Multi Degree of freedom system – [WT Thomson]

(i) Derive DEOM ✓
(ii) Find natural frequency ✓
(iii) Find mode shape. ✓
(iv) Orthogonality properties of mode shape
(v) Modal matrix. ✓
(vi) Decoupling of DEOM
(vii) Modal space.

⇒ $[M]\{\dot{x}\} + [K]\{x\} = \{0\}$

$[M]$ → mass matrix

$[K]$ → stiffness matrix

$$\det\left(K - \omega^2 M\right) = 0$$

$$(\omega^2)^n + (\quad)(\omega^2)^{n-1} \ldots\ldots = 0$$

⇒

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\det\left(K - \omega^2 M\right) = 0 \quad , \quad \omega = 0,\ \sqrt{\frac{2k}{m}}$$

$$\frac{A_1}{A_2} = \frac{1}{1}$$

$$\Rightarrow \frac{A_{11}}{A_{21}} = \frac{k - m\omega^2}{k} = \frac{k}{k - m\omega^2} \quad \Big\} \text{ First mode shape.}$$

$$\{X_1\} = \begin{Bmatrix} A_{11} \\ A_{21} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \rightarrow \text{first mode shape.}$$

for 2nd mode shape,

$$\omega = \sqrt{\frac{2K}{M}}$$

$$X_2 = \begin{Bmatrix} A_{12} \\ A_{22} \end{Bmatrix} = \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} \rightarrow \text{second mode shape.}$$

$$\Rightarrow \left.\begin{aligned} \{X_i\}^T_{1\times2} [K]_{2\times2} \{X\}_{2\times1} &= 0 \quad i \neq j \\ \underset{1\times2}{\{X_i\}^T} \cdot \underset{2\times2}{[M]} \underset{2\times1}{\{X\}} &= 0 \quad i \neq j \end{aligned}\right\} \text{Orthogonality properties}$$

$$\Rightarrow \text{Modal matrix} = \underset{2\times2}{[P]} = \begin{bmatrix} \{X_1\} & \{X_2\} \end{bmatrix}$$

$$\underset{2\times1}{\{X_1\}} = \begin{Bmatrix} A_{11} \\ A_{21} \end{Bmatrix} \qquad \underset{2\times1}{\{X_2\}} = \begin{Bmatrix} A_{12} \\ A_{22} \end{Bmatrix}$$

$$[P] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$\Rightarrow$ <u>Decoupling</u> -

$$m\ddot{x}_1 + 0\ddot{x}_2 + kx_1 - kx_2 = 0$$

$$m\ddot{x}_2 + 0\ddot{x}_1 - kx_1 + kx_2 = 0$$

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix}$$

$$M_{11}\ddot{y}_1 + k_{11}y_1 = 0$$

$$M_{22}\ddot{y}_2 + k_{22}y_2 = 0$$

In general,

$$M\ddot{X} + KX = 0$$

$$\text{let } \underset{2\times1}{X} = \underset{2\times2}{P}\,\underset{2\times1}{Y}$$

$$\Rightarrow \underset{2\times2}{[M]}\,\underset{2\times2}{[P]}\,\underset{2\times1}{\ddot{Y}} + \underset{2\times2}{[K]}\,\underset{2\times2}{[P]}\,\underset{2\times1}{Y} = 0$$

Multiply above eq^n with (P^T).

$$\underbrace{P^T[M][P]}_{\begin{bmatrix} M_{11} & 0 \\ 0 & M_{22} \end{bmatrix}}\ddot{Y} + \underbrace{P^T[K][P]}_{\begin{bmatrix} K_{11} & 0 \\ 0 & K_{22} \end{bmatrix}}Y = 0$$

$$\Rightarrow M_{11} = X_1^T \cdot M X_1 \qquad K_{11} = X_1^T \cdot K \cdot X_1$$

$$M_{22} = X_2^T \cdot M \cdot X_2 \qquad K_{22} = X_2^T \cdot M \cdot X_2$$

$$M_{11}\ddot{y}_1 + K_{11}\cdot y_1 = 0 \quad , \quad \omega_1 = \sqrt{\frac{K_{11}}{M_{11}}}$$

$$M_{22}\cdot\ddot{y}_2 + K_{22}\cdot y_2 = 0 \quad , \quad \omega_2 = \sqrt{\frac{K_{22}}{M_{22}}}$$

$\Rightarrow$ Modal space —