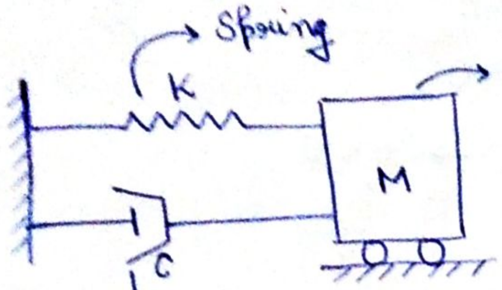
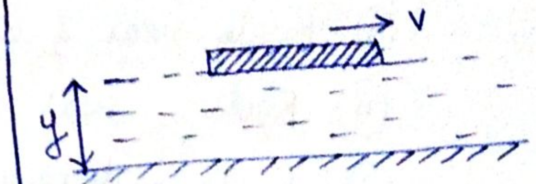
# Single Degree of Freedom Spring mass Damper System –




$T = \mu \cdot \frac{du}{dy}$

$F = T \times A$

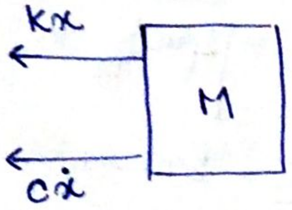
$= \mu \cdot A \cdot \frac{du}{dy}$

Damper
↓
Used to dissipate energy

$\boxed{F = CV}$

C → damping coeff. $= \frac{\mu \cdot A}{y}$ $\left(\frac{N \cdot s}{m}\right)$

F → Damper force




$\Sigma F_x = m\vec{a_x}$

$-kx - c\dot{x} = m\ddot{x}$

$\boxed{m\ddot{x} + c\dot{x} + kx = 0}$ → Second order ordinary homogenous diff. eqn.

$c > 0$

↳ EOM of SDOF spring mass damper system

∴ In case of undamped, $c = 0$

" " " Damped, $c \neq 0$

Let $x = e^{st}$

$\dot{x} = \cancel{s.e^{st}} \; \frac{de^{st}}{d(st)} \cdot \frac{d(st)}{dt}$

$\dot{x} = s.e^{st}$

$\ddot{x} = s^2.e^{st}$

- Putting the values of $x$, $\ddot{x}$ & $\dot{x}$ –

$\underbrace{(ms^2 + cs + k)}_{s_1 \,\&\, s_2} e^{st} = 0$

$x(t) = Ae^{s_1 t} + B.e^{s_2 t}$.

$\Rightarrow ms^2 + cs + k = 0$

$x = \frac{-b}{2a} \pm \frac{D}{2a}$.

(a) Roots - real & unequal $\rightarrow D > 0$

(b) Roots - real & equal $\rightarrow D = 0$

(c) Roots - Imaginary & unequal $\rightarrow D < 0$

$\Rightarrow$ If $D > 0$

$b^2 - 4ac > 0$

$\Rightarrow c^2 - 4mk > 0$

$\boxed{c^2 > 4km} \rightarrow$ overdamped motion $(\xi > 1)$

$\Rightarrow$ If $D = 0$

$b^2 - 4ac = 0$

$c^2 = 4mk$

$\boxed{c = \sqrt{4mk}} \rightarrow$ critically damped.

$(\xi = 1)$

* We know

$\omega_n = \sqrt{\frac{k}{m}}$

$c = \sqrt{4mk}$

$= \sqrt{4 \frac{k}{m} . m^2}$

$c = 2m\omega_n$

$\Rightarrow$ If $D < 0$

$c^2 - 4km < 0$

$\boxed{c < \sqrt{4km}} \rightarrow$ underdamped motion.

$(\xi < 1)$

# $\boxed{C_c = 2m.\omega_n}$

$\hookrightarrow$ critically damping coefficient

Damping factor $(\xi)$ - Ratio ~~of~~

$$\boxed{\xi = \frac{c}{C_c} = \frac{c}{2m\omega_n}} = \frac{2\sqrt{mk}}{2m\omega_n}$$

$$= \frac{\sqrt{\frac{k}{m}}}{\omega_n}$$

→ In overdamped condition

$$c^2 > 4km$$

$$c^2 > c_c^2$$

$$c > c_c$$

$$\frac{c}{c_c} > 1$$

$$\boxed{\xi > 1}$$

→ In critically damping condition -

$$\boxed{\xi = 1}$$

→ In underdamped condition -

$$\boxed{\xi < 1}$$

⇒ $m\ddot{x} + c\dot{x} + kx = 0$

$$\ddot{x} + \left(\frac{c}{m}\right)\cdot\dot{x} + \left(\frac{k}{m}\right)x = 0$$

$$\boxed{\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2\cdot x = 0}$$

$$\xi = \frac{c}{2m\omega_n}$$

$$\frac{c}{m} = 2\xi\omega_n$$