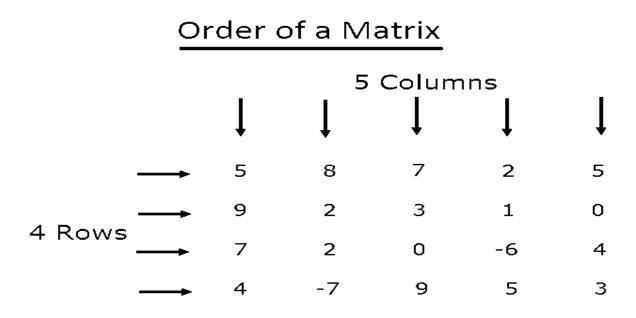
WHAT IS MATRIX ?

DEFINATION

a set of numbers arranged in rows and columns so as to form a rectangular array. the numbers are called the elements, or entries of the matrix.



VARIOUS TYPES OF MATRICES

ROW MATRIX:

A matrix having a single row and any number of columns it is called row matrix. Eg; [2 3 7 6] **COLUMN MATRIX :**

A matrix having one column and any number of rows is called a column matrix.

Eg:

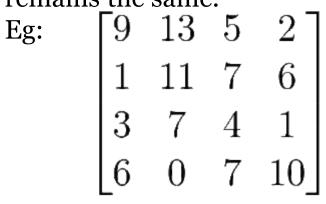
Null matrix;

A zero matrix is a matrix that has all its elements equal to zero. Since a zero matrix contains only zeros as its elements, therefore, it is also called a null matrix.

Eg;
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

square matrix:

a matrix that has an equal number of rows and columns. In mathematics, m × m matrix is called the square matrix of order m. If we multiply or add any two square matrices, the order of the resulting matrix remains the same.



Diagonal matrix:

A square matrix in which every element except the principal diagonal elements is zero is called a **Diagonal Matrix.**

A square matrix $D = [d_{ij}]_{n \times n}$ will be called a diagonal matrix if d_{ij} 0, whenever i is not equal to j.

Diagonal Matrix

$$A = \begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 \\ 0 & 0 & 0 & 0 & a_{55} \end{bmatrix}$$

scalar matrix:

a square matrix in which all of the principal diagonal elements are equal and the remaining elements are zero. It is a special case of a diagonal matrix and can be obtained when an identity matrix is multiplied by a constant numeric value. eg

Constant × Identity Matrix = Scalar Matrix

$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{vmatrix}$

unit matrix or identity matrix:

a square matrix whose all elements are zeros except the main diagonal elements which are ones .

eg

$$\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$2 \times 2 \qquad 3 \times 3$$

Symmetric Matrix:

A symmetric matrix is a matrix that is equal to its transpose.

They contain three properties, including:

Real eigenvalues, eigenvectors corresponding to the eigenvalues that are orthogonal and the matrix must be diagonalizable

$$A^{T} = A$$

1 1 -1
1 2 0
-1 0 5

Skew symmetric matrix:

If the transpose of a matrix is equal to the negative of itself, the matrix is said to be skew symmetric. This means that for a matrix to be skew symmetric, A'=-A.

$$A^{T} = -A$$

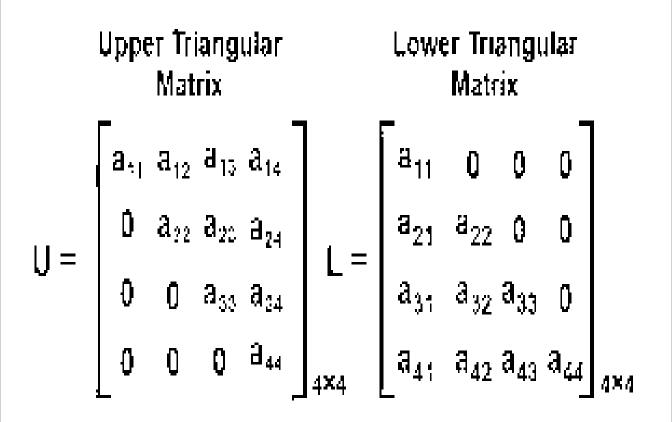
 $0 \quad 1 \quad -2$
 $-1 \quad 0 \quad 3$
 $2 \quad -3 \quad 0$

Triangular matrix

Triangular matrices have the same number of rows as they have columns; that is, they have n rows and n columns.

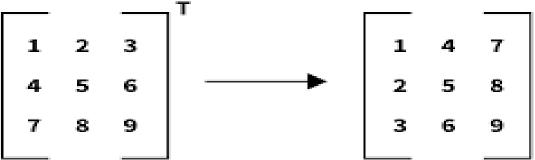
A matrix U is an upper triangular matrix if its nonzero elements are found only in the **upper triangle of the matrix**, including the main diagonal; that is: $u_{ij} = 0$ if i > j.

matrix which has elements above the main diagonal as zero is called **a lower triangular matrix**



Transpose matrix

the transpose of a matrix is found by interchanging its rows into columns or columns into rows. The transpose of the matrix is denoted by using the letter "T"



orthogonal matrix

the product of a square matrix and its transpose gives an identity matrix, then the square matrix is known as an orthogonal matrix

$$AA' = \frac{1}{9} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}$$

$$AA' = \frac{1}{9} \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow AA' = I$$

Unitary matrix

A unitary matrix is a complex square matrix whose columns (and rows) are orthogonal. It has the remarkable property that its inverse is equal to its conjugate transpose.

$$AA^{*} = \begin{bmatrix} 1-i & 1+i \\ 1+i & 1-i \end{bmatrix} \begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix}$$
$$= \begin{bmatrix} 1-i^{2}+1-i^{2} & 1+i^{2}-2i+1+i^{2}+2i \\ 1+i^{2}+2i+1+i^{2}-2i & 1-i^{2}+1-i^{2} \end{bmatrix}$$
$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

Hermitian matrix

A square matrix $A = [a_{ij}]_{n \times n}$ such that $A^* = A$, where A^* is the conjugate transpose of A; that is, if for every $a_{ij} \in A$, a ij - = a ij. ($1 \le i, j \le n$), then A is called a Hermitian Matrix.

$$A = \begin{pmatrix} 3i & 2+4i \\ -2+4i & -7i \end{pmatrix} \longrightarrow A^H = \begin{pmatrix} -3i & -2-4i \\ 2-4i & 7i \end{pmatrix}$$

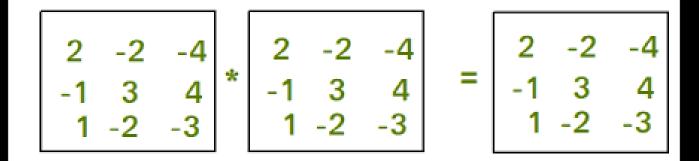
Skew hermitian matrix

A complex square matrix is said to be a skew-Hermitian matrix if the conjugate transpose matrix is equal to the negative of the original matrix. A square matrix " $A_{n\times n} = [a_{ij}]$ is said to be a Hermitian matrix if $A^H = -A$, where A^H is the conjugate transpose of matrix A

$$\begin{bmatrix} 0 & 2-j \\ -2-j & 0 \end{bmatrix}^{H} = \left(\begin{bmatrix} 0 & 2-j \\ -2-j & 0 \end{bmatrix}^{*} \right)^{T} = - \begin{bmatrix} 0 & 2-j \\ -2-j & 0 \end{bmatrix}^{H}$$

<u>Idempotent matrix</u>

A matrix M is said to be an idempotent matrix if $M^2 = M$. Further every identity matrix can be termed as an idempotent matrix. The idempotent matrix is a singular matrix and can have non-zero elements.



<u>periodic matrix.</u>

When a square matrix where the number of rows and the number of columns are equal, satisfies a relation A k + 1 = A for some positive integer k, then it is called a periodic matrix.

$$B = \begin{pmatrix} 2 & 5 & 14 \\ 1 & 3 & 8 \\ -1 & -2 & -6 \end{pmatrix} \rightarrow B^4 = B \rightarrow B \text{ is periodic of order 3}$$

<u>Nilpotent Matrix</u>

A nilpotent matrix is a square matrix A such that A^k = 0. Here k is called the index or exponent of the matrix, and 0 is a null matrix with the same order as that of <u>matrix A</u>

Show that
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}$$
 is nilponent.

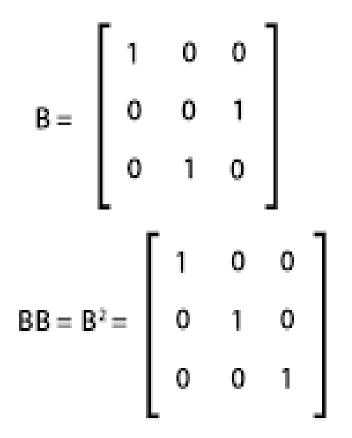
$$A^{2} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2-3 & 2+4-6 & 3+6-9 \\ 1+2-3 & 2+4-6 & 3+6-9 \\ 1+2-3 & 2+4-6 & 3+6-9 \\ -1-2+3 & -2-4+6 & -3-6+9 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
 Then A is nilponent Ans

<u>involuntary matrix</u>

a square matrix that is its own inverse. That is, multiplication by the matrix A is an involution if and only if A² = I, where I is the n × n identity matrix. Involuntary matrices are all square roots of the identity matrix.



So, B is an involutory matrix.

Properties of a matrix:

The important properties of a matrix are:

1. Properties of matrix addition:

The matrix addition is the addition of corresponding elements of the matrices.

For the matrices A and B,

- Commutative property: A + B = B + A
- Associative property: A + (B + C) = (A + B) + C
- Additive identity: A + 0 = 0 + A = A
- Additive inverse: A + (-A) = (-A) + A = 0

2. Properties of matrix multiplication

The matrix multiplication is a product of two matrices that produce a single matrix.

For the matrices ${\cal A}, {\cal B}$ and ${\cal C}$

- Associative property: (AB)C = A(BC)
- Distributive property: A(B+C) = AB + AC
- Multiplicative identity: AI = IA = A , where I is an identity matrix
- Multiplicative property of zero: A0 = 0A = 0

3. Properties of scalar multiplication

If A is a matrix and k any constant, then the product of a constant with the matrix is equal to the product

of the constant and all the elements of a matrix.

- Commutative property: kA = Ak
- Distributive property: k(A + B) = kA + kB and (k+m)A = kA + mA, where k, m are scalars
- Associative property: k(mA) = (km)A

4. Properties of Transpose Matrix

Transpose matrix is obtained by interchanging the rows and columns. The transpose of matrix A is denoted as A^T or AI.

•
$$(A^T)^T = A$$

•
$$(A+B)^T = A^T + B^T$$

•
$$(A \times B)^T = B^T \times A^T$$

•
$$(kA)^T = kA^T$$

5. Properties of Inverse Matrix.

The inverse of a matrix A is denoted by

 $A^{-1} = rac{1}{|A|} \cdot adj.(A)$, where adj.(A) is the adjoint

matrix and |A| is the determinant of the matrix A.

•
$$\left(A^{-1}\right)^{-1} = A$$

•
$$(A \times B)^{-1} = B^{-1} \times A^{-1}$$

•
$$\left(A^{T}\right)^{-1} = \left(A^{-1}\right)^{T}$$