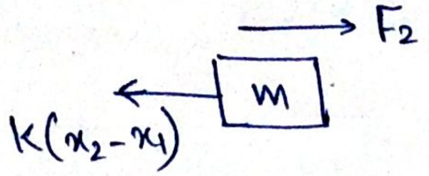


$$F_2 = -kx_1 + kx_2$$

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} 2k & -k \\ -k & +k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

$$\{F\} = [k]_{2\times 2}\{x\}$$

Multiplying both side by k^{-1}

$$[k^{-1}]\{F\} = [k^{-1}][k]_{2\times 2}\{x\}$$

$$\underbrace{[k^{-1}]}\{F\} = I\{x\}$$

↳ flexibility coefficient matrix. $\{a\}$

Physical Interpretation of stiffness coefficient -

$$[k^{-1}]\{F\} = \{x\}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

$\Rightarrow a_{11}F_1 + a_{12}F_2 = x_1$

Let $F_1 = 1N$

$F_2 = 0N$

$\Rightarrow a_{11} = x_1$

Similarly,

$\Rightarrow a_{21}F_1 + a_{22}F_2 = x_2$

$a_{21} = x_2$

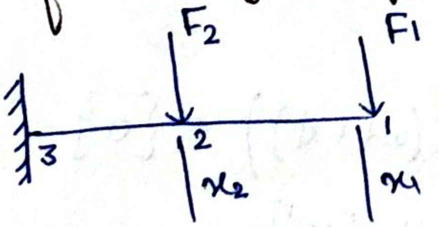
$\boxed{a_{11}}$ → It is displacement of first node when a unit force applied on the first node and remaining force that acts on other node should be zero.

$\boxed{a_{21}}$ → It is displacement of ~~first~~ second ~~second~~ node when a unit force is applied on ~~second~~ first node and remaining force that acts on other node should be zero.

$\boxed{a_{12}}$ → It is the displacement of ~~second~~ first node when a unit force is applied on the second node and remaining force that acts on the other node should be zero.

$\boxed{a_{22}}$ → It is the displacement of second node when a unit force is applied on the second node and remaining force that acts on the other node should be zero.

__flexibility coefficient in cantilever beam__ –



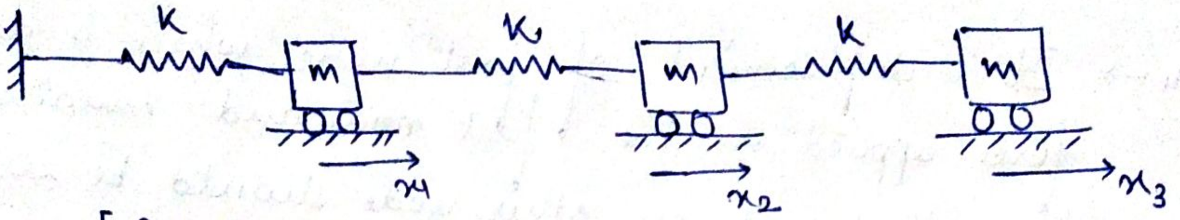
$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}$$

$\delta = PL^3/3EI$

When $F_1 = 1N$, $F_2 = 0N$

$x_1 = a_{11} = L^3/3EI$

(Dunkerely method)

⇒ 

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Apply unit force on block (I), (II) & (III) simultaneously.

Dunckerly method –

Duckerly method is an approximate method to find first natural frequency of a dynamical system.

$$\boxed{\frac{1}{\omega_1^2} = \frac{1}{\omega_{11}^2} + \frac{1}{\omega_{22}^2} + \cdots\cdots}$$

Imp

Derivation –

$$\underset{n\times n}{[m]}\underset{n\times 1}{\{\ddot{x}\}} + \underset{n\times n}{[k]}\underset{n\times 1}{\{x\}} = \underset{n\times 1}{\{0\}}$$

$$\Rightarrow \underset{n\times 1}{\{x\}} = \underset{n\times 1}{\{A\}} \sin(\omega t + \phi)$$

$$\Rightarrow \left[-\omega^2[m] + [k]\right]\{A\}\left(\sin(\omega t + \phi)\right) = \{0\}$$

Multiplying the above eq^n by $[k^{-1}]$ –

$$\left[-\omega^2[k^{-1}][m] + [I]\right]\left(\sin(\omega t + \phi)\right) = \{0\}$$

$$\left[[d][m] - \frac{1}{\omega}[I]\right]\left(\sin(\omega t + \phi)\right) = \{0\}$$

$$\det\left([a][m] - \frac{1}{\omega^2}[I]\right) = 0$$

[as the above eq^n was divided by ω, $d/\omega = a$]

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & a_{22} & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} m_{11} & \cdots & \\ \vdots & m_{22} & \\ & \cdots & m_{nn} \end{bmatrix} - \begin{bmatrix} 1/\omega^2 & & \\ & 1/\omega^2 & \\ & & 1/\omega^2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a_{11}m_{11} - \frac{1}{\omega^2} & a_{12}m_{22} & \cdots \\ \vdots & a_{22}m_{22} - \frac{1}{\omega^2} & \cdots \\ \vdots & & a_{nn} \cdot m_{nn} - \frac{1}{\omega^2} \end{bmatrix}$$

$$\Rightarrow \left(\frac{1}{\omega^2}\right)^n - \left(a_{11}m_{11} + a_{22}m_{22} + \cdots\cdots a_{nn} \cdot m_{nn}\right)\left(\frac{1}{\omega^2}\right)^{n-1} + C_{n-2}\left(\frac{1}{\omega^2}\right)^{n-2}$$

$$\Rightarrow \left(\frac{1}{\omega^2} - \frac{1}{\omega_1^2}\right)\left(\frac{1}{\omega^2} - \frac{1}{\omega_2^2}\right) \cdots\cdots \left(\frac{1}{\omega^2} - \frac{1}{\omega_n^2}\right) = 0$$

$$\Rightarrow \left(\frac{1}{\omega^2}\right)^n - \left(\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} + \cdots\cdots + \frac{1}{\omega_n^2}\right)\left(\frac{1}{\omega^2}\right)^{n-1}$$

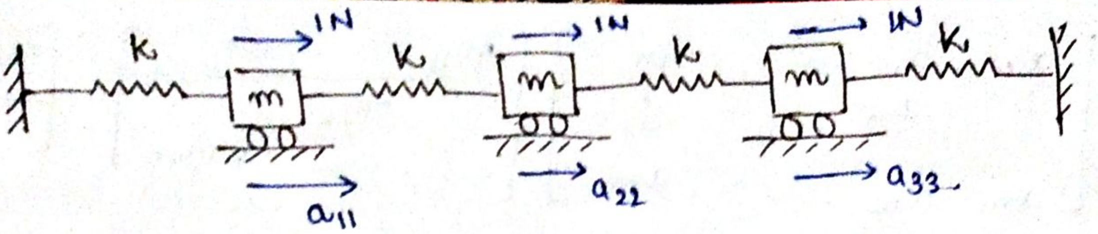
$$\Rightarrow \left(\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} + \cdots\cdots\right)$$

$$\frac{1}{\omega_1^2} >>> \underbrace{\left(\frac{1}{\omega_2^2} + \frac{1}{\omega_3^2} + \cdots\cdots \frac{1}{\omega_n^2}\right)}_{\text{This part is neglected.}}$$

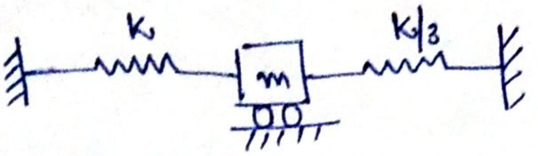
$$\Rightarrow \frac{1}{\omega_1^2} = a_{11}m_{11} + a_{22}m_{22} + \cdots\cdots + a_{nn} \cdot m_{nn}.$$

$$\Rightarrow \boxed{\frac{1}{\omega_1^2} = \sum_{i=1}^{n} a_{ii}\, m_{ii}}$$

Q)



$\Rightarrow$



$$k_{eq.} = k + \frac{k}{3}$$

$$k_{eq} = \frac{4k}{3}$$

$$\because \quad F = k_{eq.} x$$

$$1N = \frac{4k}{3} . x$$

$$x = a_{11} = \frac{3}{4k}.$$

When one mass of block is considered, other masses are considered to be zero.

$\Rightarrow$ Now, On block 2 -

$$F = 1 N.$$

$$k_{eq.} = \frac{k}{2} + \frac{k}{2} = k.$$

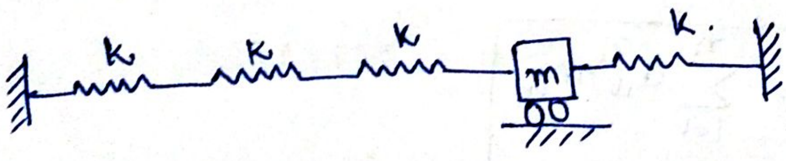
$$F = k_{eq.} \times x.$$

$$1 = k \times x$$

$$x = a_{22} = 1/k.$$

$\Rightarrow$ Similarly, on block 3 -

$$F = 1N$$



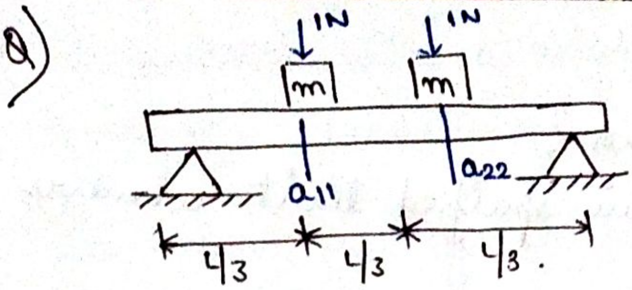
$$k_{eq.} = k + \frac{k}{3} = 4k/3.$$

$$F = k_{eq.} x$$

$$1 = 4k/3 \times x$$

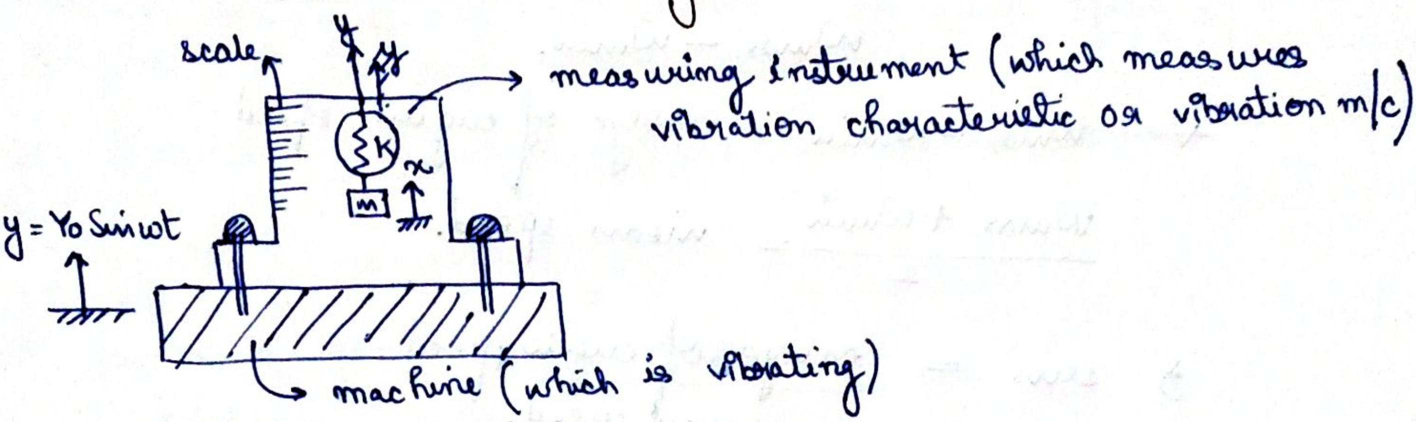
$$a_{33.} = x = 3/4k.$$

Q)



$$\Rightarrow \quad \frac{1}{\omega_1^2} = a_{11} m_{11} + a_{22} m_{22}$$

Vibration measuring instruments



$$a_{max} = -Y_0 . \omega^2$$

If $x > y$, the spring will be compressed.

$$\downarrow k(x-y)$$
$$\boxed{m}$$

$$\Sigma F_x = ma.$$
$$-k(x-y) = m\ddot{x}$$
$$m\ddot{x} + k(x-y) = 0$$
$$\text{Let } (x-y) = z.$$
$$\Rightarrow m\ddot{x} + kz = 0$$
$$\Rightarrow m\ddot{z} + kz = m\omega^2 Y_0 \cdot \sin \omega t$$
$$z = z_0 \sin(\omega t + \phi)$$
$$\text{where, } z_0 = \frac{m\omega^2 Y_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

$$(x-y) = z$$
$$\ddot{x} - \ddot{y} = \ddot{z}$$
$$\ddot{x} = \ddot{z} + \ddot{y}$$
$$= \ddot{z} - Y_0 \omega^2 \sin \omega t$$

→ Article 16 Governor

→ What is the function of governor?

Maintain speed of an engine within specified limit whenever there is variation of load.

→ Distinguish b/w governor & flywheel.

→ Sensitiveness of governor -

$$\text{Sensitiveness} = \frac{\frac{\omega_{max} + \omega_{min}}{2}}{\omega_{max.} - \omega_{min.}}$$

∵ $\omega_{max} - \omega_{min}$ = range of engine speed.

$\frac{\omega_{max.} + \omega_{min}}{2}$ = mean speed.

$$\Rightarrow \text{sen.} = \frac{\text{range of engine speed}}{\text{mean speed.}}$$

- A governor is said to be sensitive when it readily ~~ss~~ response to a small change of speed. The movement of sleeve for a fractional change of speed is the measure of sensitiveness.

→ Hunting - Sensitivity of governor is a desired quantity, however if a governor is too sensitive, it may fluctuate continuously because when the load on the engine falls, the sleeve rises apparently to a maximum position, this shut off the fuel supply to ~~e~~ the extent to effect falling speed.

→ Isochronism - When the range of speed for governor is zero i.e. $\omega_{max} = \omega_{min}$.

- A governor with a range of speed zero is c/d <u>isochronous governor</u>. (sensitivity $= \infty$)

This means that for all position of sleeve ~~or~~ or wall, the governor has the same speed.

- A pendulum-type governor can't possibly be isochronous.

→ <u>Stability</u> – A governor is said to be stable if it bring the speed of engine to the required value. The ball masses occupy a definite position for each speed of the engine within the working range.

* Stability & sensitiveness are two opposite terms.

→ Effect of governor

→ Power of governor

→ Controlling force