# Forced Vibration -

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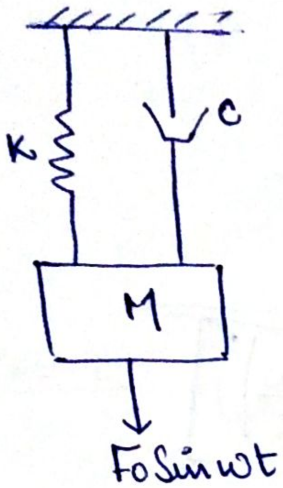



EOM:

$$m\ddot{x} + kx + c\dot{x} = F_o \sin\omega t$$

$x(t)$ = complementary sol$^n$ + steady state sol$^n$

$$x(t) = x_c(t) + x_{ss}(t)$$

$$x_c(t) \to 0 \text{ as } t \to \infty$$

$$x_{ss}(t) = X \sin(\omega t - \phi)$$

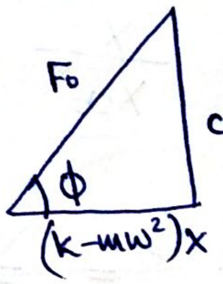
X → amplitude

ω → Excitation frequency

φ → phase angle

$$X = \frac{F_o}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \quad \text{——— (1)}$$

$$\tan\phi = \frac{p}{b} = \frac{c\omega X}{(k - m\omega^2)X}$$




$$\tan\phi = \frac{c\omega}{k - m\omega^2}$$

$$\Rightarrow F_o^2 = (c\omega X)^2 + [(k - m\omega^2)X]^2$$

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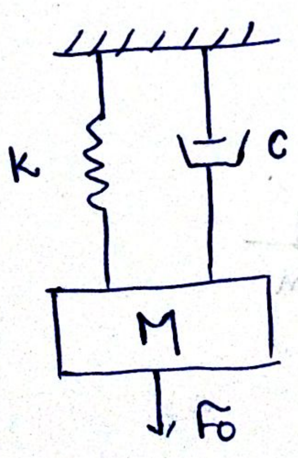
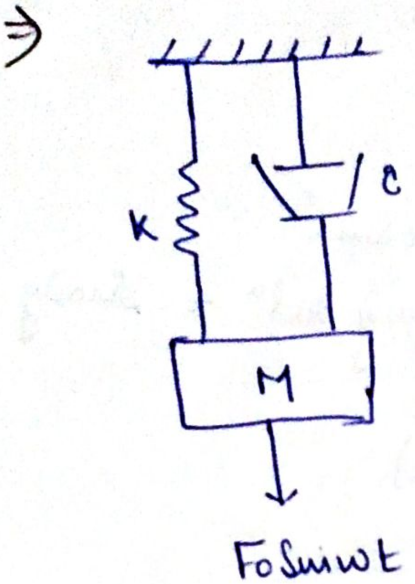



$$m\ddot{x} + c\dot{x} + kx = F_o \quad (m\ddot{x} \to 0,\ c\dot{x} \to 0)$$

$$\boxed{x_{st} = \frac{F_o}{k}}$$

$x_{st}$ → Static state deflection

⇒ 


## Magnification factor -

$$MF = \frac{X}{X_{st}}$$

from eqn (1). -

$$X = \frac{F_0}{\sqrt{\left[\left[1-\frac{\omega^2}{K/m}\right]^2 K^2 + \left(\frac{c\omega}{K}\right)^2\right]}}$$

$$X = \frac{(F_0/K) \rightarrow X_{st}}{\sqrt{\left(1-\frac{\omega^2}{K/m}\right)^2 + \left(\frac{c\omega}{K}\right)^2}}$$

Xs

$$\frac{X}{X_{st}} = \frac{1}{\sqrt{\left(1-\frac{\omega^2}{K/m}\right)^2 + \left(\frac{c\omega}{K}\right)^2}}$$

$$\frac{c\omega}{C_c K} \times C_c = \xi \cdot \frac{\omega}{K/m} \cdot 2\omega_n = 2\xi \cdot \frac{\omega}{\omega_n}$$

$$\frac{X}{X_{st}} = \frac{1}{\sqrt{\left[1-\frac{\omega^2}{\omega_n^2}\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$MF = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

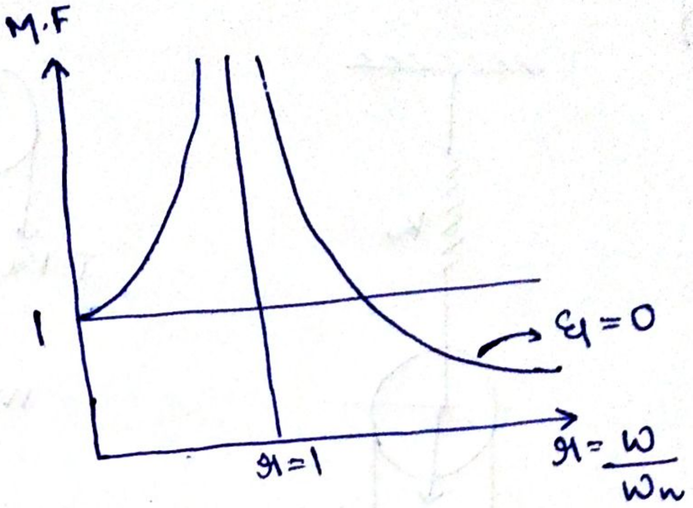
where,

frequency ratio 'r' $= \frac{\omega_f}{\omega_n}$

Graph - M.F v/s r

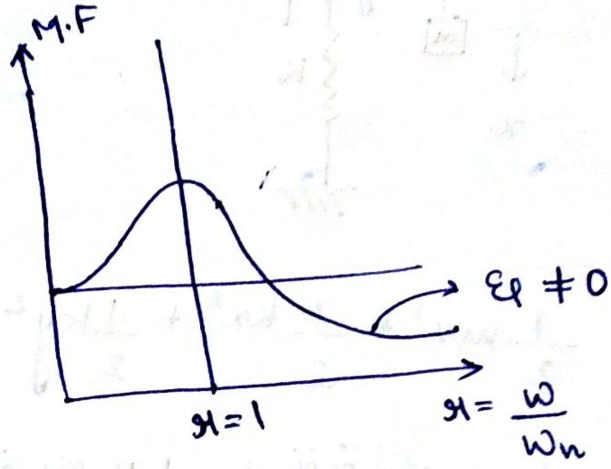
Case-1 Let $\xi = 0$

$$M.F = \frac{1}{|1-r^2|}$$




Case-2 - Let $\xi \neq 0$

$$M.F = \frac{1}{2\xi r} \quad |x| \sqrt{x^2}$$




$\phi$ v/s r

Case-1 :

$$\tan\phi = \frac{c\omega}{K - m\omega^2}$$

$$= \frac{2\xi r}{1 - r^2}$$