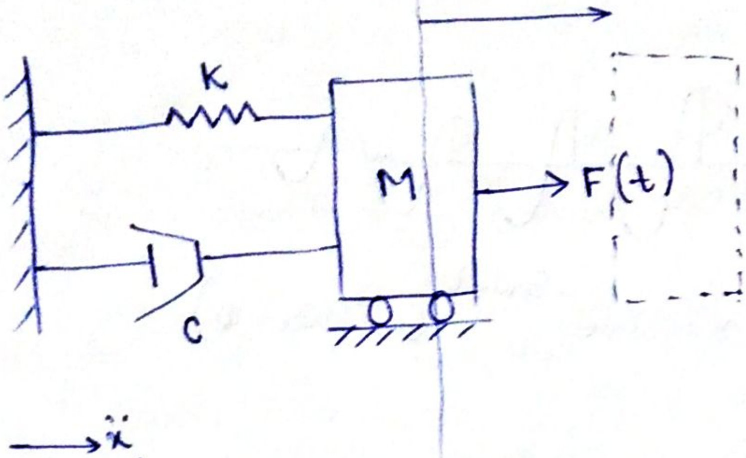
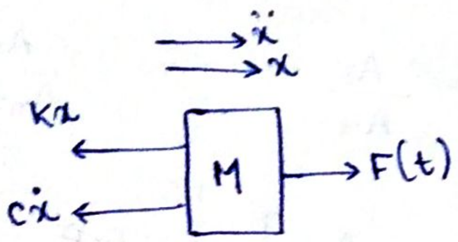
Forced Vibration of SDOF spring + mass + damper system





$$M\ddot{x} = F(t) - kx - c\dot{x}$$

$$\boxed{M\ddot{x} + c\dot{x} + kx = F(t)} \text{ EOM.}$$

↳ Non-homogenous Second-order differential eq^n.

D-Alembert's principle

Let $F(t) = F_0 \sin \omega t$ or $F_0 \cos \omega t$

where, $\omega \rightarrow$ exication frequency.

$$\Rightarrow M\ddot{x} + c\dot{x} + kx = F_0(\sin \omega t)$$

$$x(t) = x_c(t) + x_{ss}(t)$$

↳ steady state sol^n

↳ complementary sol^n.

$$x_{ss} = X_0 \sin(\omega t + \phi)$$

$$\Rightarrow M(\ddot{x}_c + \ddot{x}_{ss}) + c(\dot{x}_c + \dot{x}_{ss}) + k(x_c + x_{ss}) = F_0 \sin \omega t.$$

$$\underbrace{\left[M\ddot{x}_c + c\dot{x}_c + kx_c\right]}_{0} + \underbrace{\left[M\ddot{x}_{ss} + c\dot{x}_{ss} + kx_{ss}\right]}_{F_0 \sin \omega t} = F_0 \sin \omega t$$

$$(\sin \omega t+\phi)\left[kX_0 - m\omega^2 X_0\right] + c\omega X_0 \cos(\omega t+\phi) = F_0 \sin \omega t$$

Let $k - m\omega^2 = a$

$c\omega = b$

$$\sqrt{a^2+b^2}\left(X_0 \left[\frac{a \sin(\omega t+\phi)}{\sqrt{a^2+b^2}} + \frac{b \cos(\omega t+\phi)}{\sqrt{a^2+b^2}}\right]\right) = F_0 \sin \omega t$$

$$\cos\theta = \frac{a}{\sqrt{a^2+b^2}}$$

$$\sin\theta = \frac{b}{\sqrt{a^2+b^2}}$$

$$X_0 \cdot \sqrt{a^2+b^2}\left[\cos\theta \cdot \sin(\omega t+\phi) + \sin\theta \cdot \cos(\omega t+\phi)\right] = F_0 \sin \omega t$$

$$X_0 \cdot \sqrt{a^2+b^2}\left[\sin(\omega t+\phi+\theta)\right] = F_0 \sin \omega t$$

$$\Rightarrow \quad X_0 \cdot \sqrt{a^2+b^2} = F_0 .$$

$$X_0 = \frac{F_0}{\sqrt{a^2+b^2}} .$$

$$\Rightarrow \cancel{\omega t} + \phi + \theta = \cancel{\omega t}$$

$$\phi + \theta = 0$$

$$\phi = -\theta \cdot = -\tan^{-1}\left(\frac{b}{a}\right)$$