d}r,$$

$$= \frac{\pi R^2 V_c}{2}. \tag{16}$$

Substituting V_c in Eq. (16), we obtain

$$Q = \frac{\pi D^4 \Delta p}{128 \mu L}. \tag{17}$$

Average velocity of flow:

The average velocity of flow, V is given by

$$V = \frac{Q}{A}. \tag{18}$$

Substituting Q from Eq. (17) in Eq. (18) and cross-sectional area $A=\pi D^2$, we get

$$V = \frac{\Delta p D^2}{32 \mu L}. \tag{19}$$

Loss of pressure head (h_f):

$$h_f = \frac{\Delta p}{\rho g}. \tag{20}$$

Using Eq. (17) or Eq. (19) for Δp in Eq. (20), one obtains

$$h_f = \frac{32 \mu V L}{\rho g D^2} = \frac{128 \mu L Q}{\pi \rho g D^4}. \tag{21}$$

Eq. (21) is called the **Hagen Poiseuille** equation.