Subject : Dynamics of Machine & Vibration

Course :

- → Dynamics of machine & vibration
- → Class timing : 11 – 12 AM
- → No. of classes/week : 3
- → Book :

For vibration –

- VP Singh
- WT Thomson (for practice)
- MIT video lectures
- SS Rattan } Dynamics

(VP Singh, WT Thomson, MIT video lectures } Vibration)

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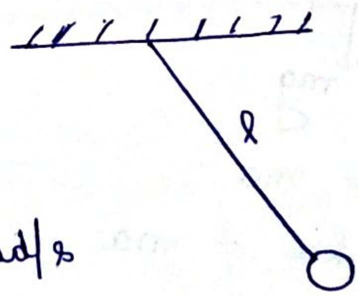
* Natural frequency -

Eg 1 : In pendulum

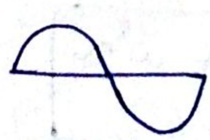
$$T = 2\pi\sqrt{\frac{l}{g}}$$



Ang. frequency $(\omega) = \frac{2\pi}{T}$ rad/s

frequency $(f) = \frac{1}{T}$ Hz. (cycle/s)

Time taken in 1 oscillation (⇄) → time period.

1 complete oscillation  $= 2\pi$

Therefore, 2π rad ⟶ T sec

$\omega = \frac{2\pi}{T}$ ⟶ 1 sec

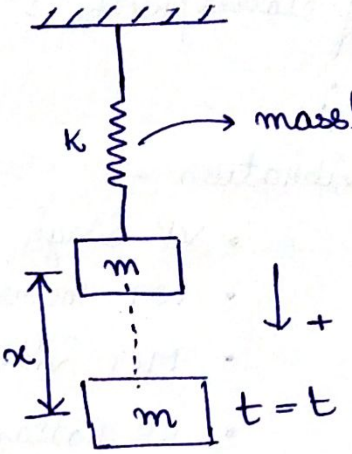
Similarly,

$$1 \text{ cycle} \longrightarrow T \text{ sec}$$

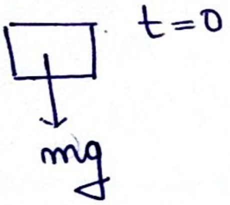
$$f = \frac{1}{T} \text{ cycle} \longrightarrow 1 \text{ sec}$$

Eg : 2 : Spring-mass system

∵ When spring is at its original length, then no force is applied by it.



→ Equation of motion -

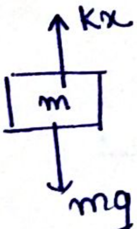


$$\Sigma F = ma$$

$$\cancel{m}g = \cancel{m}a$$

$$\boxed{a = g}$$

At $t = t$,



If $x \to$ ~~exten~~ expansion
then F is upwards $= kx$

If $x \to$ compression
then F is downwards $= k|x|$

$$\Sigma F = ma$$

$$mg - kx = ma$$

$$mg = ma + kx$$

$$g = a + \left(\frac{k}{m}\right) \cdot x$$

$$g = a + \omega^2 x$$

$$\boxed{g = \ddot{x} + \omega^2 x}$$

↳ Differential Eqn of motion

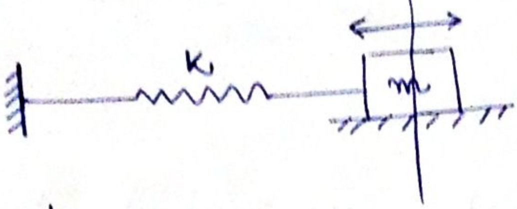
k : spring constant (N/m)

$\left(\frac{k}{m}\right) = \omega^2$
= ang. natural freq.
= rad/s

$$v = \frac{dx}{dt} = \dot{x}$$

$$a = \frac{d^2x}{dt^2} = \ddot{x}$$

$$\boxed{x(t) = A \sin(\omega t + \phi)}$$



$a_y = 0$

$ma_y = 0$

$\Sigma F_y = 0$

$N - mg = 0$

$\boxed{N = mg}$

Mean position :-

Summation of forces = 0

$$\Rightarrow \ddot{x} + \omega^2 g = g$$

By solving, we get-

$$x = A \sin(\omega t + \phi)$$

↳ phase angle (rad)

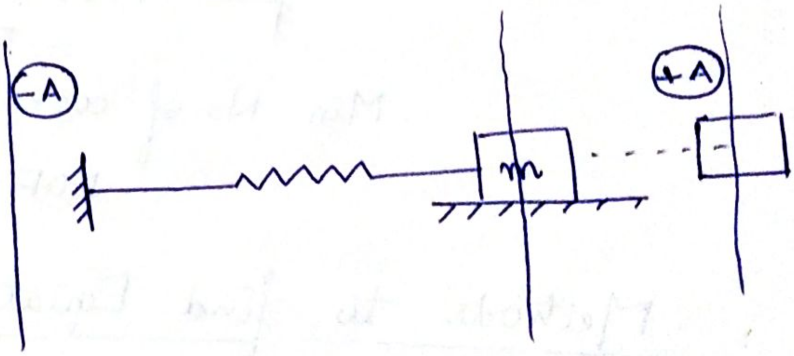
→ time (s)

→ rad/s

→ Amplitude.

Amplitude :-

Amplitude and phase angle depends on initial condition $[x(0), \dot{x}(0)]$



$x(0) = A \sin\phi$

$x = A\omega \cos \omega t + \phi$

$\dot{x}(0) = A\omega \cos\phi$

$\omega = \sqrt{\frac{k}{m}}$

$$\Rightarrow x^2(0) + \dot{x}^2(0) = (\sin^2\phi + \cos^2\phi)A^2 = A^2$$

$$A = \sqrt{x^2(0) + \dot{x}^2(0)}$$

where, $x(0) \rightarrow$ Initial displacement

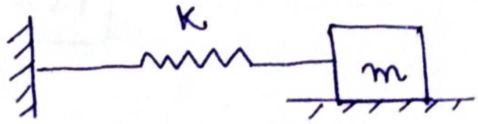
$\dot{x}(0) \rightarrow$ Initial velocity

$$\sin\phi = \frac{x(0)}{\sqrt{x(0) + \frac{\dot{x}(0)}{\omega^2}}}$$

Degree of freedom -

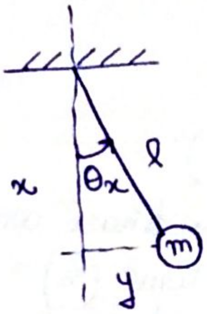
Min^m. number of coordinates required to specify position of a given dynamical system.

Ex. 1 -



DOF = 1

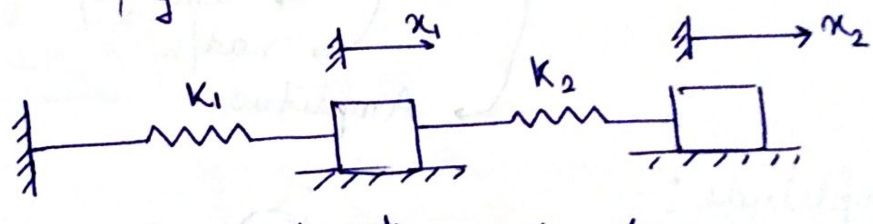
Ex. 2 -



DOF = 1

$x^2 + y^2 = l^2$

Ex. 3 -



Min. No. of coordinates $= x_1, x_2$

DOF = 2

Methods to find Equation of motion -

→ Newtonian method

→ Energy method

→ Lagrangian method.