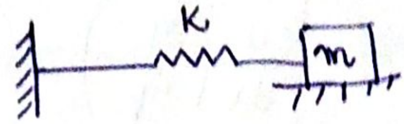
Q) A spring-mass system has natural period of 0.25 sec. What will be the new period, if spring constant is -

(i) Increased by 60 %. 0.198 sec

(ii) Decreased by 30%. 0.298 sec.



$$T = \frac{2\pi}{\omega}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{m}{k}}.$$

$$T = 0.25\ sec = 2\pi\sqrt{\frac{m}{k}}.$$

$\cancel{T}$ Now, $k' = 1.6k$.

(i)

$$T' = 0.25\ sec = 2\pi\sqrt{\frac{m}{1.6k}}$$

$$= 2\pi\sqrt{\frac{m}{k}} \cdot \sqrt{\frac{1}{1.6}}$$

$$\cancel{= 0.25\times}\ 0.198\ sec.$$

(ii)

$$k' = k - 0.3k = 0.7k$$

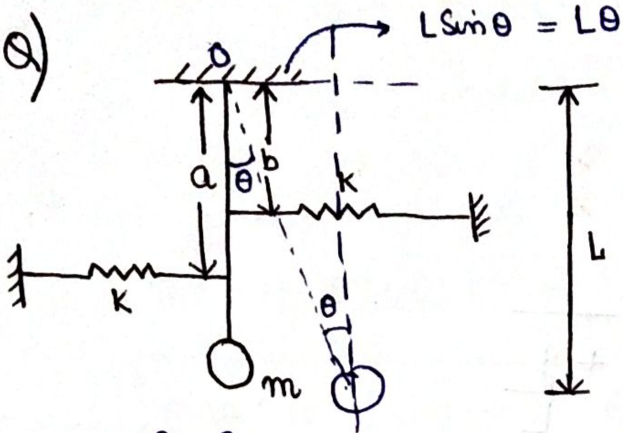
$$T' = 2\pi\sqrt{\frac{m}{0.7k}}$$

$$= 2\pi\sqrt{\frac{m}{k}} \times \sqrt{\frac{1}{0.7}}$$

$$= 0.25 \times 1.194$$

$$= 0.298\ sec.$$

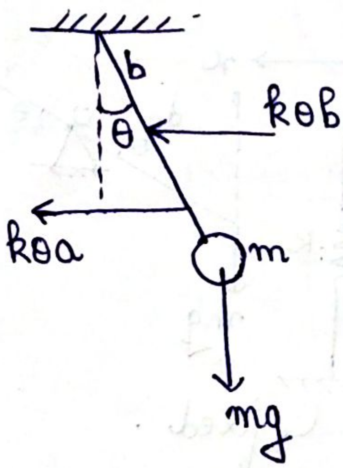
Q)

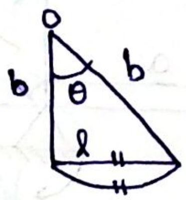


Find natural frequency (ω)?

Degree of freedom = 1

Displace pendulum by θ [$\theta \approx$ small]

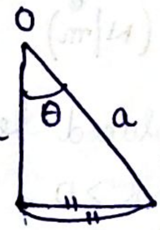




$\theta = \frac{l}{R}$

$\theta = \frac{l}{b}$

$l = \theta b$ (Compression)

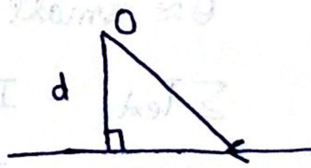


$l = a\theta$ (Expansion)

Force = stiffness × displacement

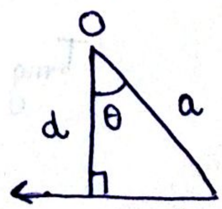
$= k\theta b$.

For torque: $\tau = \vec{r} \times \vec{F}$



$\tau = k\theta b \times d$

$\tau = k\theta b^2$ (−) ; $b\cos\theta \approx b$



$d = a\cos\theta$
$\approx a$

$\tau = k\theta a^2$ ($\mp$)

Due to mg: $\sin\theta \approx k\theta$

$\tau = -mgL\theta$

$$\Rightarrow \sum T_0 = I\alpha$$

$$-kb^2\theta \mp ka^2\theta - mgL\theta = (mL^2)\cdot\alpha$$

$$-k(a^2+b^2)\theta - mgL\theta = mL^2\alpha$$

$$\alpha = -\underbrace{\left[\frac{k(a^2+b^2) + mg}{mL^2}\right]}_{\text{+ve number}}\theta$$

$$\alpha = -\omega^2\theta$$

+ve number
⇓
can be written as square of any real No.

$$\because \quad \alpha = \frac{d\omega}{dt}$$

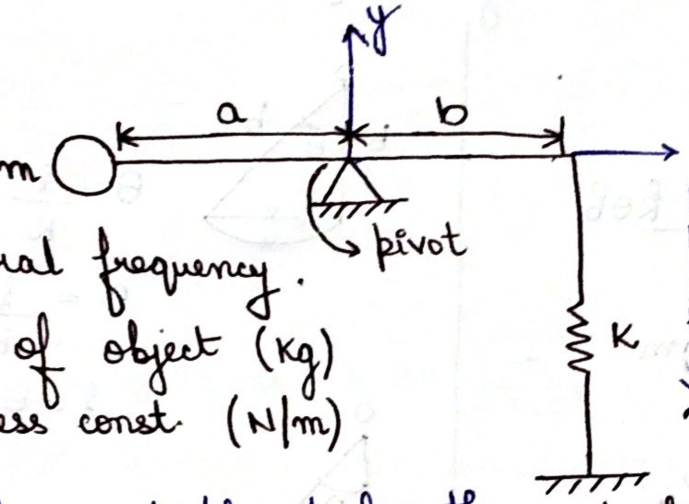
$$\omega = \frac{d\theta}{dt}$$

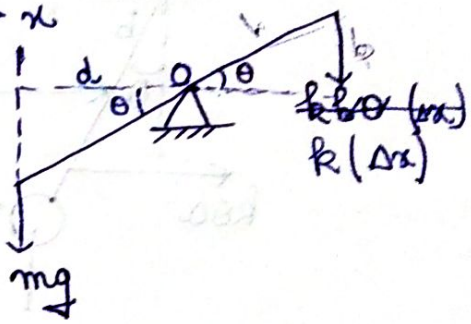
$$\alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}$$

$$\Rightarrow \quad \ddot{\theta} = -\omega^2\theta.$$

$$\forall \quad \omega = \sqrt{\frac{k}{m}(a^2+b^2) + \frac{g}{L}}$$

Q)





Find natural frequency.

m = mass of object (Kg)

K = stiffness const. (N/m).

Let the spring displaced length be Δx. $\forall \ \Delta x > 0$,

$$\therefore \quad F_s = k(\Delta x)$$

$\theta \approx$ small.

$\forall$ I : mass moment of inertia

α : angular accl^n.

$$= \frac{d^2\theta}{dt^2} = \ddot{\theta}$$

$$\Sigma T_{ext.} = I\alpha$$

$$\Rightarrow \Sigma T_{ext} = I_0 \cdot \ddot{\theta}$$

Torque due to mg –

$$T_{mg} = \cancel{\vec{F} \times \vec{d}} \quad \vec{r} \times \vec{F}$$

$$= mg \times a\cos\theta.$$

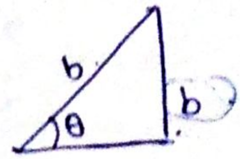
$$= mga \ (+)$$

Torque due to spring -

$$T_s = k\cdot(\Delta x)\times d$$

$$= k(\Delta x)\times b\cos\theta$$

$$= k(\Delta x)\, b\cdot(-)$$



$\sin\theta = \frac{p}{b}$

$p = b\sin\theta$

$p = b$

$\Rightarrow$ $$\Sigma T_{ext} = I_o\cdot\alpha$$

$$mga - k(\Delta x)\cdot b = I_\theta\cdot\ddot{\theta}$$

$$\boxed{mga = I_o\cdot\ddot{\theta} + kb^2\theta}$$ "DEOM"

$$\frac{mga}{I_o} = \ddot{\theta} + \underbrace{\frac{kb^2}{I_o}}_{\omega^2}\dot{\theta}$$

$$\omega^2 = \frac{kb}{ma^2}$$

$$\omega = \sqrt{\frac{kb}{ma^2}}$$

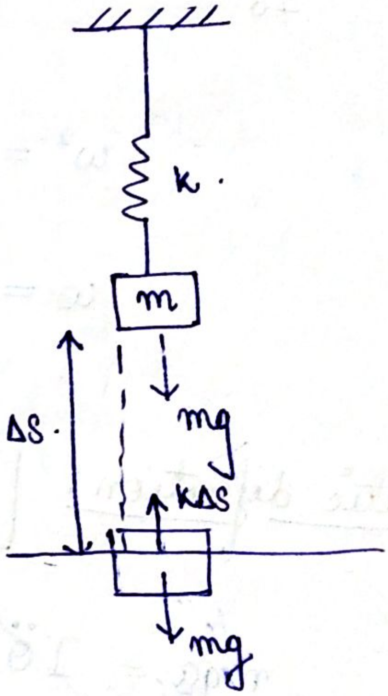
Static deflection: $\boxed{\text{'}\theta\text{' for which } T_{net} = 0}$

$$mga = \cancelto{0}{I\ddot{\theta}} + kb^2\theta$$

$$\underbrace{\theta_s}_{\text{static position}} = \frac{mga}{kb^2}$$

Static deflection -

When spring is at its initial position, it isn't the mean position.



$$k \cdot \Delta S = mg$$

$$\Delta S = \frac{mg}{k}$$

↳ static deflection.

$$\frac{\Delta S}{g} = \frac{m}{k} \quad \text{or} \quad \frac{k}{m} = \frac{g}{\Delta S}.$$

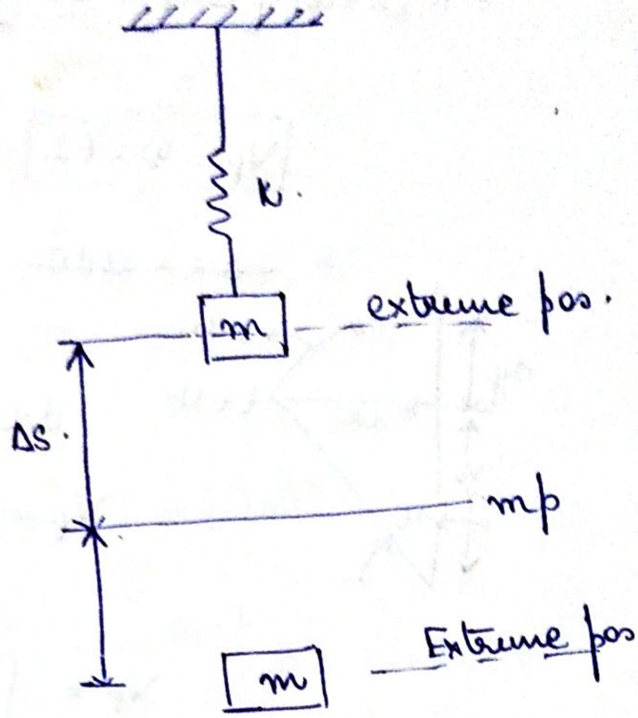
Therefore,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\Delta S}}.$$

When $t=0$, $x(0)=0$

then,

ΔS = amplitude.



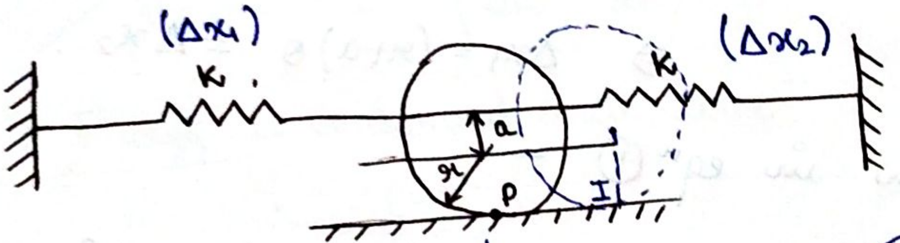
To determine the value of amplitude and ~~its~~ phase angle, initial conditions of the system must be known.

$$\vec{a} = -\omega^2 x \rightarrow \text{Amplitude}$$

$$\boxed{g = a = -\omega^2 \Delta S}$$

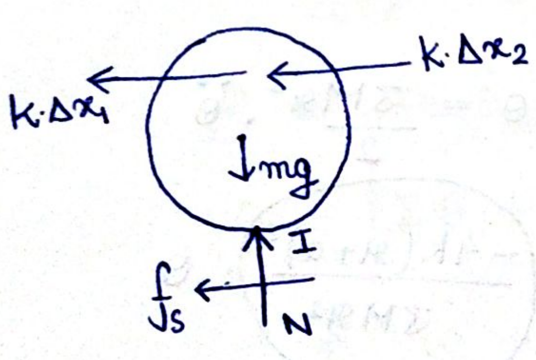
$$|g| = |-\omega^2 \Delta S|$$

Q)



Find ω?

$$|\Delta x_1| = |\Delta x_2|$$

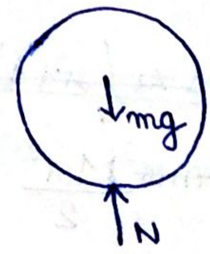


$a_y = 0$

$ma_y = \Sigma F_y$

$0 = N - mg$

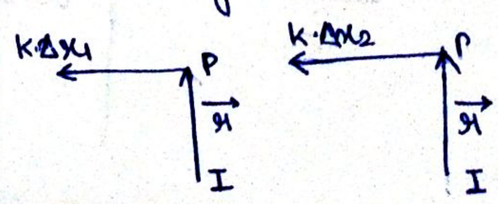
$$\boxed{N = mg}$$



$$\Rightarrow \Sigma \tau_I = I \cdot \alpha$$

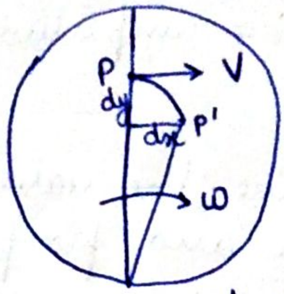
$$-k \cdot \Delta x_1 (r+a) - k \cdot \Delta x_2 (r+a) = I \cdot \ddot{\theta}$$

$$\tau_N = \tau_{mg} = \tau_{fs} = 0$$



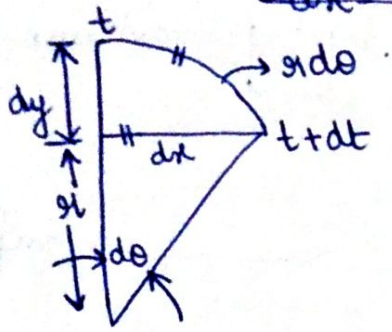
$$-k(r+a)\left[\Delta x_1+\Delta x_2\right] = I\ddot{\theta} \quad \text{———} \; (1)$$

$$\boxed{V_p = \omega \times PI}$$



I → Instantaneous center

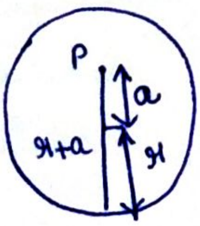
~~dx = r dθ~~



$$dx = r\,d\theta$$

$$V_p = \frac{dx}{\cancel{dt}} = \frac{d\theta}{\cancel{dt}}\cdot R$$

$$V_p = \int dx = \int d\theta \cdot R$$



X : displacement in point P.

$$X = R\theta$$

$$X = (r+a)\theta$$

$$\Rightarrow \Delta x_1 = (r+a)\theta = \Delta x_2 .$$

Now in eqn (1) —

$$\cancel{-k\,\Delta x_1 (r+a) - k} \; -k(r+a)^2\theta - k(r+a)^2\theta = I\cdot\ddot{\theta}$$

$$I_c = \cancel{mr}\,\frac{Mr^2}{2}$$

By ||el axis theorem -

$$I_I = I_{CM} + Md^2$$

$$= \frac{Mr^2}{2} + Mr^2$$

$$= \frac{3Mr^2}{2}.$$

$$\Rightarrow -2k(r+a)^2\theta = I\ddot{\theta}$$

$$\Rightarrow -2k(r+a)^2\theta = \frac{3Mr^2}{2}\cdot\ddot{\theta}$$

$$\ddot{\theta} = \underbrace{\left(\frac{-4k(r+a)^2}{3Mr^2}\right)}_{\omega^2}\cdot\theta$$

where,

$$\omega = \sqrt{\frac{-4k(r+a)^2}{3Mr^2}}.$$