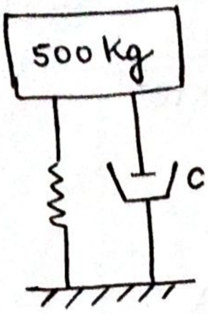
Q)

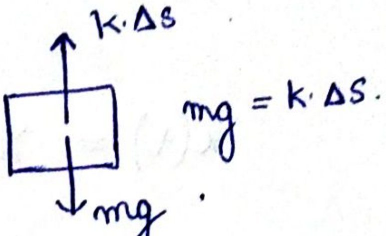


$\Delta s = 1.5 mm$.

$C = C_c$

$= 2m\omega_n = \sqrt{4km}$

$\Delta S = \frac{mg}{K}$.



$\frac{500 \times 10}{k} = 1.5 \times 10^{-3}$.

$K = \frac{\cancel{50}\ \cancel{5} \times 10^{3}}{3\ \cancel{15} \times 10^{-4}}$

$K = \cancel{0.33}\ 3.34$

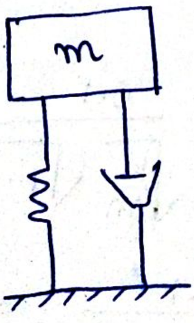
$\Rightarrow c = \sqrt{4 \times 3.34 \times 500}$

$C = 81.731\ Ns/m$.

Q) A machine weight 15 kg and is supported on spring and damper. The total stiffness of string is 10 N/mm and damping coefficient is 0.2 NS/mm. The system is initially at rest, a velocity of 100 mm/s. is imparted to the mass. Determine –

i) Displacement and velocity of mass as funct^n of time.

ii) The displacement and velocity of mass after 0.5 sec.



$m = 15\ kg$

$k = 10\ N/mm$

$c = 0.2\ Ns/mm$

$\dot{x}(0) = 100\ mm/s. = 0.1 m/s$.

$x(0) = 0$

~~$\Rightarrow \sqrt{4km} = \sqrt{4\times10\times15} =$~~

$$C_c = \sqrt{4km} = \sqrt{4\times10\times10^3\times0.2\times10^3}$$

$$\xi = \frac{c}{C_c} = 0.258$$

$$\because \xi < 1 \quad (\text{Underdamped})$$

$$x(t) = X_0 e^{-\xi\omega_n t} \sin(\omega_d t + \phi)$$

$$\because \dot{x}(0) = 100 \text{ mm/s}$$

$$x(0) = 0$$

$$\Rightarrow \quad x(0) = X_0 \sin\phi = 0$$

$$X_0 \neq 0$$

$$\Rightarrow \sin\phi = 0$$

$$\phi = 2n\pi$$

$$\underline{\phi = 0}, 2\pi, 4\pi \ldots\ldots$$

| Take $\phi = 0$

$$\Rightarrow \dot{x}(t) = X_0 \cdot e^{-\xi\omega_n t}(-\xi\omega_n)\cdot\sin(\omega_d t) + X_0 e^{-\xi\omega_n t}\cdot\cos(\omega_d t)\cdot\omega_d.$$

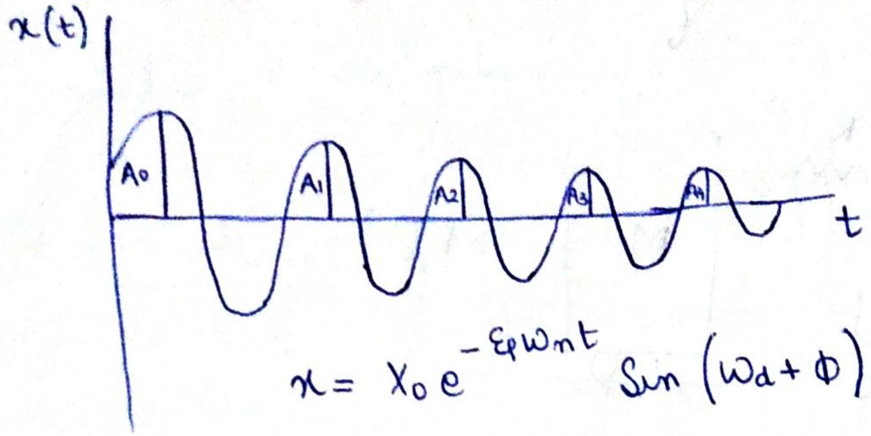
$$\dot{x}(0) = 0.1 \text{m/s}.$$

$$\Rightarrow \quad 0.1 = X_0(-\xi\omega_n)\times0 + X_0\cdot\omega_d.$$

$$X_0 = \frac{0.1}{\omega_d} = \frac{0.1}{\omega_n\sqrt{1-\xi^2}} = \frac{0.1}{\sqrt{\frac{K}{m}}\cdot\sqrt{1-\xi^2}}$$

$$X_0 = \frac{0.1}{\sqrt{\frac{10^{-2}}{15}}\cdot\sqrt{1-(0.258)^2}} = \frac{0.1}{0.26\times0.966} = \underline{\underline{0.398}}$$

Logrithmic Decrement -



$$x = X_0 e^{-\xi\omega_n t} \sin(\omega_d + \phi)$$

$$\frac{A_0}{A_1} = e^{2\pi\xi/\sqrt{1-\xi^2}} = \frac{A_1}{A_2} = \frac{A_2}{A_3} = \frac{A_3}{A_4} = \cdots\cdots = \frac{A_n}{A_{n\pm1}}$$

$$\ln\frac{A_0}{A_4} = \ln\left[\frac{A_0}{A_1}\times\frac{A_1}{A_2}\times\frac{A_2}{A_3}\times\frac{A_3}{A_4}\right] \quad \Bigg| \because GP$$

$$= 4\ln\frac{A_0}{A_1}$$

$$= \frac{4\cdot2\pi\xi}{\sqrt{1-\xi^2}}$$