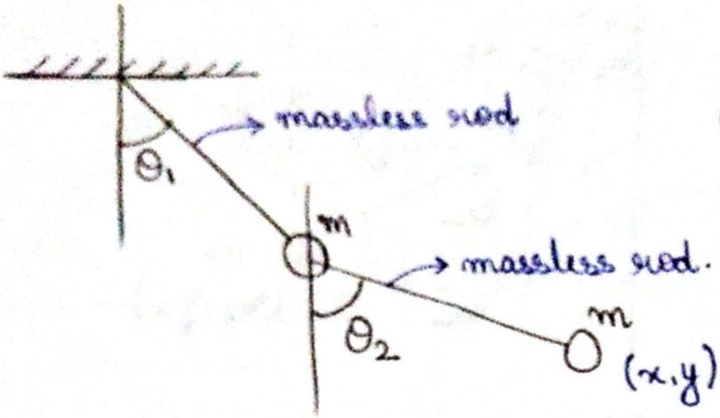
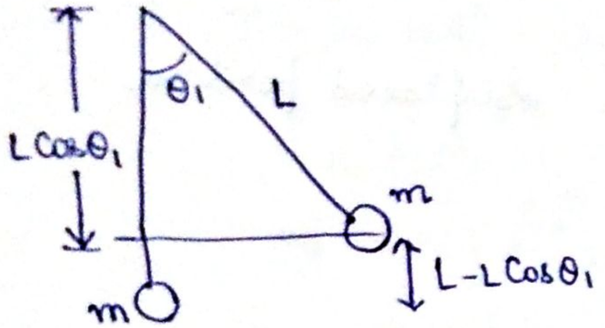
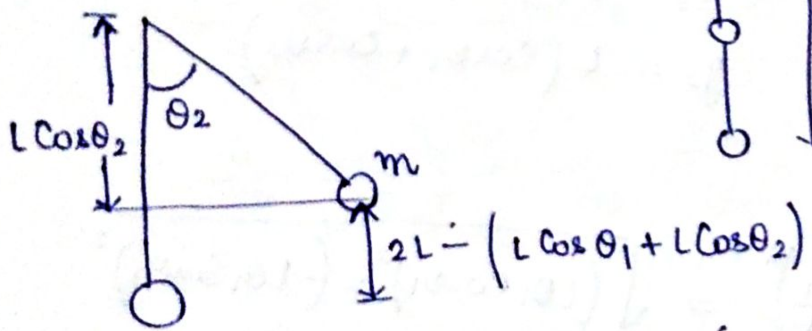
Double pendulum problem

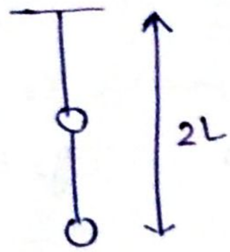


(a) Derive DEOM
(b) Natural frequency.



$$\Delta V_1 = mgL(1 - \cos\theta_1)$$





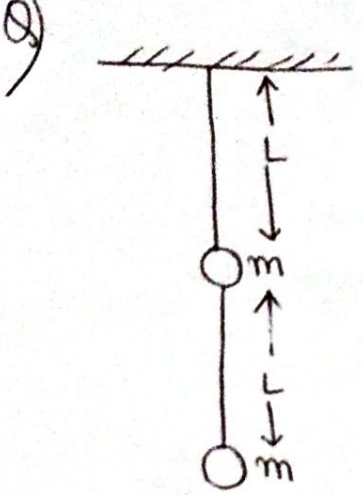
$$\Delta V_2 = mgL(2 - \cos\theta_1 - \cos\theta_2)$$

$$T_1 = \frac{1}{2} m (L\theta_1)^2$$

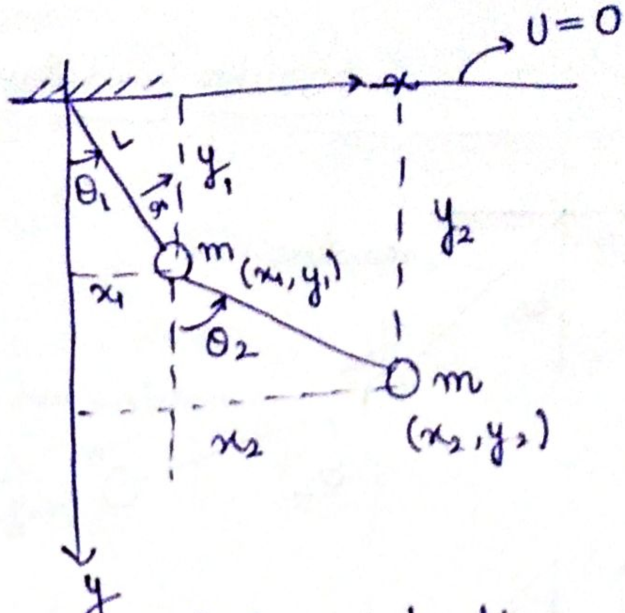
$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\frac{d\vec{r}}{dt} = \frac{dx}{dt}.\hat{i} + \frac{dy}{dt}\hat{j}$$

Q)



Equilibrium position



Displaced position.

$$\sin\theta_1 = \frac{p}{h} = \frac{x_1}{L}$$

$$\cos\theta_1 = \frac{b}{h} = \frac{y_1}{L}$$

$$\Rightarrow x_1 = L\sin\theta_1 \qquad x_2 = L(\sin\theta_1 + \sin\theta_2)$$

$$y_1 = L\cos\theta_1 \qquad y_2 = L(\cos\theta_1 + \cos\theta_2)$$

Velocity of 1^{st} mass -

$$V_1 = \sqrt{\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dy_1}{dt}\right)^2} = \sqrt{(L\dot\theta_1\cos\theta_1)^2 + (-L\dot\theta_1\sin\theta_1)^2}$$

$$= L\dot\theta_1$$

$$\frac{dx_1}{dt} = \frac{dx_1}{d\theta_1} \times \frac{d\theta_1}{dt}$$

$$\dot x_1 = \frac{d(L\sin\theta_1)}{d\theta_1} \times \frac{d\theta_1}{dt}$$

$$\dot x_1 = L\cos\theta_1 \cdot \dot\theta_1$$

$$\dot x_1 = L\dot\theta_1 \cdot \cos\theta_1$$

$$\frac{dy_1}{dt} = \frac{dy_1}{d\theta_1} \times \frac{d\theta_1}{dt}$$

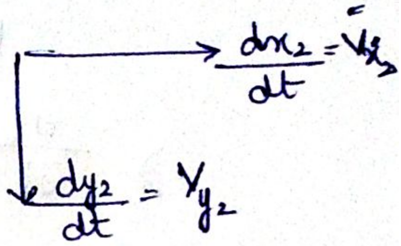
$$= \frac{d(L\cos\theta_1)}{d\theta_1} \cdot \dot\theta_1$$

$$= L(-\sin\theta_1) \cdot \dot\theta_1$$

$$= -L\dot\theta_1\sin\theta_1$$

Similarly,

Velocity of 2nd mass –



$$V_2 = \sqrt{\left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dy_2}{dt}\right)^2}$$

↳ velocity along y-dir.

↳ velocity along x-dir.

$$\Rightarrow \quad T = \frac{1}{2} m V_1^2 + \frac{1}{2} m V_2^2$$

$$V_1 = mgh_1 = -mgy_1$$

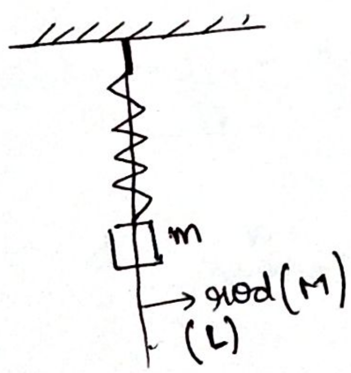
$$V_2 = mgh_2 = -mgy_2$$

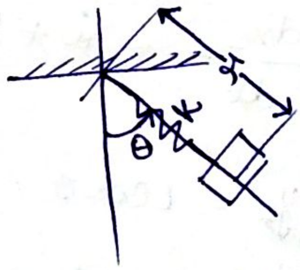
$$L = T - V$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \theta_1}\right) - \frac{\partial L}{\partial \theta_1} = 0 \quad \text{—— (I)}$$

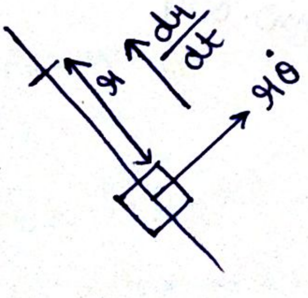
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \theta_2}\right) - \frac{\partial L}{\partial \theta_2} = 0 \quad \text{—— (II)}$$

Q)





K.E of rod $= \frac{1}{2} I_0 \omega^2 \quad \left[\omega = \frac{d\theta}{dt} = \dot{\theta}\right]$



$$T = \frac{1}{2} I_0 \omega^2 + \frac{1}{2} m \left(\dot{r}^2 + (r\dot{\theta})^2\right)$$

$$U_1 = \frac{1}{2} k (r - l_0)^2$$

$$U_2 = -mgy = -mg(r\cos\theta)$$

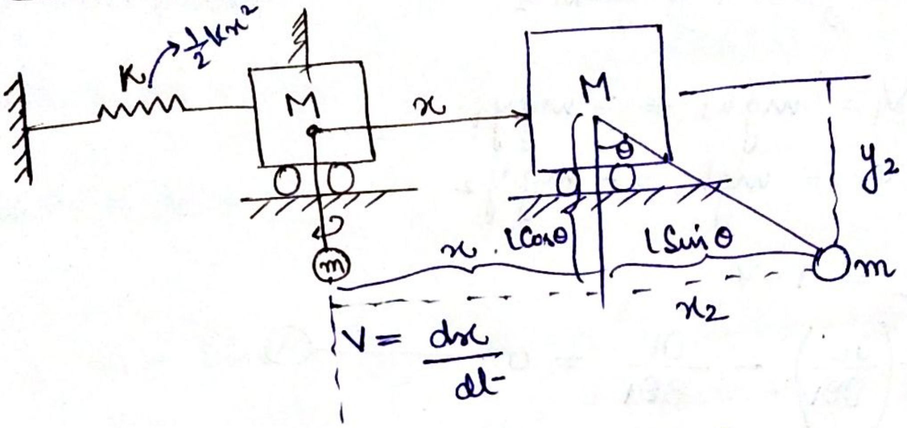
$$L = T - V$$

$$\Rightarrow \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$\Rightarrow \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$

CART- pendulum problem -

Q)



$$T_1 = \frac{1}{2}MV^2 = \frac{1}{2}M\dot{x}^2$$

$$x_2 = x + L\sin\theta$$

$$V_{x_2} = \frac{dx_2}{dt} = \dot{x} + L\cos\theta \cdot \dot{\theta}$$

$$y_2 = L\cos\theta$$

$$V_{y_2} = \frac{dy_2}{dt} = -L\sin\theta \cdot \dot{\theta}$$

$$V_2 = \sqrt{(V_{x_2})^2 + (V_{y_2})^2}$$

$$= \sqrt{(L\cos\theta)^2 + (-L\sin\theta \cdot \dot{\theta}^2)}$$

$$T_2 = \frac{1}{2}mV_2^2$$

$$U_2 = -mgy_2$$
$$= -mg(L\cos\theta)$$

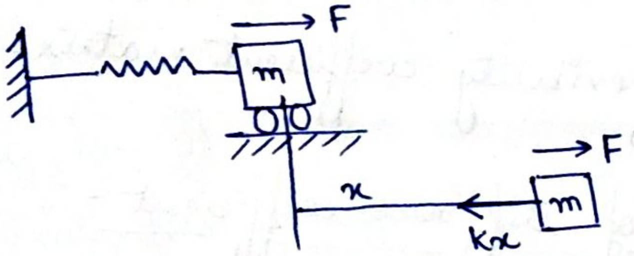
$$L = T - V$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial x}\right) - \frac{\partial L}{\partial x} = 0$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$

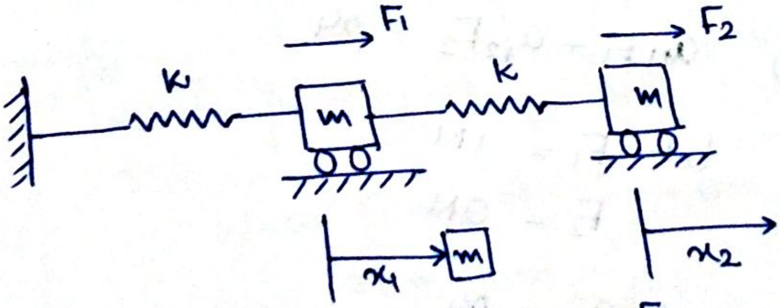
31/10/22

→ <u>flexibility coefficient matrix</u> –

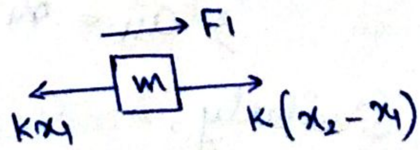


$$F = kx$$
$$x = \left(\frac{1}{K}\right)F$$

flexibility coeff. = $d = 1/k$



Assume, $x_2 > x_1$



$$F_1 + k(x_2 - x_1) = kx_1$$
$$F_1 = kx_1 + kx_1 - kx_2$$
$$F_1 = 2kx_1 - kx_2 \quad \text{——} \quad (1)$$