Vibration

- Free vibration
  - Undamped
    $\left(m\ddot{x} + Kx = 0 \atop c = 0 \right)$
  - Damped
    $\left(m\ddot{x} + c\dot{x} + kx = 0 \right)$
    - Overdamped
      $\left[x(t) = Ae^{S_1 t} + Be^{S_2 t} \right]$
    - Critically damped
      $\left[(A+Bt)e^{-c/2m\, t} \right]$
    - Underdamped
      $\left[x(t) = X_0 e^{-\xi \omega_n t} \sin(\omega_d t + \phi) \right]$
- Forced vibration

Underdamped free vibration –

$\Rightarrow$ $$ms^2 + cs + K = 0$$

$$D < 0 \quad ; \quad c < c_c$$

$$S_{1,2} = \alpha \pm i\beta$$

$$\alpha = \frac{-c}{2m} \quad , \quad \beta = \sqrt{\frac{4mk - c^2}{4m^2}}$$

$$= \sqrt{\frac{K}{m} - \left(\frac{c}{2m}\right)^2} = \sqrt{\omega_n^2 - \xi^2 \cdot \omega_n^2}$$

$$\beta = \omega_n \sqrt{1 - \xi^2}$$

$\Rightarrow$ $$x(t) = Ae^{S_1 t} + Be^{S_2 t}$$

$$\boxed{x(t) = X_0 \cdot e^{-\xi \omega_n t} \sin(\omega_d t + \phi)}$$

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

ω_d → rad/s ω_n → rad/s.

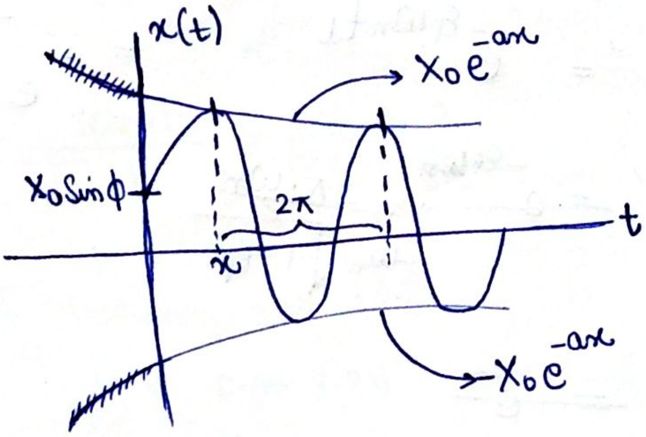
where,

ω_n = Underdamped Natural frequency

ω_d = Damped natural frequency.

→ Graph -

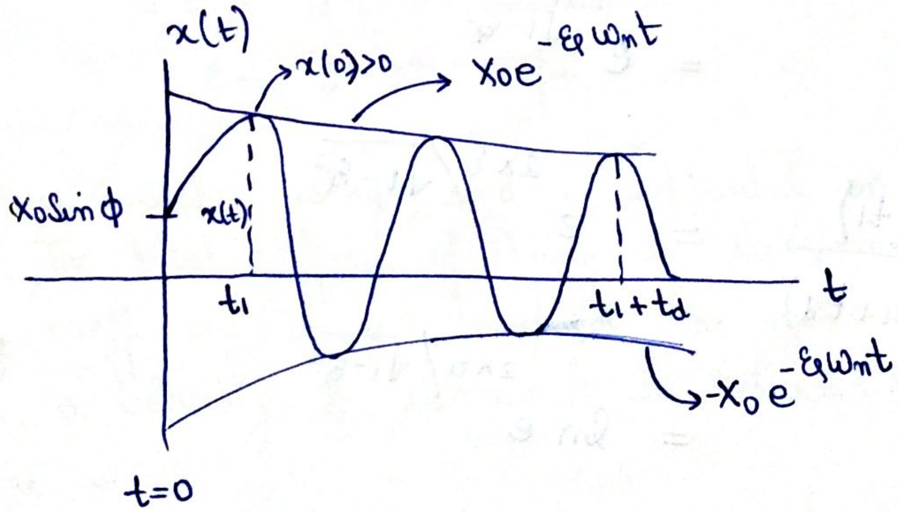
$$f(x) = [X_0 e^{-ax}][\sin(bx+\phi)]$$



$x = 2n\pi + \frac{\pi}{2}$

$T = \frac{2\pi}{b}$

⇒



$\Rightarrow \sin(\omega_d t_1 + \phi) = 1$

$\Rightarrow \sin\left(\omega_d\left(t_1 + \frac{2\pi}{\omega_d}\right) + \phi\right)$

$\Rightarrow \sin(\omega_d t_1 + \phi + 2\pi)$

$\Rightarrow \sin(2\pi + x) = \sin x$

$$\Rightarrow \frac{x(t_1+t_d)}{x(t_1)} = \frac{X_0 e^{-\xi\omega_n(t_1+t_d)} \cdot \sin(\omega_d(t_1+t_d)+\phi)}{X_0 e^{-\xi\omega_n t_1} \cdot \sin(\omega_d t_1+\phi)}$$

$$\because \quad t_d = \frac{2\pi}{\omega_d}$$

$$= e^{-\xi\omega_n t_d} = e^{-\xi\omega_n \cdot \frac{2\pi}{\omega_n\sqrt{1-\xi^2}}}$$

$$= e^{-\frac{\xi 2\pi}{\sqrt{1-\xi^2}}}$$

$$\frac{x(t_1)}{x(t_1+t_d)} = e^{2\pi\xi/\sqrt{1-\xi^2}}$$

$$= \ln e^{2\pi\xi/\sqrt{1-\xi^2}}$$

$$\boxed{\frac{x(t_1)}{x(t_1+t_d)} = \frac{2\pi\xi}{\sqrt{1-\xi^2}}}$$

LOGARITHMIC DECREMENT !