ENERGY METHOD -

Total Energy = Total K.E + Total P.E

$$E_T = K.E_I + P.E_I.$$

For previous question -

$$E_T = \frac{1}{2} I\omega^2 + \frac{1}{2} k.(\Delta x)^2 + \frac{1}{2} k(-\Delta x)^2$$

$$= \frac{1}{2} I\omega^2 + k.(\Delta x)^2 \qquad \Big| \; \Delta x = (r+a)\theta$$

$$= \frac{1}{2} I\omega^2 + k(r+a)^2.\theta^2$$

~~dE_T~~ Diff. wrt time -

$$\cancel{\frac{dE_T}{dt}}^{\,0} = \frac{1}{2} . I\left(\frac{d\omega^2}{d\omega} . \frac{d\omega}{dt}\right) + \frac{(r+a)^2}{2} . k\left(\frac{d\theta^2}{d\theta} . \frac{d\theta}{dt}\right)$$

$\because$ Energy is conserved with time,

$$\therefore \frac{dE_T}{dt} = 0.$$

$$\Rightarrow 0 = \frac{1}{\cancel{2}} I.\cancel{2}\omega.\ddot{\theta} + \frac{(r+a)^2}{\cancel{2}}.k(\cancel{2}\theta).\omega.$$

$$0 = I\omega.\ddot{\theta} + (r+a)^2.k\theta.\omega.$$

$$0 = \left[I\ddot{\theta} + (r+a)^2 k\theta\right].\omega$$

$$\Rightarrow \omega \neq 0$$

$$\Rightarrow I\ddot{\theta} + (r+a)^2.k\theta = 0$$

$$\Rightarrow \theta + \left(\frac{(r+a)^2 K}{I}\right) \cdot \theta = 0$$

ω^2.

$$\omega = \sqrt{\frac{(r+a)^2 K}{I}} = \sqrt{\frac{(r+a)^2 \cdot K}{\frac{3}{2} M r^2}} = \sqrt{\frac{2(r+a)^2 K}{3 M r^2}}$$

a)