(1) Lagrangian method :-

$$\mathcal{L} = T - V \quad [\text{Diff b/w KE and PE}]$$

(Lagrange) (K.E) (P.E)

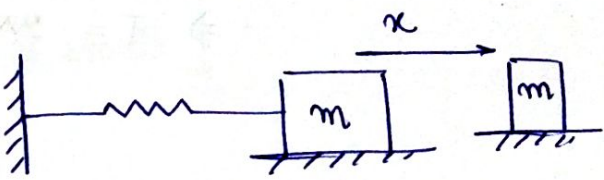
Linear KE $= \frac{1}{2}mv^2$
Rotational KE $= \frac{1}{2}I\omega^2$
Spring PE $= \frac{1}{2}kx^2$
Gravitational PE $= mgh$

$$\Rightarrow \boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = 0}$$

↳ Lagrangian Eqn.

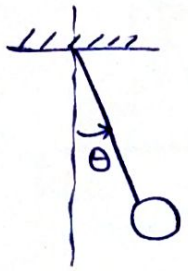
q : generalised coordinate.

$x \longrightarrow q.$



$$P.E = \frac{1}{2}kx^2$$

$$K.E = \frac{1}{2}m\dot{x}^2$$



$\theta \longrightarrow q.$

$\Rightarrow$ For spring-mass system -

$$\mathcal{L} = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$$

$$\frac{\partial \mathcal{L}}{\partial \dot{x}} = \frac{1}{2}m(2\dot{x}) - 0$$

$$\frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x}$$

$$\frac{\partial L}{\partial x} = 0 - \frac{k}{2}(2x) = kx$$

$$\Rightarrow \frac{d(m\dot{x})}{dt} - m\ddot{x} = 0$$

$$\Rightarrow \frac{d(m\dot{x})}{dt} - kx = 0$$

$$ma - kx = 0$$

$$\boxed{ma = kx}$$