FLOW THROUGH PIPES – NUMERICAL EXAMPLES

Problem: Compute the head loss due to pipe friction and the power required to maintain flow in a circular pipe of 40 *mm* diameter and 750 *m* laid horizontal when water flows at a rate: (a) 4 litres per minute; (b) 30 litres per minute. Take dynamic viscosity of water equal to 1.14×10^{-3} $N\,s\,m^{-2}$. Assume that for the pipe absolute roughness, *k* is 0.00008 *m*.

Solution.

(a) Diameter of pipe, $D = 40\ mm = 40 \times 10^{-3}\ m$
Length of pipe, L = 750 *m*
Orientation of pipe: horizontal
Rate of flow, $Q = 4\ lpm = 4 \times 10^{-3}\ m^3 / 60\ s = 66.7 \times 10^{-6}\ m^3 s^{-1}$
Cross-sectional area of pipe, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(40x10^{-3})^2 = 1.26 \times 10^{-3}\ m^2$

Average velocity of flow in the pipe, $V = Q/A = \frac{66.7x10^{-6}}{1.26x10^{-3}} = 52.9 \times 10^{-3}\ m\ s^{-1}$

In order to determine the type of flow occurring in the pipe, let us determine the Reynolds number of flow.

$$Re = \frac{\rho VD}{\mu} = \frac{(1000)(52.9x10^{-3})(40x10^{-3})}{1.14x10^{-3}} = 1856$$

As Reynolds number of flow is less than 2000, the type of flow occurring in the pipe is laminar. Hence, the loss of head due to pipe friction can be computed by using either Hagen-Poiseuille's equation or the Darcy-Weisbach's equation.

Hagen-Poiseuille's equation:

$$\Delta p = \frac{128\mu LQ}{\pi D^4} = \frac{128x(1.14x10^{-3})x750x(66.7x10^{-6})}{\pi x(40x10^{-3})^4} = 907.6\ N\ m^{-2}$$

Hence, the head lost due to friction in pipe is given by

$$h_f = \frac{\Delta p}{\gamma} = \frac{907.6}{9810} = 92.4 \times 10^{-3}\ m \text{ of water}$$

Darcy-Weisbach's equation:

$$h_f = \frac{4fL}{D}\frac{V^2}{2g}$$

$f = \frac{16}{Re} = \frac{16}{1856} = 0.00862$

Hence, $h_f = \frac{4x0.00862x750}{40x10^{-3}} x \frac{(52.9x10^{-3})^2}{(2x9.81)}$ = 92.4 x 10^{-3} *m* of water.

Power required to maintain flow, $P = \gamma Q h_f$

= 9810 x (66.7 x 10^{-6}) x (92.4 x 10^{-3})

= 0.0605 *Nm/s* (or) 0.0605 *Watts*

(b) Rate of flow, Q = 30 *lpm* = 30 x 10^{-3} m^3 / 60 *s* = 0.5 x 10^{-3} $m^3 s^{-1}$

Average velocity of flow in the pipe, $V = Q/A = \frac{0.5x10^{-3}}{1.26x10^{-3}} = 0.4\ m\ s^{-1}$

In order to determine the type of flow occurring in the pipe, let us determine the Reynolds number of flow.

$$Re = \frac{\rho VD}{\mu} = \frac{(1000)x0.4x(40x10^{-3})}{1.14x10^{-3}} = 14035$$

As Reynolds number of flow is more than 4000, the type of flow occurring in the pipe is turbulent. Hence, the loss of head due to pipe friction can be computed by using he Darcy-Weisbach's equation.

To determine the value of friction factor:

Relative roughness = $\frac{k}{D} = \frac{0.00008}{40x10^{-3}} = 0.002$

From the Moody's chart, for Reynolds number, *Re*, equal to 14035 and relative roughness equal to 0.002, friction factor, $f = 0.008$

Hence, $h_f = \frac{4fL}{D}\frac{V^2}{2g} = \frac{4x0.008x750}{40x10^{-3}} x \frac{(0.4)^2}{(2x9.81)}$ = 4.89 *m* of water

Power required o maintain flow, $P = \gamma Q h_f$

= 9810 x (0.5 x 10^{-3}) x (4.89)

= 24 $N\ m\ s^{-1}$ (or) 24 *Watts*

Problem: Oil of specific gravity 0.9 and kinematic viscosity 0.00033 $m^2 s^{-1}$ is pumped over a distance of 1.5 *km* through a 75 *mm* diameter tube at a rate of 25 x 10^3 $kg\ h^{-1}$. Determine whether the flow is laminar and calculate the pumping power required, assuming 70 per cent mechanical efficiency.

Solution:

Specific gravity of oil, $S = 0.9$
Mass density of oil, ρ = specific gravity of oil x mass density of water
$= S\,\rho_w = 0.9 \times 1000 = 900\ kg\ m^{-3}$
Specific weight of oil, $\gamma = S\,\gamma_w = 0.9 \times 9810 = 8829\ N\ m^{-3}$
Kinematic viscosity of oil, $\nu = 0.00033\ m^2\ s^{-1}$
Length of pipe, $L = 1.5\ km = 1500\ m$
Diameter of pipe, $D = 75\ mm = 75 \times 10^{-3} = 0.075\ m$
Cross-sectional area of pipe, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(75x10^{-3})^2 = 4.419643 \times 10^{-3}\ m^2$
Mass rate of pumping, $\rho Q = 25 \times 10^3\ kg\ h^{-1}$
Rate of flow, Q = mass rate of pumping / mass density of oil

$$= \frac{\rho Q}{\rho} = \frac{25x10^3}{900} = 27.778\ m^3\ h^{-1} = 27.778 / (60 \times 60)$$

$$= 0.007716\ m^3\ s^{-1}$$

Average velocity of flow in the pipe, $V = Q/A = \frac{0.007716}{4.419643 \times 10^{-3}} = 1.746\ m\ s^{-1}$
Dynamic viscosity of oil, $\mu = \nu\,\rho = 0.00033 \times 900 = 0.297\ N\ s\ m^{-2}$

To determine whether the flow is laminar, let us compute the Reynolds number of flow.

$$Re = \frac{\rho VD}{\mu} = \frac{900x1.746x0.075}{0.297} = 396.8$$

As Reynolds number of flow is less than 2000, the type of flow occurring in the pipe is laminar. Hence, the loss of head due to pipe friction can be computed by using either Hagen-Poiseuille's equation or the Darcy-Weisbach's equation.

Hagen-Poiseuille's equation:

$$\Delta p = \frac{128\mu LQ}{\pi D^4} = \frac{128x0.297x1500x0.007716}{\pi x(0.075)^4} = 4424663\ N\ m^{-2}$$

Hence, the head lost due to friction in pipe is given by

$$h_f = \frac{\Delta p}{\gamma} = \frac{4424663}{8829} = 501.1511\ m \text{ of water}$$

Power required to maintain flow, $P = \frac{\gamma Q h_f}{\eta}$

where η = mechanical efficiency = 70 % = 0.7

Hence, $P = \frac{\gamma Q h_f}{\eta} = \frac{8829 x 0.007716 \text{ x } 501.1511}{0.70}$ = 48772.43 W = 48.772 kW

Problem: Water at a density of 998 $kg\ m^{-3}$ and kinematic viscosity of 1 x 10^{-6} $m^2\ s^{-1}$ flows through smooth tubing at a mean velocity of 2 $m\ s^{-1}$. If the tube diameter is 30 mm, calculate the pressure gradient per unit length necessary. Assume that the friction factor for a smooth pipe is given by 16/Re for laminar flow and 0.079/$Re^{1/4}$ for turbulent flow.

Solution.

Density of water, ρ = 998 $kg\ m^{-3}$
Kinematic viscosity of water, ν = 1 x 10^{-6} $m^2\ s^{-1}$
Dynamic viscosity of water, $\mu = \nu\,\rho$ = 1 x 10^{-6} x 998 = 0.000998 $N\ s\ m^{-2}$
Mean velocity of flow, V = 2 $m\ s^{-1}$
Diameter of tube, D = 30 mm = 30 x 10^{-3} = 0.030 m
Cross-sectional area of pipe, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(30 x 10^{-3})^2$ = 0.707143 x 10^{-3} m^2
Pressure gradient per unit length, $\frac{\Delta p}{L} = ?$
Rate of flow, $Q = A\ V$ = (0.707143 x 10^{-3}) x 2 = 1.414286 x 10^{-3} $m^3\ s^{-1}$

To determine whether the flow is laminar or turbulent, let us compute the Reynolds number of flow.

$$Re = \frac{\rho V D}{\mu} = \frac{998 x 2 x 0.030}{0.000998} = 60000$$

As Reynolds number of flow is more than 2000, the type of flow occurring in the pipe is turbulent. Hence, the loss of head due to pipe friction can be computed by using the Darcy-Weisbach's equation.

Darcy-Weisbach's equation: $h_f = \frac{4 f L}{D} \frac{V^2}{2g}$

where f = friction factor = $\frac{0.079}{Re^{1/4}} = \frac{0.079}{60000^{1/4}} = 0.005$

$$h_f = \frac{4x0.005xL}{0.030} x \frac{2^2}{2x9.81}$$

$$\Rightarrow \frac{h_f}{L} = \frac{4x0.002}{0.030} x \frac{2^2}{2x9.81} = 0.136\ m\ m^{-1}$$

$$\Rightarrow \frac{\left(\frac{\Delta p}{\gamma}\right)}{L} = 0.136\ m\ m^{-1}$$

$$\Rightarrow \frac{\Delta p}{L} = 0.136\gamma = 0.136x9810 = 1333.333\ N\ m^{-2}\ m^{-1} = 1.333\ kN\ m^{-2}\ m^{-1}$$

Problem: Water flows through a pipe 25 *mm* in diameter at a velocity of 6 $m\ s^{-1}$. Determine whether the flow is laminar or turbulent. Assume that the dynamic viscosity of water is 1.30 x 10^{-3} $kg\ m^{-1}\ s^{-1}$ and its density is 1000 $kg\ m^{-3}$. If oil of specific gravity 0.9 and dynamic viscosity 9.6 x 10^{-2} $kg\ m^{-1}\ s^{-1}$ is pumped through the same pipe, what type of flow will occur?

Data given:

Diameter of pipe, D = 25 *mm* = 0.025 *m*
Velocity of flow of water in pipe, V = 6 $m\ s^{-1}$
Dynamic viscosity of water, μ = 1.30 x 10^{-3} $kg\ m^{-1}\ s^{-1}$
Density of water, ρ = 1000 $kg\ m^{-3}$
Specific gravity of oil, S_{oil} = 0.9
Dynamic viscosity of oil, μ_{oil} = 9.6 x 10^{-2} $kg\ m^{-1}\ s^{-1}$

Required:

Type of flow when water is flowing through the pipe
Type of flow when oil is flowing through the pipe

Solution.

Case (i): When water is flowing through the pipe

Cross-sectional area of pipe, $A = \frac{\pi}{4}D^2 = \frac{\pi}{4} \text{ x } (0.025)^2 = 0.000491\ m^2$

Reynolds number of flow, $Re = \frac{\rho VD}{\mu} = \frac{1000 \text{ kg m}^{-3} \text{ x } 6 \text{ m s}^{-1} \text{ x } 0.025 \text{ m}}{1.30 \text{ x } 10^{-3} \text{ kg m}^{-1}\text{s}^{-1}}$

$= 115385$

As the Reynolds umber of flow is greater than 4000, the flow is categorized as turbulent.

Case (ii) When oil is flowing through the pipe

Mass density of oil, $\rho_{oil} = S_{oil} \times \rho_{water}$ = 0.9 x 1000 $kg\ m^{-3}$ = 900 $kg\ m^{-3}$

Reynolds number of flow, $Re = \dfrac{\rho_{oil} VD}{\mu_{oil}} = \dfrac{900\ \text{kg m}^{-3} \times 6\ \text{m s}^{-1} \times 0.025\ \text{m}}{9.60 \times 10^{-2}\ \text{kg m}^{-1}\text{s}^{-1}}$

= 1406

As the Reynolds number of flow is less than 2000, the flow is categorized as laminar.

Problem: In a 0.6 m diameter duct carrying air the velocity profile was found to follow the law $u = -5r^2 + 0.45$ m s^{-1}, where u is the velocity at radius r. Determine the volume flow rate of the air and the mean velocity of flow of air.

Data given:

Diameter of duct, D = 0.6 m
Velocity profile, $u = -5r^2 + 0.45$ m s^{-1}
Velocity of flow at radius r = u

Required:

Volume rate of flow of air, Q = ?
Mean velocity of flow of air, $\bar{u}$ = ?

Solution.

Volume rate of flow of air, $Q = 2\pi \int_{r=0}^{r=R} ur.dr$

$$= 2\pi \int_{r=0}^{r=R} \left(-5r^2 + 0.45\right) r.dr$$

$$= 2\pi \int_{r=0}^{r=R} \left(-5r^3 + 0.45r\right) dr$$

$$= 2\pi \int_{r=0}^{r=R} -5r^3 dr + 2\pi \int_{r=0}^{r=R} 0.45 r dr$$

$$= -10\pi \left[\frac{r^4}{4}\right]_{r=0}^{r=0.3} + 0.90\pi \left[\frac{r^2}{2}\right]_{r=0}^{r=0.3}$$

$$= -10\pi\left[\frac{0.3^4}{4} - 0\right] + 0.90\pi\left[\frac{0.3^2}{2} - 0\right]$$

$$= 0.0636 \text{ m}^3 \text{ s}^{-1}$$

Mean velocity, $\bar{u} = \frac{Q}{A}$

Cross-sectional area of duct, A = $\frac{\pi}{4}D^2 = \frac{\pi}{4} \text{ x } (0.6)^2 = 0.282857 \text{ m}^2$

Hence, $\bar{u} = \frac{0.0636 \text{ m}^3\text{s}^{-1}}{0.282857 \text{ m}^2} = 0.225 \text{ m s}^{-1}$

Problem: A fluid flows through a circular duct of diameter 2 *m* such that the fluid velocity is zero at the duct surface and 6 $m\ s^{-1}$ on the axis of the duct. The flow rate is 9 $m^3\ s^{-1}$. Assuming the velocity distribution is given by $v = c_1 - c_2 r^n$, where v is the fluid velocity at any radius *r*. Determine the constants c_1, c_2 and *n*. ***Specify the units of c_1 and c_2***. Evaluate the mean velocity of fluid and determine the radial position at which a Pitot tube must be placed to measure this mean velocity.

Data given:

Diameter of circular duct, $D = 2\ m$
Fluid velocity at the surface of duct (i.e., at the boundary of the duct, $r = R$) = 0
Fluid velocity on the axis of the duct (i.e., at $r = 0$) = 6 $m\ s^{-1}$
Fluid velocity in the duct is assumed to follow the equation $v = c_1 - c_2 r^n$

Required:

Values of constants c_1, c_2 and *n* in the equation $v = c_1 - c_2 r^n$
Units of c_1 and c_2
Mean velocity of fluid in the duct, $\bar{v}$
Radial position from the axis at which this mean velocity occurs

Solution.

Radius of duct, $R = \frac{D}{2} = \frac{2 \text{ m}}{2} = 1 \text{ m}$

Let us use the boundary conditions. At $r = R = 1\ m$, the fluid velocity $v = 0$. Hence, $v = 0 = c_1 - c_2 r^n = c_1 - c_2 (1)^n$

$\Rightarrow\ c_1 = c_2(1)^n$

$\Rightarrow\ c_1 = c_2$ …… (1)

At r = 0, fluid velocity v = 6 m s^{-1}. Hence,

$v = 6\,\mathrm{m\,s^{-1}} = c_1 - c_2 r^n$

$\Rightarrow\ 6\,\mathrm{m\,s^{-1}} = c_1 - c_2(0)^n$

$\Rightarrow\ 6\,\mathrm{m\,s^{-1}} = c_1$ - 0

$\Rightarrow\ 6\,\mathrm{m\,s^{-1}} = c_1$

Therefore, from (1), $c_2 = c_1 = 6$

Units of c_1 and c_2:

The velocity distribution in the duct is given by $v = c_1 - c_2 r^n$

Here, the units of velocity *v* are $m\ s^{-1}$

Hence, the units of c_1 and the quantity $c_2 r^n$ must be m s^{-1}

In the quantity $c_2 r^n$, *r* represents the radius of duct having unit of *m* and *n* is simply a number whose value is found to 1.826. Hence, c_2 must have units of $m^{-0.826}\ s^{-1}$, so that $c_2\ m^{-0.826}\ s^{-1}$ x $(r\ m)^{1.826} = (c_2 r^{1.826})\ m^{-0.826}\ s^{-1}\ \mathrm{m}^{1.826}$

$= (c_2 r^{1.826})\ m\ s^{-1}$

Volume flow rate is given by

$$Q = 2\pi \int_{r=0}^{r=R} v r\,dr$$

$$\Rightarrow\ Q = 9\,\mathrm{m^3\,s^{-1}} = 2\pi \int_{r=0}^{r=1} \left(c_1 - c_2 r^n\right) r\,dr$$

$$\Rightarrow\ 9\,\mathrm{m^3\,s^{-1}} = 2\pi\left[c_1 \frac{r^2}{2}\right]_{r=0}^{r=1} - 2\pi\left[c_2 \frac{r^{n+2}}{n+2}\right]_{r=0}^{r=1}$$

$$\Rightarrow\ 9\,\mathrm{m^3\,s^{-1}} = 2\pi\left[c_1 \frac{1^2}{2} - 0\right] - 2\pi\left[c_2 \frac{1^{n+2}}{n+2} - 0\right] = 2\pi\left[\frac{c_1}{2} - \frac{c_2}{n+2}\right]$$

$$\Rightarrow \left[\frac{c_1}{2} - \frac{c_2}{n+2}\right] = \frac{9}{2\pi} = 1.431818$$

$$\Rightarrow \left[\frac{6}{2} - \frac{6}{n+2}\right] = 1.431818$$

$$\Rightarrow \left[3-\frac{6}{n+2}\right]=1.431818$$

$$\Rightarrow \frac{6}{n+2}=3-1.431818=1.568182$$

$$\Rightarrow n+2=\frac{6}{1.568182}=3.826$$

$$\Rightarrow n=3.826-2=1.826$$

Mean velocity of flow, $\bar{v}=\frac{Q}{A}=\frac{Q}{\left(\frac{\pi}{4}D^2\right)}=\frac{9\,\mathrm{m}^3\,\mathrm{s}^{-1}}{\left\{\frac{\pi}{4}\,\mathrm{x}\,(2\,\mathrm{m})^2\right\}}=2.863\ m\ s^{-1}$

To find the radial position at which this mean velocity occurs, let us put v = 2.863 $m\ s^{-1}$ in the velocity distribution profile.

$$v=2.863\,\mathrm{m\,s^{-1}}=c_1-c_2r^n=6-6r^{1.826}$$

$$\Rightarrow 6r^{1.826}=6-2.863=3.136364$$

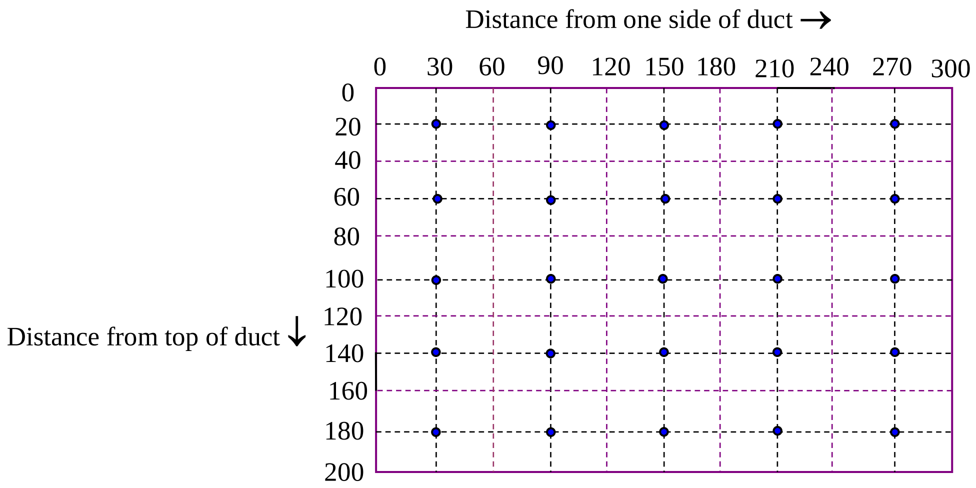
$$\Rightarrow r^{1.826}=\frac{3.136364}{6}=0.522727$$

$$\Rightarrow r=0.701\ m$$

Problem: Air flows through a rectangular duct having a cross-section of 300 mm width and 200 mm depth. In order to determine the volume rate of flow of air through the duct experimentally, the cross-section of the duct is divided into a number of imaginary rectangular elements of equal area and the velocity measured at the centre of each element. The results obtained are given below.

Distance from top of duct (mm)	Distance from side of duct (mm)				
	30	90	150	210	270
	Velocity ($\mathrm{m\ s^{-1}}$)				
20	1.6	2.0	2.2	2.0	1.7
60	1.9	3.4	6.9	3.7	2.0
100	2.1	6.8	10.0	7.0	2.3
140	2.0	3.5	7.0	3.8	2.1
180	1.8	2.0	2.3	2.1	1.9

Determine the volume flow rate and the mean velocity of flow in the duct.



Width of each imaginary rectangular element = 60 mm
Depth of each imaginary rectangular element = 40 mm
Velocity of flow of air is measured at the centre of each rectangular element

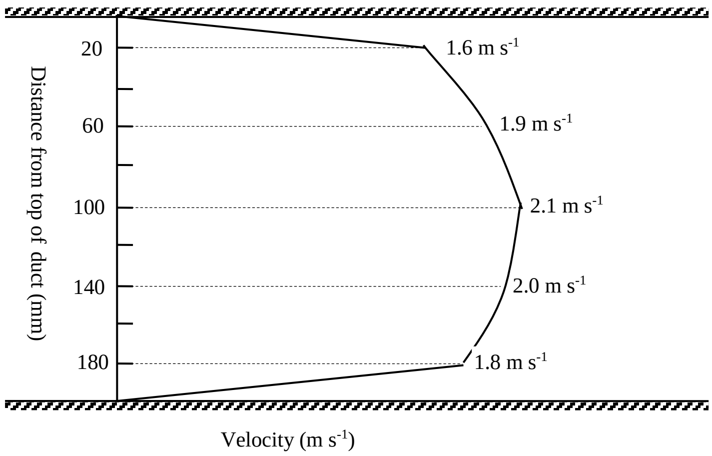


Figure. Velocity profile at the vertical section 30 mm from one side of duct

Mean velocity at vertical section 30 mm from one side (left side) of duct is given by the area under the graph shown in Figure above. The area under the graph may be determined by the mid-ordinate method. As the cross-section of the duct is divided into a number of imaginary rectangular elements of equal area and the velocity measured at the centre of each element, the given velocity profile at the section conforms to mid ordinate values. Hence,

$$\overline{u_{30}} = \frac{\sum \text{Mid} - \text{ordinates}}{5} = \frac{u_{30,20} + u_{30,60} + u_{30,100} + u_{30,140} + u_{30,180}}{5}$$

$$= \frac{1.6+1.9+2.1+2.0+1.8}{5} = \frac{9.4}{5} = 1.88 \text{ m s}^{-1}$$

Similarly, the mean velocity at vertical sections 90 mm, 150 mm, 210 mm and 270 mm can be determined using the mid-ordinate method.

$$\overline{u_{90}} = \frac{\sum \text{Mid}-\text{ordinates}}{5} = \frac{u_{90,20}+u_{90,60}+u_{90,100}+u_{90,140}+u_{90,180}}{5}$$

$$= \frac{2.0+3.4+6.8+3.5+2.0}{5} = \frac{17.7}{5} = 3.54 \text{ m s}^{-1}$$

$$\overline{u_{150}} = \frac{\sum \text{Mid}-\text{ordinates}}{5} = \frac{u_{150,20}+u_{150,60}+u_{150,100}+u_{150,140}+u_{150,180}}{5}$$

$$= \frac{2.2+6.9+10.0+7.0+2.3}{5} = \frac{28.5}{5} = 5.7 \text{ m s}^{-1}$$

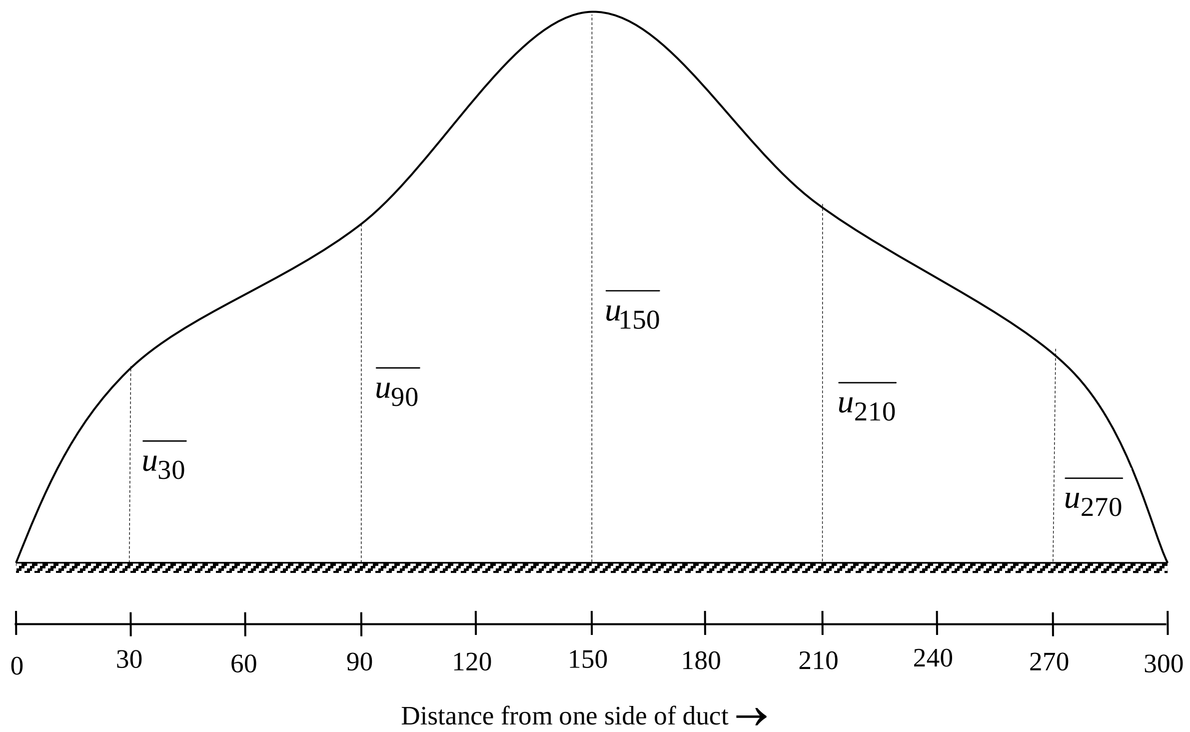
$$\overline{u_{210}} = \frac{\sum \text{Mid}-\text{ordinates}}{5} = \frac{u_{210,20}+u_{210,60}+u_{210,100}+u_{210,140}+u_{210,180}}{5}$$

$$= \frac{2.0+3.7+7.0+3.8+2.1}{5} = \frac{18.6}{5} = 3.72 \text{ m s}^{-1}$$

$$\overline{u_{270}} = \frac{\sum \text{Mid}-\text{ordinates}}{5} = \frac{u_{270,20}+u_{270,60}+u_{270,100}+u_{270,140}+u_{270,180}}{5}$$

$$= \frac{1.7+2.0+2.3+2.1+1.9}{5} = \frac{10.0}{5} = 2.0 \text{ m s}^{-1}$$

To find the mean velocity of flow in the duct, let us construct the velocity profile using the mean velocities worked out earlier at the vertical sections located at 30 mm, 90 mm, 150 mm, 210 mm and 270 mm from one side of duct as shown below.



Mean velocity of flow in the duct is given by

$$\bar{u} = \frac{\sum \text{Mid} - \text{ordinates}}{5} = \frac{\overline{u_{30}} + \overline{u_{90}} + \overline{u_{150}} + \overline{u_{210}} + \overline{u_{270}}}{5} = \frac{1.88 + 3.54 + 5.7 + 3.72 + 2.0}{5}$$

$$= \frac{16.84}{5} = 3.368 \text{ m s}^{-1}$$

Problem: In a laboratory, the water supply is drawn from a roof storage tank 25 *m* above the water discharge point. If the friction factor is 0.008, the pipe diameter is 5 *cm* and the pipe is assumed vertical, calculate the maximum volume of flow achievable, if separation losses are ignored.

Solution.

Head of water available, $H = 25\ m$
Diameter of pipe, $D = 5\ cm = 0.05\ m$
Cross-sectional area of pipe, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(5x10^{-2})^2 = 1.964286 \times 10^{-3}\ m^2$
Friction factor for pipe, $f = 0.008$
Pipe is laid vertical from the storage tank; hence, length of pipe, $L = 25\ m$
Ignoring the separation losses, head lost due to pipe friction, $h_f = H = 25\ m$

As per Darcy-Weisbach's equation for head loss due to pipe friction, we have,

$$h_f = \frac{4fL}{D}\frac{V^2}{2g}$$

$$\Rightarrow 25 = \frac{4x0.008x25}{0.05} x \frac{V^2}{2x9.81}$$

$\Rightarrow V^2 = 30.65625$

$\Rightarrow V = 5.536809\ m\ s^{-1}$

Hence, $Q = A\ V = (1.964286 \text{ x } 10^{-3}) \text{ x } (5.536809) = 0.010876\ m^3\ s^{-1}$

Problem: The friction factor applicable to turbulent flow in a smooth glass pipe is given by $0.079/Re^{1/4}$. Calculate the pressure loss per unit length necessary to maintain a flow of 0.02 $m^3\ s^{-1}$ of kerosene, specific gravity 0.82, viscosity 1.9 x $10^{-3}\ N\ s\ m^{-2}$, in a glass pipe of 8 *cm* diameter. If the tube is replaced by a galvanized steel pipeline, wall roughness 0.15 *mm*, calculate the increase in pipe diameter to handle this flow with the same pressure gradient.

Solution.

For turbulent flow in a smooth glass pipe, friction factor, $f = \frac{0.079}{Re^{1/4}}$

Flow in pipeline, $Q = 0.02\ m^3\ s^{-1}$

Specific gravity of kerosene, $S = 0.82$

Mass density of kerosene, $\rho = S\ \rho_w = 0.82 \text{ x } 1000 = 820\ kg\ m^{-3}$

Specific weight of kerosene, $\gamma = S\ \gamma_w = 0.82 \text{ x } 9810 = 8044\ N\ m^{-3}$

Viscosity of kerosene, $\mu = 1.9 \text{ x } 10^{-3}\ N\ s\ m^{-2}$

Diameter of glass pipe, $D = 8\ cm = 0.08\ m$

Cross-sectional area of pipe, $A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(8x10^{-2})^2 = 5.029 \text{ x } 10^{-3}\ m^2$

Mean velocity of flow, $V = Q / A = \frac{0.02}{5.029x10^{-3}} = 3.98\ m\ s^{-1}$

Reynolds number of flow, $Re = \frac{\rho VD}{\mu} = \frac{820x3.98x0.08}{1.9x10^{-3}} = 137415$

As Reynolds number of flow is more than 2000, the type of flow occurring in the pipe is turbulent. Hence, the loss of head due to pipe friction can be computed by using the Darcy-Weisbach's equation.

Darcy-Weisbach's equation for head loss due to pipe friction:

$$h_f = \frac{4fL}{D}\frac{V^2}{2g}$$

$$f = \frac{0.079}{\text{Re}^{1/4}} = \frac{0.079}{(137415)^{1/4}} = 0.004$$

Head loss per unit length of pipe is given by

$$\frac{h_f}{L} = \frac{4f}{D}\frac{V^2}{2g} = \frac{4x0.004}{0.08}x\frac{(3.98)^2}{2x9.81} = 0.161472\ m\ m^{-1}$$

$$\Rightarrow \frac{\left(\frac{\Delta p}{\gamma}\right)}{L} = 0.161472\ m\ m^{-1}$$

$$\Rightarrow \frac{\Delta p}{L} = 0.161472\gamma = 0.161472x8044 = 1298\ N\ m^{-2}\ m^{-1}$$

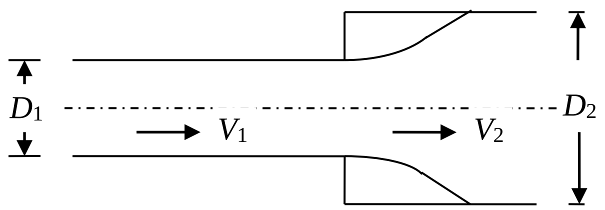
<u>When the glass tube is replaced by galvanized steel pipeline:</u>

Absolute wall roughness, k = 0.15 *mm* = 0.00015 *m*
Flow in pipeline, $Q = 0.02\ m^3\ s^{-1}$

Solution incomplete

Problem: In a water pipeline there is an abrupt change in diameter from 140 *mm* to 250 *mm*. If the head lost due to separation when the flow is from the smaller to the larger pipe is 0.6 m greater than the head lost when the same flow is reversed, determine the flow rate.

Solution.



D_1 = diameter of smaller pipe = 140 *mm*
D_2 = diameter of larger pipe = 250 *mm*

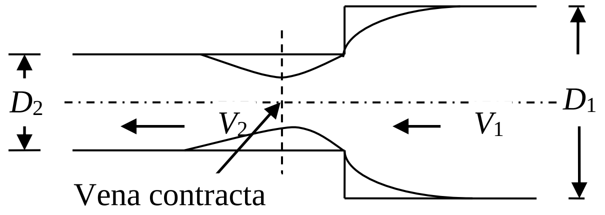
A_1 = cross-sectional area of smaller pipe = $\frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.14)^2 = 0.0154\ m^2$

A_2 = cross-sectional area of smaller pipe = $\frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.25)^2 = 0.049107\ m^2$

When the flow is from smaller pipe to larger pipe, the loss is due to sudden enlargement. The head lost due to sudden enlargement is given by equation

$$(h_L)_{S.E.} = \left(1 - \frac{A_1}{A_2}\right)^2 \left(\frac{V_1^2}{2g}\right) = \left(1 - \frac{0.0154}{0.049107}\right)^2 \left(\frac{V_1^2}{2g}\right) = 0.471\left(\frac{V_1^2}{2g}\right)$$

When the flow is reversed, the flow is from larger pipe to smaller pipe and the loss is due to sudden contraction.



D_1 = diameter of larger pipe = 250 *mm*
D_2 = diameter of smaller pipe = 140 *mm*

The head lost due to sudden contraction is given by equation

$$(h_L)_{S.C.} \approx \left(\frac{A_2}{A_c} - 1\right)^2 \left(\frac{V_2^2}{2g}\right) = \left(\frac{1}{C_c} - 1\right)^2 \left(\frac{V_2^2}{2g}\right)$$

where, A_c = cross-sectional area of vena contracta
A_2 = cross-sectional area of smaller pipe
A_1 = cross-sectional area of larger pipe
C_c = coefficient of contraction for the junction based on the smaller pipe entry diameter (D_2)

Table below shows the experimental values of C_c.

A_2/A_1	0.1	0.3	0.5	0.7	1.0
C_c	0.61	0.632	0.673	0.73	1.0

Here, $A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.14)^2 = 0.0154\ m^2$

$A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.25)^2 = 0.049107\ m^2$

The ratio, $A_2/A_1 = 0.0154 / 0.049107 = 0.3$
For $A_2/A_1 = 0.3$, $C_c = 0.632$

$$(h_L)_{S.C.} = \left(\frac{1}{C_c} - 1\right)^2 \left(\frac{V_2^2}{2g}\right) = \left(\frac{1}{0.632} - 1\right)^2 \left(\frac{V_2^2}{2g}\right) = 0.34 \left(\frac{V_2^2}{2g}\right)$$

It is given that the head lost due to separation when the flow is from the smaller to the larger pipe is 0.6 m greater than the head lost when the same flow is reversed. That is,

$$(h_L)_{S.E.} = (h_L)_{S.C.} + 0.6 \text{ m}$$

$$\Rightarrow 0.471 \left(\frac{V_1^2}{2g}\right) = 0.34 \left(\frac{V_2^2}{2g}\right) + 0.6$$

where, V_1 = velocity of flow in smaller pipe when flow occurs from smaller pipe to larger pipe

V_2 = velocity of flow in smaller pipe when flow occurs from larger ppe to smaller pipe

Therefore, we can afford to replace V_1 by V_2 or V_2 by V_1 in the above expression. Let us replace V_1 by V_2 in the above expression; then, the above expression becomes

$$0.471 \left(\frac{V_2^2}{2g}\right) = 0.34 \left(\frac{V_2^2}{2g}\right) + 0.6$$

$$\Rightarrow (0.471 - 0.340) \left(\frac{V_2^2}{2g}\right) = 0.6$$

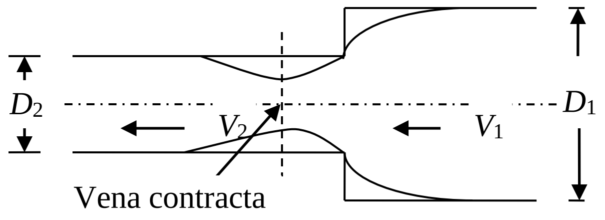
$$\Rightarrow 0.131 \left(\frac{V_2^2}{2g}\right) = 0.6$$

$$\Rightarrow \left(\frac{V_2^2}{2g}\right) = 0.6 / 0.131 = 4.58 \text{ m of water}$$

$\Rightarrow V_2$ = velocity of flow in smaller pipe = 9.48 m s^{-1}

Problem: A 150 *mm* diameter pipe reduces in diameter abruptly to 100 *mm*. If the pipe carries water at 30 *litres* s^{-1}, calculate the pressure loss across the contraction and express this as a percentage of the loss to be expected if the flow was reversed. Take the coefficient of contraction as 0.6.

Solution.



Discharge carried in the pipe,

Q = 30 lps

= 30 x 10^{-3} m^3 s^{-1}

= 0.03 m^3 s^{-1}

Loss of head
due to sudden contraction,
$(h_L)_{S.C.}$ = ?

D_1 = diameter of larger pipe = 150 *mm*
D_2 = diameter of smaller pipe = 100 *mm*

Loss of head due to sudden expansion, $(h_L)_{S.E.}$ = ?
Coefficient of contraction, C_c = 0.6

The head lost due to sudden contraction is given by equation

$$(h_L)_{S.C.} \approx \left(\frac{A_2}{A_c} - 1\right)^2 \left(\frac{V_2^2}{2g}\right) = \left(\frac{1}{C_c} - 1\right)^2 \left(\frac{V_2^2}{2g}\right)$$

where, A_c = cross-sectional area of vena contracta
A_2 = cross-sectional area of smaller pipe
A_1 = cross-sectional area of larger pipe
C_c = coefficient of contraction for the junction based on the smaller pipe entry diameter (D_2)

Here, $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.10)^2 = 0.007857\ m^2$

$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.15)^2 = 0.017679\ m^2$

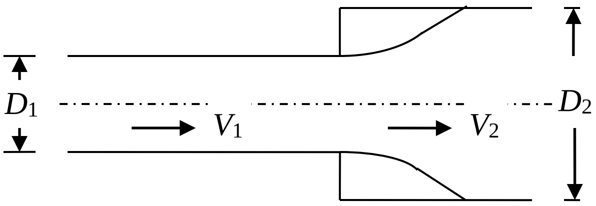
C_c = 0.6 (given)

$$(h_L)_{S.C.} = \left(\frac{1}{C_c} - 1\right)^2 \left(\frac{V_2^2}{2g}\right) = \left(\frac{1}{0.6} - 1\right)^2 \left(\frac{V_2^2}{2g}\right) = 0.444 \left(\frac{V_2^2}{2g}\right)$$

V_2 = velocity of flow in smaller pipe = Q/A_1 = 0.03 / 0.017679 = 1.7 $m\ s^{-1}$

Hence, $(h_L)_{S.C.} = 0.444 \left(\frac{1.7^2}{2x9.81}\right) = 0.065\ m$ of water

Pressure loss due to sudden contraction is given by,
$\Delta p = \gamma (h_L)_{S.C.}$ = (9810 $N\ m^{-3}$) x (0.065 m of water) = 638 $N\ m^{-2}$



D_1 = diameter of smaller pipe = 100 *mm*
D_2 = diameter of larger pipe = 150 *mm*

A_1 = cross-sectional area of smaller pipe = $\frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.10)^2 = 0.007857\ m^2$

A_2 = cross-sectional area of larger pipe = $\frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.15)^2 = 0.017679\ m^2$

V_1 = velocity of flow in smaller pipe = Q / A_1 = 0.03 / 0.007857 = 1.7 $m\ s^{-1}$

When the flow is from smaller pipe to larger pipe, the loss is due to sudden enlargement. The head lost due to sudden enlargement is given by equation

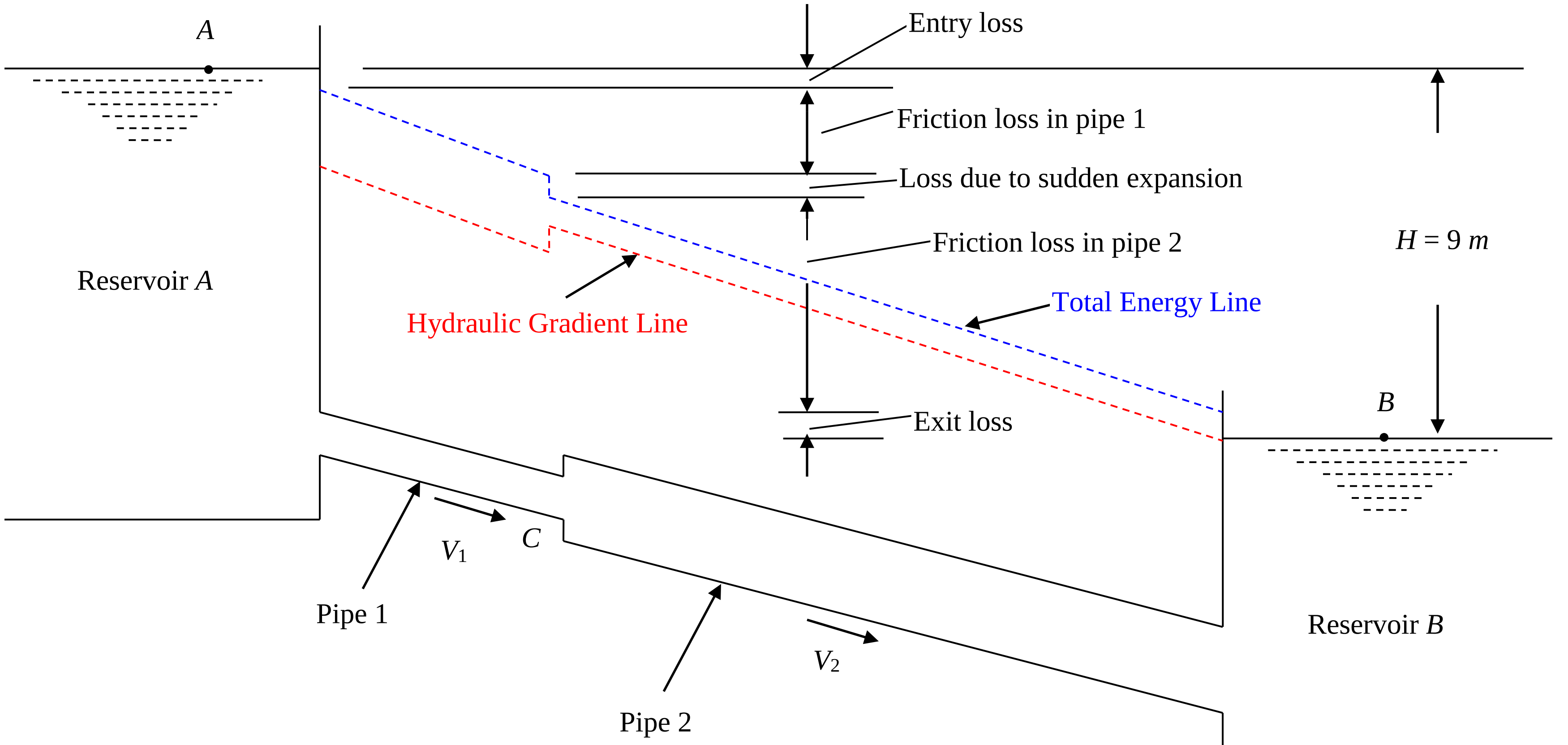
$$(h_L)_{S.E.} = \left(1 - \frac{A_1}{A_2}\right)^2 \left(\frac{V_1^2}{2g}\right) = \left(1 - \frac{0.007857}{0.017679}\right)^2 \left(\frac{V_1^2}{2g}\right) = 0.309 \left(\frac{1.7^2}{2x9.81}\right) =$$

$$= 0.0455\ m \text{ of water}$$

Expressing loss of head due to sudden contraction as a percentage of loss of head due to sudden expansion, we have,

$$\frac{(h_L)_{S.C.}}{(h_L)_{S.E}} x100 = \frac{0.065}{0.0455} x100 = 143.8\ \%$$

Problem: Two reservoirs *A* and *B* have a difference in level of 9 *m* and are connected by a pipeline 200 *mm* in diameter over the first part *AC*, which is 15 *m* long, and then 250 *mm* diameter for *CB*, the remaining 45 *m* length. The entrance to and exit from the pipes are sharp and the change of section at *C* is sudden. The friction coefficient *f* is 0.01 for both pipes.
(a) List the losses of head (energy per unit weight of flowing liquid) which occur, giving an expression for each.



A
Entry loss
Friction loss in pipe 1
Loss due to sudden expansion
Friction loss in pipe 2
H = 9 m
Reservoir A
Hydraulic Gradient Line
Total Energy Line
B
Exit loss
V1
C
Pipe 1
Reservoir B
V2
Pipe 2

Data given:
Difference of water level in two reservoirs *A* and *B*, $H = 9\ m$
Diameter of pipe 1 taking off from reservoir *A*, $D_1 = 200\ mm = 0.2\ m$
Cross-sectional area of pipe 1, $A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.2)^2 = 0.0314\ m^2$
Length of pipe 1, $L_1 = 15\ m$
Diameter of pipe 2 connecting reservoir *B*, $D_2 = 250\ mm = 0.25\ m$
Cross-sectional area of pipe 1, $A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.25)^2 = 0.049107\ m^2$
Length of pipe 2, $L_2 = 45\ m$
Friction coefficient for pipe 1, $f_1 = 0.01$
Friction coefficient for pipe 2, $f_2 = 0.01$

(a) The losses of head which occur are as follows:

(i) Loss at entrance to pipe 1 taking off from reservoir *A*. This is a separation loss and, since the entrance is described as sharp and is below the free surface of the reservoir, the value of loss coefficient *k* will be 0.5

Loss of head at entrance to pipe 1, $(h_L)_{entrance} = 0.5\left(\frac{V_1^2}{2g}\right)$

where, V_1 = mean velocity of flow in pipe 1

(ii) Loss of head due to friction in pipe 1 of length 15 *m* and diameter 0.2 *m*

$$h_{f_1} = \frac{4f_1L_1}{D_1}\frac{V_1^2}{2g}$$

(iii) Loss of head due to sudden change of section (sudden expansion) at *C* where the pipe diameter abruptly changes from 200 *mm* to 250 *mm*.

Loss of head due to sudden enlargement at *C*, $(h_L)_{S.E.} = \frac{(V_1 - V_2)^2}{2g}$

where V_1 = mean velocity of flow in pipe 1
V_2 = mean velocity of flow in pipe 2

(iv) Loss of head due to friction in pipe 2 of length 45 *m* and diameter 250 *mm*

$$h_{f_2} = \frac{4f_2L_2}{D_2}\frac{V_2^2}{2g}$$

(v) Loss of head at exit of pipe 2 (exit of pipe 2 is connected to reservoir 2)
As the exit of pipe is described as sharp and it is beneath the free surface of liquid in reservoir *B*, there will be a separation loss.

Loss of head at exit of pipe 2 is given by

$$(h_L)_{exit} = \frac{V_2^2}{2g}$$

Governing equation for solving for the flow rate in pipeline from reservoir *A* to reservoir *B*:

1. Difference in water level between reservoir *A* and reservoir *B* = loss of head at entrance to pipe 1 + loss of head due to friction in pipe 1 + loss of head due to sudden enlargement of pipe section at *C* + loss of head due to friction in pipe 2 + loss of head at exit of pipe 2

i.e., $H = 0.5\left(\frac{V_1^2}{2g}\right) + \frac{4 f_1 L_1}{D_1}\frac{V_1^2}{2g} + \frac{(V_1 - V_2)^2}{2g} + \frac{4 f_2 L_2}{D_2}\frac{V_2^2}{2g} + \frac{V_2^2}{2g}$

2. Principle of continuity

$$Q = A_1 V_1 = A_2 V_2$$

$$\Rightarrow \left(\frac{\pi}{4} D_1^2\right) V_1 = \left(\frac{\pi}{4} D_2^2\right) V_2 \Rightarrow D_1^2 V_1 = D_2^2 V_2 \Rightarrow (0.2)^2 V_1 = (0.25)^2 V_2$$

$$\Rightarrow V_1 = \frac{(0.25)^2}{(0.2)^2} V_2 = 1.5625\ V_2$$

Putting $V_1 = 1.5625\ V_2$ in equation (1), we have,

$H = 0.5\left[\frac{(1.5625V_2)^2}{2x9.81}\right] + \frac{4(0.01)(15)}{0.2}\frac{(1.5625V_2)^2}{2x9.81} + \frac{(1.5625V_2 - V_2)^2}{2x9.81} + \frac{4(0.01)(45)}{0.25}\frac{V_2^2}{2x9.81} + \frac{V_2^2}{2x9.81}$

$\Rightarrow 9 = 0.062217\ V_2^2 + 0.373304\ V_2^2 + 0.016127\ V_2^2 + 0.366972\ V_2^2 + 0.050968\ V_2^2$

$\Rightarrow 9 = 0.869589\ V_2^2$

$\Rightarrow V_2 = 3.217\ m\ s^{-1}$

Therefore, $Q = A_2 V_2 = (0.049107) \text{ x } (0.311) = 0.158\ m^3\ s^{-1}$

Problem: Two vessels in which the difference of surface levels is maintained constant at 2.4 *m* are connected by a 75 *mm* diameter pipeline 15 *m* long. If the frictional coefficient f may be taken as 0.008, determine the volume rate of flow through the pipe.

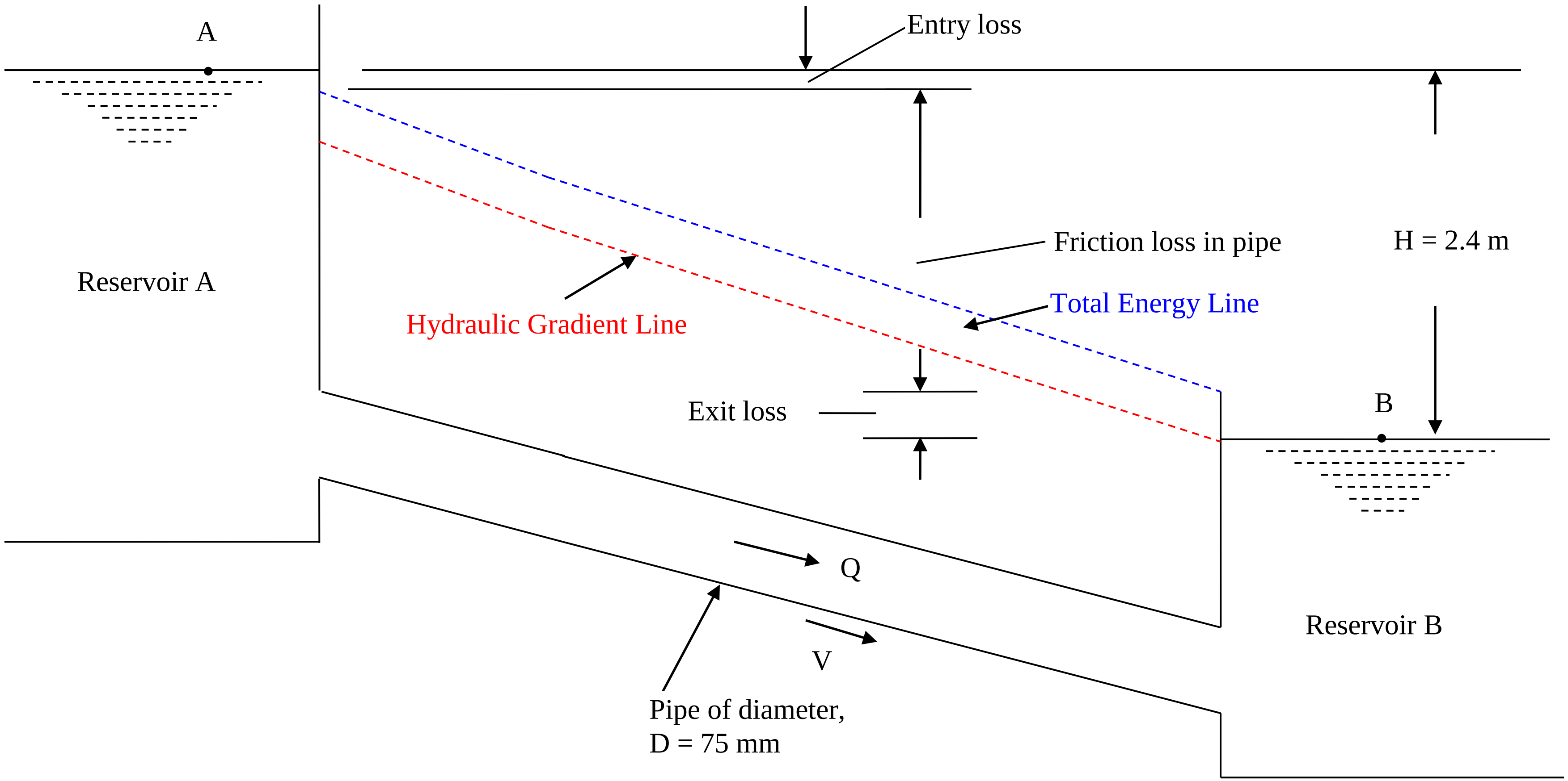
Solution.

Data given:

Difference of water surface levels in two vessels, $H = 2.4\ m$

Diameter of pipeline connecting the two vessels, $D = 75\ mm = 0.075\ m$

Length of pipeline connecting the two vessels, L = 15 m
Frictional coefficient of pipeline, f = 0.008
Required: Volume rate of flow through the pipeline, Q


A
Entry loss
Reservoir A
Friction loss in pipe
H = 2.4 m
Total Energy Line
Hydraulic Gradient Line
Exit loss
B
Q
Reservoir B
V
Pipe of diameter,
D = 75 mm

Governing equation for solving for the flow rate in pipeline from reservoir *A* to reservoir *B*:

Difference in water level between reservoir *A* and reservoir *B* = loss of head at entrance to pipe + loss of head due to friction in pipe + loss of head at exit of pipe

i.e., $H = 0.5\left(\frac{V^2}{2g}\right) + \frac{4fL}{D}\frac{V^2}{2g} + \frac{V^2}{2g}$

where, V = mean velocity of flow in pipe connecting the two vessels (reservoirs) *A* and *B*

We have, $H = 0.5\left(\frac{V^2}{2 \text{ x } 9.81}\right) + \frac{4 \text{ x } 0.008 \text{ x } 15}{0.075}\left(\frac{V^2}{2 \text{ x } 9.81}\right) + \left(\frac{V^2}{2 \text{ x } 9.81}\right)$

$\Rightarrow 2.4\ m = 0.025484\ V^2 + 0.326198\ V^2 + 0.050968\ V^2$

$= 0.40265\ V^2$

$\Rightarrow V^2 = \frac{2.4}{0.40265} = 5.961$

$\Rightarrow V = 2.44\ m\ s^{-1}$

As per equation of continuity, we have, $Q = AV = \left(\frac{\pi}{4}D^2\right)V$

$= \left(\frac{\pi}{4} \text{ x } 0.0075^2\right)2.44$

$= 0.01079\ m^3\ s^{-1}$

Problem: Water discharges from a reservoir through a 100 mm diameter pipe 15 m long which rises to its highest point at B, 1.5 m above the free surface of the reservoir, and discharges direct to the atmosphere at C, 4 m below the free surface at A. The length of pipe L_{AB} from A to B is 5 m and the length of pipe L_{BC} from B to C is 10 m. Both the entrance and exit of the pipe are sharp and the value of f is 0.08. Calculate (a) the mean velocity of water leaving the pipe at C and (b) the pressure in the pipe at B.

Solution.

Data given. Diameter of pipe, $D = 100\ mm = 0.1\ m$

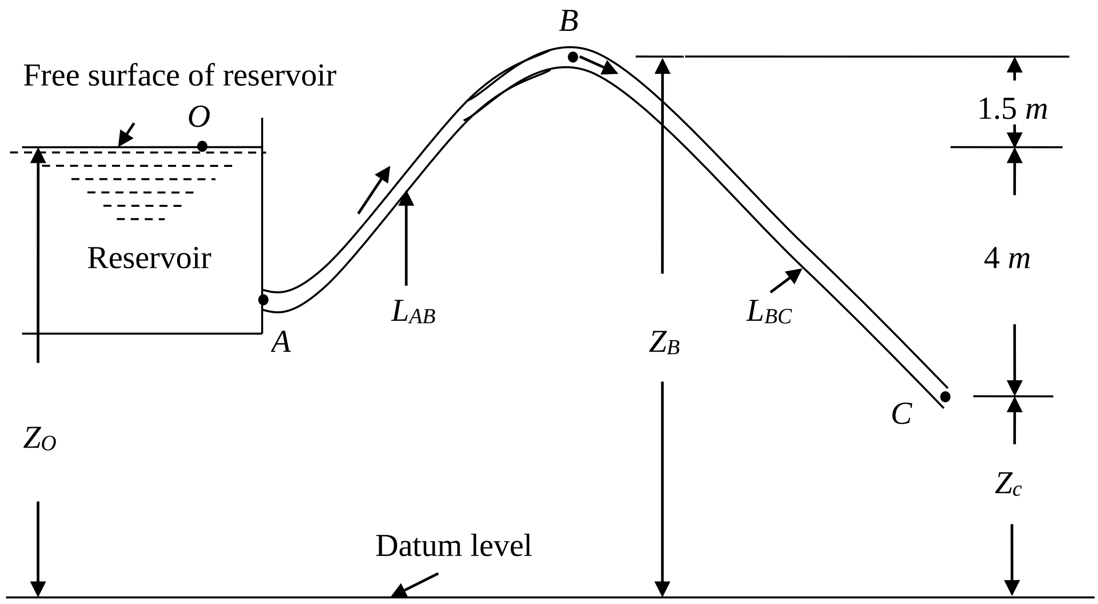
Length of pipe, $L = 15\ m$

Vertical height of the highest point *B* of the pipe above the free surface of water in reservoir = 1.5 *m*

Vertical location of the free discharge point *C* of the pipeline below the free surface of water in reservoir = 4 *m*

Length of portion *AB* of pipe, $L_{AB} = 5\ m$
Length of portion *BC* of pipe, $L_{BC} = 10\ m$
Entrance of pipe at *A* is sharp
Exit of pipe at *C* is sharp
Friction coefficient, $f = 0.08$

Required. (a) Mean velocity of water leaving the pipe at *C*, *V*
(b) Pressure in the pipe at *B*, p_B



The pressure at the free surface of water in the reservoir is atmospheric. Similarly, the pressure at the exit of pipe at *C* is also atmospheric. So, we can write, $p_O = p_C$ = zero gauge pressure.

The datum head of point *O* on the free surface of water in the reservoir = Z_A
Similarly, datum head at the exit of pipe at $C = Z_C$

As the area of free surface of reservoir is considered to be large, the velocity at point *O* on the free surface is negligible, i.e., $V_O = 0$
Velocity with which water leaves the exit of pipe at *C* is *V*, i.e., $V_c = V$
V is the mean velocity of flow in the pipe *ABC*.

It should be noted that even though the exit of pipe is sharp, there will be no loss of energy at exit because, water emerges into the atmosphere without any change of cross-section of the stream.

Since the entrance to the pipe at A is sharp, there will be an entrance loss of $\frac{0.5V^2}{2g}$.

The loss due to friction in the pipe ABC is given by the Darcy-Weisbach formula as $\frac{4f(L_{AB}+L_{BC})}{D}\frac{V^2}{2g}$

Let us now apply the steady flow energy equation between point *O* on the free surface of water in the reservoir and the exit of pipe at *C*.

Total energy per unit weight of water at *O* = Total energy per unit weight of water at *C* + Loss in energy per unit weight of water

$$Z_O + \frac{p_O}{\gamma} + \frac{V_O^2}{2g} = \left(Z_C + \frac{p_C}{\gamma} + \frac{V_C^2}{2g}\right) + \text{Losses}$$

$$\Rightarrow Z_O + 0 + 0 = Z_C + 0 + \frac{V^2}{2g} + \frac{0.5V^2}{2g} + \frac{4f(L_{AB}+L_{BC})}{D}\frac{V^2}{2g}$$

$$\Rightarrow Z_O - Z_C = \frac{1.5V^2}{2g} + \frac{4 \text{ x } 0.08 \text{ x } (5+10)}{0.1}\frac{V^2}{2g}$$

$$\Rightarrow 4\ m = 0.076453\ V^2 + 2.446483\ V^2 = 2.522936\ V^2$$

$$\Rightarrow V^2 = \frac{4}{2.522936} = 1.585$$

$$\Rightarrow V = \sqrt{1.585} = 1.26\ m\ s^{-1}$$

Now, let us apply the steady flow energy equation between the points *O* and *B*. We have,

$$\left(Z_O + \frac{p_O}{\gamma} + \frac{V_O^2}{2g}\right) = \left(Z_B + \frac{p_B}{\gamma} + \frac{V_B^2}{2g}\right) + \text{Entrance loss at A} + \text{Friction loss in segment AB of pipe}$$

$$\Rightarrow Z_O + 0 + 0 = \left(Z_B + \frac{p_B}{\gamma} + \frac{V^2}{2g}\right) + \frac{0.5V^2}{2g} + \frac{4fL_{AB}}{D}\frac{V^2}{2g}$$

$$\Rightarrow Z_O + 0 + 0 = \left(Z_B + \frac{p_B}{\gamma} + \frac{V^2}{2g}\right) + \frac{0.5V^2}{2g} + \frac{4 \text{ x } 0.08 \text{ x } 5}{0.1}\left(\frac{V^2}{2g}\right)$$

$$\Rightarrow Z_O = Z_B + \frac{p_B}{\gamma} + 0.050968\ V^2 + 0.025484\ V^2 + 0.815494\ V^2$$

$$\Rightarrow Z_O - Z_B = \frac{p_B}{\gamma} + 0.891947\ V^2$$

$$\Rightarrow \frac{p_B}{\gamma} = (Z_O - Z_B) - 0.891947V^2$$

$= - 1.5 - 0.891947\ (1.26)^2$

$= - 2.91606\ m$

$\Rightarrow p_B = (- 2.91606\ m) \times (9810\ N\ m^{-3})$

$= - 28606.5\ N\ m^{-2}$

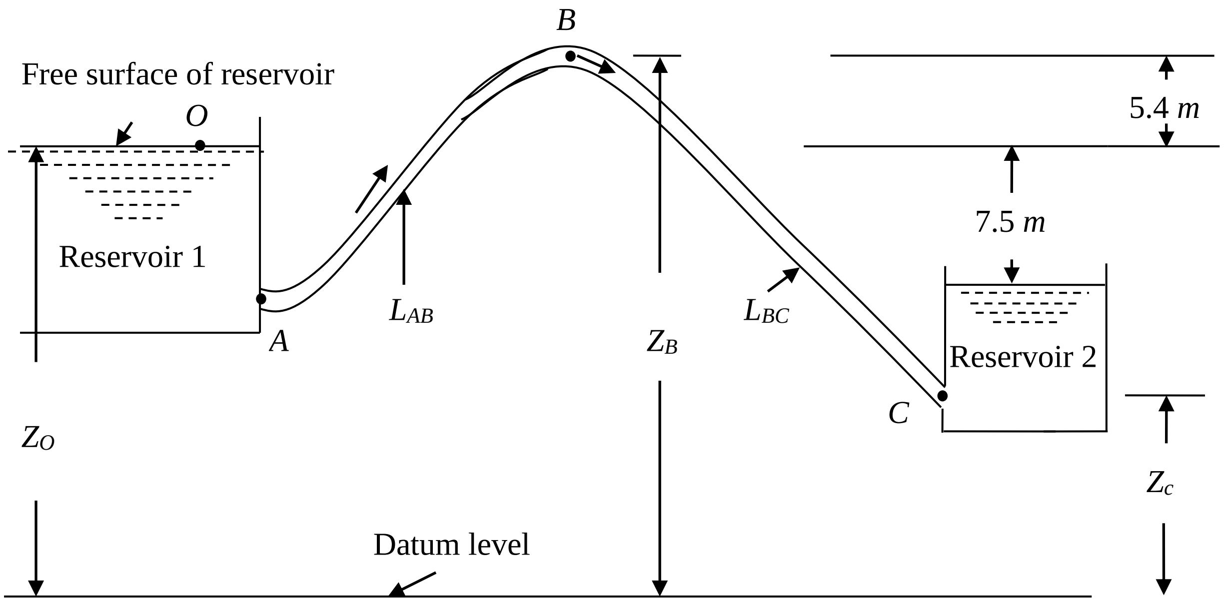
$= 28.607\ kN\ m^{-2}$ below atmospheric pressure

Problem: The difference in surface levels in two reservoirs connected by a siphon is 7.5 *m*. The diameter of the siphon is 300 *mm* and its length 750 *m*. The friction coefficient is 0.0064. If air is liberated from solution when the absolute pressure is less than 1.2 *m* of water, what will be the maximum length of the inlet leg of the siphon to run full, if the highest point is 5.4 *m* above the surface level in the upper reservoir? What will be the discharge?

Solution.

Data given: Difference in surface levels in two reservoirs, $H = 7.5\ m$
Diameter of the siphon, $D = 300\ mm = 0.3\ m$
Length of siphon, $L = 750\ m$
Friction coefficient, $f = 0.0064$
Vapour pressure of solution, $H_{vap} = 1.2\ m$ of water absolute
Vertical height of the highest point B of the siphon above the free surface of water in reservoir 1 = 5.4 *m*

Required: Maximum length of inlet leg of siphon, L_{AB}
Discharge, Q



Total energy losses per unit weight of flowing fluid as the fluid flows from reservoir 1 to reservoir 2 through the siphon *ABC* of length 750 *m* = Difference in surface levels in two reservoirs, *H*

$\Rightarrow H$ = Entrance loss at *A* + Friction loss in length of siphon *ABC* + Exit loss at *C*

$$\Rightarrow 7.5\ m = \frac{0.5V^2}{2g} + \frac{4fL}{D}\frac{V^2}{2g} + \frac{V^2}{2g}$$

$$\Rightarrow 7.5\ m = 0.025484\ V^2 + \frac{4 \text{ x } 0.0064 \text{ x } 750}{0.3}\left(\frac{V^2}{2 \text{ x } 9.81}\right) + 0.050968\ V^2$$

$$\Rightarrow 7.5\ m = 0.025484\ V^2 + 3.261978\ V^2 + 0.050968\ V^2$$

$$\Rightarrow 7.5\ m = 3.33843\ V^2$$

$$\Rightarrow V^2 = \frac{7.5}{3.33843} = 2.246565$$

$$\Rightarrow V = 1.5\ m\ s^{-1}$$

Discharge, *Q* = (area of cross-section of siphon) x (mean velocity of flow in siphon)

$$= \left(\frac{\pi}{4}D^2\right)V$$

$$= \left(\frac{\pi}{4} \text{ x } 0.3^2\right) \text{ x } 1.5$$

$$= 1.065\ m^3\ s^{-1}$$

Air is liberated from solution when the absolute pressure is less than 1.2 *m* of water, that is, the solution flowing in the siphon will get separated when the pressure in the siphon falls less than 1.2 *m* of water absolute pressure. The pressure in the siphon will fall to the maximum level below atmospheric pressure at the summit point *B* of the siphon. Hence, to avoid separation of flow in siphon, the pressure developed in the summit point *B* must not fall below 1.2 *m* of water absolute pressure. That is, $\frac{p_B}{\gamma} = -(10.3-1.2) = -9.1$ *m* of water, i.e., 9.1 *m* of water below atmospheric pressure.

Now, let us apply the steady flow energy equation between the points *O* and *B*. We have,

$$\left(Z_O + \frac{p_O}{\gamma} + \frac{V_O^2}{2g}\right) = \left(Z_B + \frac{p_B}{\gamma} + \frac{V_B^2}{2g}\right) + \text{Entrance loss at A} + \text{Friction loss in segment AB of pipe}$$

$$\Rightarrow Z_O + 0 + 0 = \left(Z_B + \frac{p_B}{\gamma} + \frac{V^2}{2g}\right) + \frac{0.5V^2}{2g} + \frac{4fL_{AB}}{D}\frac{V^2}{2g}$$

$$\Rightarrow Z_O + 0 + 0 = \left(Z_B - 9.1 + \frac{1.5^2}{2 \text{ x } 9.81}\right) + \frac{0.5 \text{ x } 1.5^2}{2 \text{ x } 9.81} + \frac{4 \text{ x } 0.0064 \text{ x } L_{AB}}{0.3}\left(\frac{1.5^2}{2 \text{ x } 9.81}\right)$$

$$\Rightarrow Z_O = Z_B - 9.1 + 0.114679 + 0.057339 + 0.009786\, L_{AB}$$

$$\Rightarrow Z_O - Z_B = -8.92798 + 0.009786\, L_{AB}$$

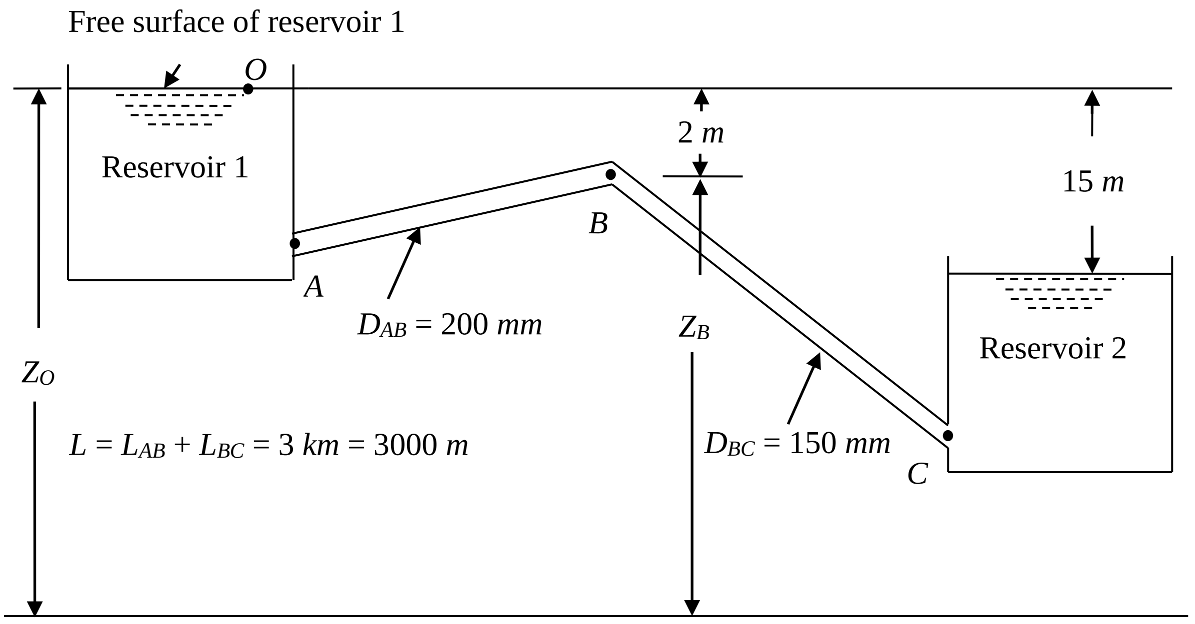
$$\Rightarrow -5.4\ m = -8.92798 + 0.009786\, L_{AB}$$

$$\Rightarrow L_{AB} = \frac{-5.4 + 8.92798}{0.009786} = 360.5\ m$$

Problem: Two reservoirs whose difference of level is 15 *m* are connected by a pipe *ABC* whose highest point *B* is 2 *m* below the level in the upper reservoir *A*. The portion *AB* has a diameter of 200 *mm* and the portion *BC* a diameter of 150 mm, the friction coefficient being the same for both portions. The total length of pipe is 3 *km*.

Find the maximum allowable length of the portion *AB* if the pressure head at *B* is not to be more than 2 *m* below atmospheric pressure. Neglect the secondary losses.

Solution.



Neglecting secondary losses as water flows from reservoir 1 through pipe ABC to reservoir 2, we have,

Difference of level between free water surface in two reservoirs, H = friction loss in segment AB of pipe ABC + friction loss in segment BC of pipe ABC

$$\Rightarrow H = 15\text{ m} = \frac{4fL_{AB}}{D_{AB}}\frac{V_{AB}^2}{2g} + \frac{4fL_{BC}}{D_{BC}}\frac{V_{BC}^2}{2g} \qquad \ldots\ldots \quad (1)$$

By the principle of continuity, we have $Q = A_{AB}V_{AB} = A_{BC}V_{BC}$

$$\Rightarrow \left(\frac{\pi}{4}D_{AB}^2\right)V_{AB} = \left(\frac{\pi}{4}D_{BC}^2\right)V_{BC}$$

$$\Rightarrow D_{AB}^2 V_{AB} = D_{BC}^2 V_{BC}$$

$$\Rightarrow (0.2)^2 V_{AB} = (0.15)^2 V_{BC}$$

$$\Rightarrow 0.04\ V_{AB} = 0.0225\ V_{BC}$$

$$\Rightarrow V_{BC} = \frac{0.04}{0.0225}V_{AB} = 1.778\ V_{AB}$$

$L_{BC} = L - L_{AB} = 3000 - L_{AB}$

Putting $L_{BC} = 3000 - L_{AB}$ and $V_{BC} = 1.778\ V_{AB}$ in equation (1), we have,

$$15\ m = \frac{4f \text{ x } L_{AB} \text{ x } V_{AB}^2}{2 \text{ x } 9.81 \text{ x } 0.2} + \frac{4f \text{ x } (3000 - L_{AB}) \text{ x } (1.778V_{AB})^2}{2 \text{ x } 9.81 \text{ x } 0.15}$$

$$\Rightarrow 15 = 0.008155\ L_{AB}\ V_{AB}^2 + 0.034373\ (3000 - L_{AB})\ V_{AB}^2$$

$$\Rightarrow 15 = 0.008155\ L_{AB}\ V_{AB}^2 + 103.1204\ V_{AB}^2 - 0.034373\ L_{AB}\ V_{AB}^2$$

$$\Rightarrow 15 = -0.02622\ L_{AB}\ V_{AB}^2 + 103.1204\ V_{AB}^2$$

$$\Rightarrow L_{AB} = \frac{103.1204\, V_{AB}^{2} - 15}{0.02622 V_{AB}^{2}} \qquad \ldots\ldots (2)$$

Now, let us apply the steady flow energy equation between the points *O* and *B*. We have,

$$\left(Z_O + \frac{p_O}{\gamma} + \frac{V_O^2}{2g}\right) = \left(Z_B + \frac{p_B}{\gamma} + \frac{V_B^2}{2g}\right) + \text{Entrance loss at A} + \text{Friction loss in segment AB of pipe}$$

Ignoring the entrance loss at *A*, we have,

$$\left(Z_O + \frac{p_O}{\gamma} + \frac{V_O^2}{2g}\right) = \left(Z_B + \frac{p_B}{\gamma} + \frac{V_B^2}{2g}\right) + \text{Friction loss in segment AB of pipe}$$

$$\Rightarrow Z_O + 0 + 0 = \left(Z_B + \frac{p_B}{\gamma} + \frac{V_{AB}^2}{2g}\right) + \frac{4fL_{AB}}{D_{AB}} \frac{V_{AB}^{2}}{2g}$$

$$\Rightarrow Z_O + 0 + 0 = \left(Z_B - 2.0 + \frac{V_{AB}^2}{2 \text{ x } 9.81}\right) + \frac{4f}{0.2}\left(\frac{103.1204\, V_{AB}^{2} - 15}{0.02622 V_{AB}^2}\right)\left(\frac{V_{AB}^2}{2 \text{ x } 9.81}\right)$$

$$\Rightarrow Z_O = Z_B - 2.0 + 0.050968\ V_{AB}^2 + 20f\left(\frac{5.255882\, V_{AB}^4 - 0.764526\, V_{AB}^2}{0.02622\, V_{AB}^2}\right)$$

$$\Rightarrow Z_O - Z_B = -2.0 + 0.050968\ V_{AB}^2 + 20f\left(\frac{5.255882\, V_{AB}^4 - 0.764526\, V_{AB}^2}{0.02622\, V_{AB}^2}\right)$$

$$\Rightarrow 2.0 \text{ m} = -2.0 + 0.050968\ V_{AB}^2 + 20f\,(200.4532\ V_{AB}^2 - 29.15812)$$

$$\Rightarrow 2.0 + 2.0 = 0.050968\ V_{AB}^2 + 32.07251\ V_{AB}^2 - 4.665299$$

$$\Rightarrow 4.0 + 4.665299 = 0.050968\ V_{AB}^2 + 32.07536\ V_{AB}^2$$

$$\Rightarrow 8.665299 = 32.12633\ V_{AB}^2$$

$$\Rightarrow V_{AB}^2 = \frac{8.665299}{32.12633} = 0.269726$$

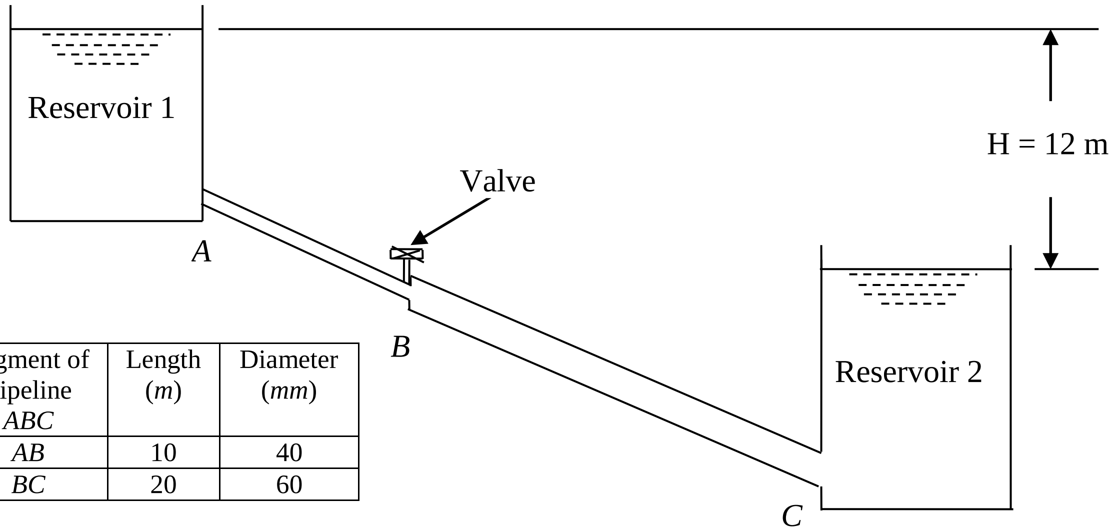
$$\Rightarrow V_{AB} = 0.519351\ m\ s^{-1}$$

Substituting $V_{AB} = 0.519351\ m\ s^{-1}$ in the expression for L_{AB} given by equation (2), we have, $L_{AB} = 1811.911\ m$

Problem: A pipeline 30 *m* long connects two tanks which have a difference of water level of 12 *m*. The first 10 *m* of pipeline from the upper tank is of 40 *mm* diameter and the next 20 *m* is of 60 *mm* diameter. At the change of section a valve is fitted. Calculate the rate of flow when the valve is fully opened assuming that its resistance is negligible and that *f* for both pipes is 0.0054. In order to restrict the flow the valve is then partially closed. If *k* for the valve is now 5.6, find the percentage reduction in flow.

Solution.

**



Segment of Pipeline *ABC*	Length (*m*)	Diameter (*mm*)
AB	10	40
BC	20	60

Difference of level between free water surface in two reservoirs, H =
Entrance loss at A + Friction loss in segment AB of pipe ABC +
Loss due to sudden expansion at B + Friction loss in segment BC of pipe ABC
+ Exit loss at C

$$\Rightarrow H = 12\ m = \frac{0.5V_{AB}^2}{2g} + \frac{4fL_{AB}}{D_{AB}}\frac{V_{AB}^{\ 2}}{2g} + \frac{(V_{AB}-V_{BC})^2}{2g} + \frac{4fL_{BC}}{D_{BC}}\frac{V_{BC}^{\ 2}}{2g} + \frac{V_{BC}^2}{2g} \quad \ldots\ldots \quad (1)$$

By the principle of continuity, we have $Q = A_{AB}V_{AB} = A_{BC}V_{BC}$

$$\Rightarrow \left(\frac{\pi}{4}D_{AB}^2\right)V_{AB} = \left(\frac{\pi}{4}D_{BC}^2\right)V_{BC}$$

$$\Rightarrow D_{AB}^2V_{AB} = D_{BC}^2V_{BC}$$

$$\Rightarrow (0.04)^2V_{AB} = (0.06)^2V_{BC}$$

$$\Rightarrow 0.0016\ V_{AB} = 0.0036\ V_{BC}$$

$$\Rightarrow V_{AB} = \frac{0.0036}{0.0016}V_{BC} = 2.25\ V_{BC}$$

Substituting $V_{AB} = 2.25\ V_{BC}$ in equation (1), we have,

$$12\ m = \frac{0.5(2.25V_{BC})^2}{2g} + \frac{4fL_{AB}}{D_{AB}}\frac{(2.25V_{BC})^2}{2g} + \frac{(2.25V_{BC}-V_{BC})^2}{2g} + \frac{4fL_{BC}}{D_{BC}}\frac{V_{BC}^{\ 2}}{2g} + \frac{V_{BC}^2}{2g}$$

$$= \frac{0.5(2.25V_{BC})^2}{2 \text{ x } 9.81} + \frac{4 \text{ x } 0.0054 \text{ x } 10}{0.004}\frac{(2.25V_{BC})^2}{2 \text{ x } 9.81} + \frac{(1.25V_{BC})^2}{2 \text{ x } 9.81} +$$

$$\frac{4 \text{ x } 0.0054 \text{ x } 20}{0.006}\frac{V_{BC}^{\ 2}}{2 \text{ x } 9.81} + \frac{V_{BC}^2}{2 \text{ x } 9.81}$$

$= 0.129014\ V_{BC}^2 + 1.393349\ V_{BC}^2 + 0.079638\ V_{BC}^2 + 0.366972\ V_{BC}^2 + 0.050968\ V_{BC}^2$

$= 2.019941386\ V_{BC}^2$

$\Rightarrow V_{BC} = 2.44\ m\ s^{-1}$

Rate of flow when the valve is fully open, $Q_{max} = A_{BC}V_{BC} = \left(\frac{\pi}{4}D_{BC}^2\right)V_{BC}$

$= \left\{\frac{\pi}{4}(0.06)^2\right\}(2.44)$

$= 0.0069\ m^3\ s^{-1}$

Approximate solution:

Neglecting the entrance loss at *A*, loss due to sudden expansion at *B* and the exit loss at *C*, we have,

$$H = 12\ m = \frac{4\,fL_{AB}}{D_{AB}}\frac{V_{AB}^{\ 2}}{2g} + \frac{4\,fL_{BC}}{D_{BC}}\frac{V_{BC}^{\ 2}}{2g}$$

$$\Rightarrow 12\ m = \frac{4\,fL_{AB}}{D_{AB}}\frac{(2.25V_{BC})^2}{2g} + \frac{4\,fL_{BC}}{D_{BC}}\frac{V_{BC}^{\ 2}}{2g}$$

$\Rightarrow 12 = 1.393349\ V_{BC}^2 + 0.366972\ V_{BC}^2$

$= 1.760321\ V_{BC}^2$

$\Rightarrow V_{BC} = 2.61\ m\ s^{-1}$

Rate of flow when the valve is fully open, $Q_{max} = A_{BC}V_{BC} = \left(\frac{\pi}{4}D_{BC}^2\right)V_{BC}$

$= \left\{\frac{\pi}{4}(0.06)^2\right\}(2.611)$

$= 0.00739\ m^3\ s^{-1}$

Alternate method:

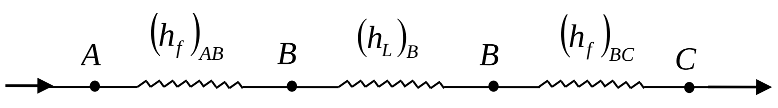


Figure above shows the given pipeline *ABC* in series. Section *AB* represents the first segment of the pipeline *ABC*; section *BC* represents the second segment of the pipeline *ABC*; section *BB* represents the valve at the junction *B* of the two segments *AB* and *BC* for controlling the flow through the pipeline *ABC*.

Total loss of head between *A* and *C* in the pipeline *ABC* can be expressed as

$$H = (h_f)_{AB} + (h_L)_B + (h_f)_{BC}$$

where, $(h_f)_{AB}$ = head loss due to friction in segment *AB* of pipeline *ABC*

$(h_L)_B$ = head loss due to partial closure of valve at junction *B*

$(h_f)_{BC}$ = head loss due to friction in segment *BC* of pipeline *ABC*

Any head loss can be expressed as $h_L = KQ^n$, where *n* is some power that depends upon the type of flow. For turbulent flow, *n* will be equal to two and, if separation losses are negligible, the above equation becomes $h_L = KQ^2$.

Darcy-Weisbach equation for head loss due to friction is given by

$$h_f = \frac{4fL}{D}\frac{V^2}{2g}$$

Putting $V = \frac{Q}{A}$, we have,

$$h_f = \frac{4fL}{D}\frac{\left(\frac{Q}{A}\right)^2}{2g} = \frac{4fL}{2gD}\left\{\frac{Q}{\left(\frac{\pi}{4}D^2\right)}\right\}^2 = \frac{4fL}{2gD}\left\{\frac{16Q^2}{\pi^2 D^4}\right\} = \frac{32fLQ^2}{\pi^2 gD^5} = \frac{fLQ^2}{3.028D^5}$$

$$= \left(\frac{fL}{3.028D^5}\right)Q^2$$

$$= KQ^2$$

where, $K = \left(\frac{fL}{3.028D^5}\right)$

Head loss due to friction in segment *AB* of pipeline *ABC*, $(h_f)_{AB} = K_{AB}Q^2$

where, $K_{AB} = \left(\frac{fL_{AB}}{3.028D_{AB}{}^5}\right) = \left(\frac{0.0054 \text{ x } 10}{3.028 \text{ x } 0.04^5}\right) = 174155.8$

Head loss due to partial closure of valve at junction *B*, $(h_L)_B = k\frac{(V_{AB} - V_{BC})^2}{2g} =$

$$= k\frac{(2.25V_{BC} - V_{BC})^2}{2g}$$

$$= k\frac{(1.25V_{BC})^2}{2g}$$

$$= 5.6\frac{(1.25V_{BC})^2}{2g}$$

Putting $V_{BC} = \left\{\frac{Q}{\left(\frac{\pi}{4}D_{BC}^{\ 2}\right)}\right\}$, we have, $(h_L)_B = 5.6\left(\frac{1.25^2}{2g}\right)\left\{\frac{Q}{\left(\frac{\pi}{4}D_{BC}^{\ 2}\right)}\right\}^2$

$$= 55741\ Q^2 = K_B Q^2$$

where, K_B = resistance coefficient for the control valve at the junction B
= 55741

Head loss due to friction in segment BC of pipeline ABC, $(h_f)_{BC} = K_{BC}Q^2$

where, $K_{BC} = \left(\frac{fL_{BC}}{3.028D_{BC}^{\ 5}}\right) = \left(\frac{0.0054 \text{ x } 20}{3.028 \text{ x } 0.06^5}\right) = 45868.19$

Hence, $H = (h_f)_{AB} + (h_L)_B + (h_f)_{BC}$

$\Rightarrow 12 = 174156\ Q^2 + 55741\ Q^2 + 45868\ Q^2$

$\Rightarrow 12 = 275765\ \ Q^2$

$\Rightarrow Q^2 = 12 / 275765$

$\Rightarrow Q = 0.0066\ m^3\ s^{-1}$

Percent reduction in flow = $\left(\frac{Q_{max} - Q}{Q_{max}}\right)100 = \left(\frac{0.00739 - 0.0066}{0.00739}\right)100 = 10.69$

Problem: A smooth walled tube having a friction coefficient $f = 0.004$ is used in a 3000 m long pipeline carrying water at 15° C between two reservoirs whose surface elevations are 6 m apart. Entry is sharp edged and the outlet is also abrupt to the downstream reservoir. The pipeline contains six 45° bends and two globe valves. Determine the necessary pipe diameter so that the discharge should be 28 *litres* s^{-1} to the lower reservoir.

Take the equivalent length of each bend as 26.5 diameters, the valve as 75 diameters and the entry as 30 diameters.

Solution.

Mass density of water at 15°C = 999.1 $kg\ m^{-3}$

Specific weight of water, γ = 999.1 x 9.81 = 9801.2 $N\ m^{-3}$

Length of pipeline, L = 3000 m

Difference between free water surface in the two reservoirs, H = 6 m

Loss at entrance due to sharp edged entry = $\frac{0.5V^2}{2g}$

Equivalent length of entry, $(L_{equi})_{entry}$ = 30 diameters = 30D

where, D = diameter of the pipe connecting the two reservoirs

Loss at abrupt (sharp edged) exit = $\frac{V^2}{2g}$

Equivalent length of exit can be taken as twice the equivalent length of entry since the head loss at exit is twice the head loss at entrance.

Equivalent length of exit, $(L_{equi})_{exit}$ = 2(30D) = 60D

No. of bends in the pipeline = 6

Equivalent length of each bend = 26.5 diameters = 26.5D

Hence, equivalent length of six bends, $(L_{equi})_{bends}$ = 6(26.5D) = 159D

Equivalent length of each valve = 75 diameters = 75D

Hence, equivalent length of two valves, $(L_{equi})_{valves}$ = 2(75D) = 150D

As all the bends and valves are located in series along the pipeline of length 3000 *m*, equivalent length of pipeline taking into account the equivalent lengths of entry, bends, valves and exit will be

$$L_{equi} = L + (L_{equi})_{entry} + (L_{equi})_{bends} + (L_{equi})_{valves} + (L_{equi})_{exit}$$

$$= 3000 + 30D + 159D + 150D + 60D$$

$$= 3000 + 399D$$

$H = KQ^2$

$\Rightarrow 6\ m = K\,(28.2/1000)^2$

$\Rightarrow K = \frac{6}{0.0282^2} = 7544.892$

$\Rightarrow K = 7544.892 = \left(\frac{fL_{equi}}{3.028D^5}\right) = \frac{0.004(3000 + 399D)}{3.028D^5} = \frac{12 + 1.596D}{3.028D^5}$

$\Rightarrow 7544.892\ (3.028D^5) = 12 + 1.596D$

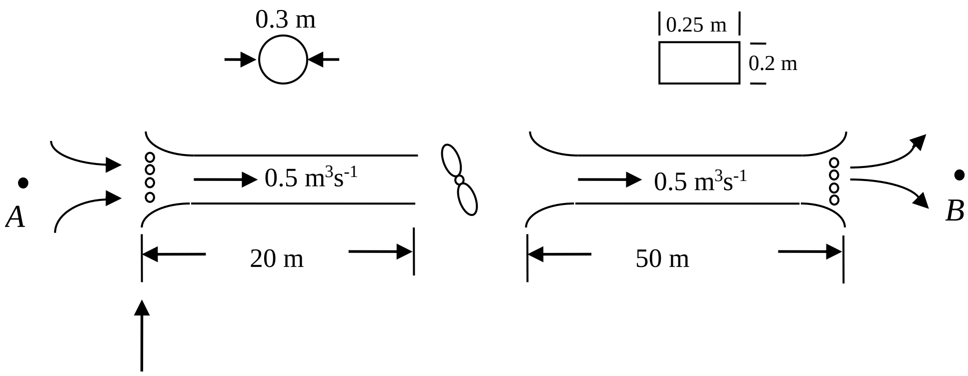
$\Rightarrow 22846\ D^5 = 12 + 1.596D$

$\Rightarrow 22846\ D^5 - 1.596\ D - 12 = 0$

Solving by trial and error, we have, D = 0.222 *m*

Problem: A horizontal duct system draws atmospheric air into a circular duct of 0.3 *m* diameter, 20 *m* long, then through a centrifugal fan and discharges it to atmosphere through a rectangular duct 0.25 *m* x 0.20 *m*, 50 *m* long. Assuming that the friction factor for each duct is 0.01 and accounting for an inlet loss of one-half of the velocity head and also for the kinetic energy at outlet, find the total pressure rise across the fan to produce a flow of 0.5 $m^3\ s^{-1}$.

Sketch also the total energy and hydraulic gradient lines putting in the most important values. Assume the density of air to be 1.2 $kg\ m^{-3}$.


0.3 m
0.25 m
0.2 m
A
0.5 m3s-1
0.5 m3s-1
B
20 m
50 m