WHAT IS MATRIX ?

DEFINATION

a set of numbers arranged in rows and columns so as to form a rectangular array. the numbers are called the elements, or entries of the matrix.

Order of a Matrix

5 Columns

	↓	↓	↓	↓	↓
→	5	8	7	2	5
→	9	2	3	1	0
4 Rows →	7	2	0	-6	4
→	4	-7	9	5	3

VARIOUS TYPES OF MATRICES

ROW MATRIX:

A matrix having a single row and any number of columns it is called row matrix.

Eg ; [2 3 7 6]

COLUMN MATRIX :

A matrix having one column and any number of rows is called a column matrix.

Eg:

$$A = \begin{bmatrix} -1 \\ 2 \\ -4 \\ 5 \end{bmatrix}$$

Null matrix;

A zero matrix is a matrix that has all its elements equal to zero. Since a zero matrix contains only zeros as its elements, therefore, it is also called a null matrix.

Eg;

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

square matrix:

a matrix that has an equal number of rows and columns. In mathematics, m × m matrix is called the square matrix of order m. If we multiply or add any two square matrices, the order of the resulting matrix remains the same.

Eg:

$$\begin{bmatrix} 9 & 13 & 5 & 2 \\ 1 & 11 & 7 & 6 \\ 3 & 7 & 4 & 1 \\ 6 & 0 & 7 & 10 \end{bmatrix}$$

Diagonal matrix :

A square matrix in which every element except the principal diagonal elements is zero is called a Diagonal Matrix.

A square matrix D = $[d_{ij}]_{n \times n}$ will be called a diagonal matrix if d_{ij} 0, whenever i is not equal to j.

Diagonal Matrix

$$A = \begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 \\ 0 & 0 & 0 & 0 & a_{55} \end{bmatrix}_{5\times 5}$$

scalar matrix:

a square matrix in which all of the principal diagonal elements are equal and the remaining elements are zero. It is a special case of a diagonal matrix and can be obtained when an identity matrix is multiplied by a constant numeric value.

eg

Constant × Identity Matrix = Scalar Matrix

$$k \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$$

unit matrix or identity matrix:

a square matrix whose all elements are zeros except the main diagonal elements which are ones .

eg

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2 x 2 3 x 3

Symmetric Matrix:

A symmetric matrix is a matrix that is equal to its transpose.

They contain three properties, including:
Real eigenvalues, eigenvectors corresponding to the eigenvalues that are orthogonal and the matrix must be diagonalizable

$$A^T = A$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 5 \end{bmatrix}$$

Skew symmetric matrix:

If the transpose of a matrix is equal to the negative of itself, the matrix is said to be skew symmetric.
This means that for a matrix to be skew symmetric, A'=-A.

$$A^T = -A$$

$$\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$$

Triangular matrix

Triangular matrices have the same number of rows as they have columns; that is, they have n rows and n columns.

A matrix U is an upper triangular matrix if its nonzero elements are found only in the **upper triangle of the matrix**, including the main diagonal; that is: $u_{ij} = 0$ if $i > j$.

matrix which has elements above the main diagonal as zero is called **a lower triangular matrix**

Upper Triangular Matrix

$$U = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{24} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}_{4\times4}$$

Lower Triangular Matrix

$$L = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}_{4\times4}$$

Transpose matrix

the transpose of a matrix is found by interchanging its rows into columns or columns into rows. The transpose of the matrix is denoted by using the letter "T"

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}^T \longrightarrow \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

orthogonal matrix

the product of a square matrix and its transpose gives an identity matrix, then the square matrix is known as an orthogonal matrix

$$AA' = \frac{1}{9}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}$$

$$AA' = \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow AA' = I$$

Unitary matrix

A unitary matrix is a complex square matrix whose columns (and rows) are orthogonal.

It has the remarkable property that its inverse is equal to its conjugate transpose.

$$AA^*$$

$$= \begin{bmatrix} 1-i & 1+i \\ 1+i & 1-i \end{bmatrix} \begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix}$$

$$= \begin{bmatrix} 1-i^2+1-i^2 & 1+i^2-2i+1+i^2+2i \\ 1+i^2+2i+1+i^2-2i & 1-i^2+1-i^2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

Hermitian matrix

A square matrix $A = [a_{ij}]_{n \times n}$ such that $A^* = A$, where A^* is the conjugate transpose of A; that is, if for every $a_{ij} \in A$, a i j — = a i j. ($1 \leq i, j \leq n$), then A is called a Hermitian Matrix.

$$A = \begin{pmatrix} 3i & 2+4i \\ -2+4i & -7i \end{pmatrix} \rightarrow A^H = \begin{pmatrix} -3i & -2-4i \\ 2-4i & 7i \end{pmatrix}$$

Skew hermitian matrix

A complex square matrix is said to be a skew-Hermitian matrix if the conjugate transpose matrix is equal to the negative of the original matrix. A square matrix "$A_{n\times n}$ = [a_{ij}] is said to be a Hermitian matrix if A^H = -A, where A^H is the conjugate transpose of matrix A

$$\begin{bmatrix} 0 & 2-j \\ -2-j & 0 \end{bmatrix}^H = \left(\begin{bmatrix} 0 & 2-j \\ -2-j & 0 \end{bmatrix}^* \right)^T = -\begin{bmatrix} 0 & 2-j \\ -2-j & 0 \end{bmatrix}^H$$

Idempotent matrix

A matrix M is said to be an idempotent matrix if $M^2 = M$. Further every identity matrix can be termed as an idempotent matrix. The idempotent matrix is a singular matrix and can have non-zero elements.

$$\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} * \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

periodic matrix.

When a square matrix where the number of rows and the number of columns are equal, satisfies a relation A k +1 = A for some positive integer k, then it is called a periodic matrix.

$$B = \begin{pmatrix} 2 & 5 & 14 \\ 1 & 3 & 8 \\ -1 & -2 & -6 \end{pmatrix} \rightarrow B^4 = B \rightarrow B \text{ is periodic of order } 3$$

Nilpotent Matrix

A nilpotent matrix is a square matrix A such that $A^k = 0$. Here k is called the index or exponent of the matrix, and 0 is a null matrix with the same order as that of matrix A

Show that $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}$ is nilponent .

$$A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2-3 & 2+4-6 & 3+6-9 \\ 1+2-3 & 2+4-6 & 3+6-9 \\ -1-2+3 & -2-4+6 & -3-6+9 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ Then A is nilponent <u>Ans</u>

involuntary matrix

a square matrix that is its own inverse. That is, multiplication by the matrix A is an involution if and only if $A^2 = I$, where I is the $n \times n$ identity matrix. Involuntary matrices are all square roots of the identity matrix.

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$BB = B^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So, B is an involutory matrix.

Properties of a matrix:

The important properties of a matrix are:

1. Properties of matrix addition:

The matrix addition is the addition of corresponding elements of the matrices.

For the matrices A and B,

- Commutative property: $A+B=B+A$
- Associative property: $A+(B+C)=(A+B)+C$
- Additive identity: $A+0=0+A=A$
- Additive inverse: $A+(-A)=(-A)+A=0$

2. Properties of matrix multiplication

The matrix multiplication is a product of two matrices that produce a single matrix.

For the matrices A, B and C

- Associative property: $(AB)C = A(BC)$
- Distributive property: $A(B+C) = AB + AC$
- Multiplicative identity: $AI = IA = A$, where I is an identity matrix
- Multiplicative property of zero: $A0 = 0A = 0$

3. Properties of scalar multiplication

If A is a matrix and k any constant, then the product of a constant with the matrix is equal to the product of the constant and all the elements of a matrix.

- Commutative property: $kA = Ak$
- Distributive property: $k(A+B) = kA + kB$ and $(k+m)A = kA + mA$, where $k,\ m$ are scalars
- Associative property: $k(mA) = (km)A$

4. Properties of Transpose Matrix

Transpose matrix is obtained by interchanging the rows and columns. The transpose of matrix A is denoted as A^T or $A\prime$.

- $\left(A^T\right)^T = A$
- $(A+B)^T = A^T + B^T$
- $(A \times B)^T = B^T \times A^T$
- $(kA)^T = kA^T$

5. Properties of Inverse Matrix.

The inverse of a matrix A is denoted by

$A^{-1} = \frac{1}{|A|} \cdot adj.(A)$, where $adj.(A)$ is the adjoint matrix and $|A|$ is the determinant of the matrix A.

- $(A^{-1})^{-1} = A$
- $(A \times B)^{-1} = B^{-1} \times A^{-1}$
- $(A^{T})^{-1} = (A^{-1})^{T}$